Acoustics II

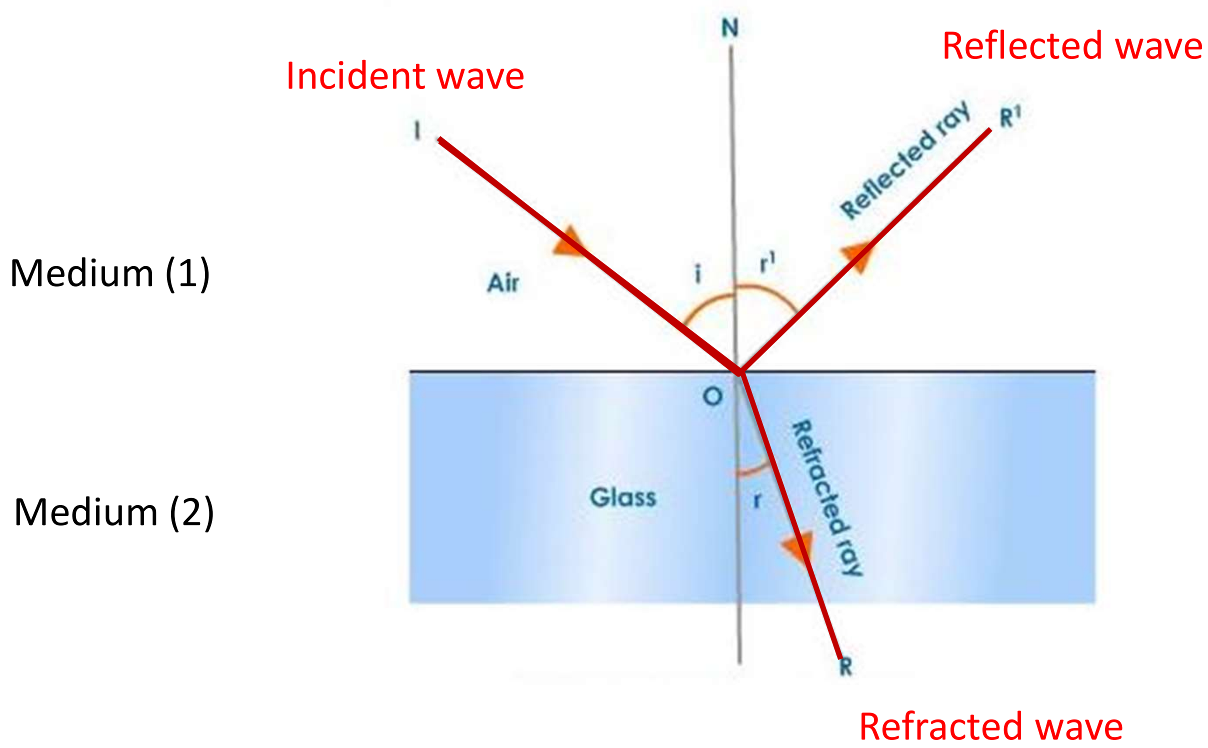
Elements of wave optics

All the concepts addressed here remain valid in case of light (electromagnetic wave)

(1) Reflexion and refraction of (sound) waves

When a (elastic) wave reaches the boundary surface separating two (elastic) media, the incident wave is divided into two components:

- Reflected wave (returns into same medium)
- Refracted wave (passes into the second medium)



Acoustic impedance

Physical quantity describing the opposition of a medium against the propagation of waves through it

$$Z = \rho v \qquad v = \sqrt{\frac{B}{\rho}}$$

$[Z]=kg/m^3 * m/s = kg/(m^2 s) = 1$ rayl (Rayleigh)

Air:	Z=430 rayl
Expanded polystyrene:	$6\ 10^3$ rayl
Full brick:	$2.4\ 10^6$ rayl
concrete:	$7.2\ 10^6$ rayl
wood:	$2.7\ 10^6$ rayl

The **reflexion** and **refraction** phenomena appear at the boundary between two media with different acoustic impedances Z1 and Z2

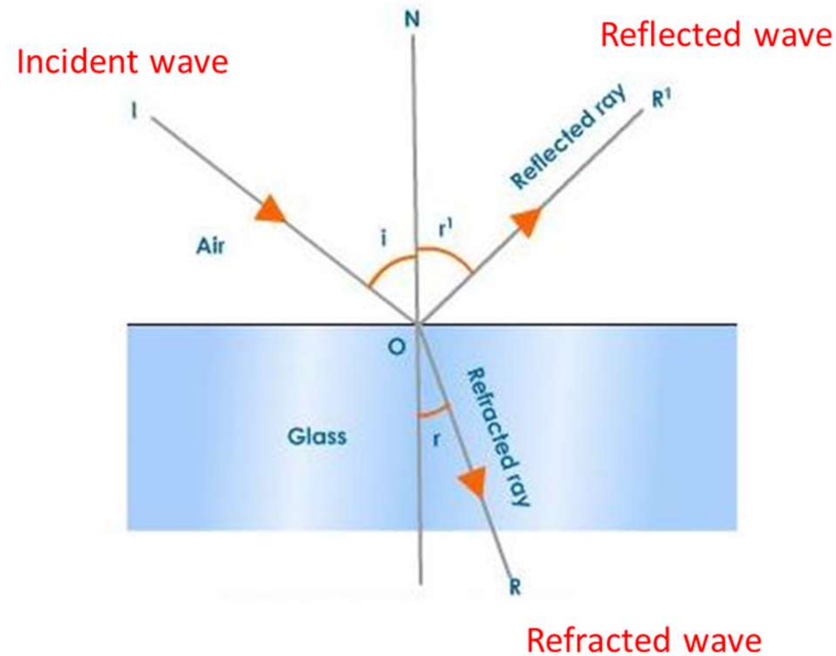
The principle of Fermat

The waves propagate between two points A and B along a path that requires the minimum time,

Upon geometrical considerations, based on the principle of Fermat, one can deduce and enounce the laws of reflexion and refraction.

The laws of reflexion

The laws of refraction



(1) The incident wave, the reflected wave and the normal at the surface are in the same plane.

(2) The incident angle is equal to the reflection angle: **i = i'**

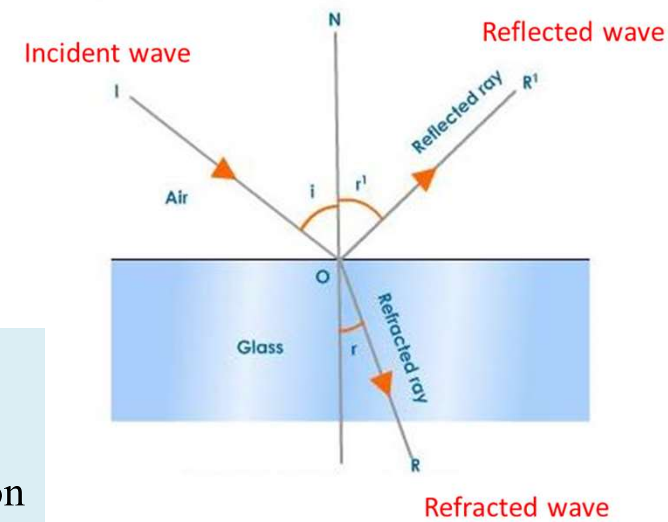
(1) The incident wave, the refracted wave and the normal at the surface are in the same plane.

(2) The angles of incidence and refraction satisfy the following equation:

$$\frac{\sin i}{\sin r} = \frac{v_1}{v_2}$$

Transmission (T) and reflexion (R) coefficients

Quantify the amount of energy that passes from the medium (1) to the medium (2)



$$R = \frac{\text{Intensity of the reflected wave}}{\text{Intensity of the incident wave}} = \frac{I_r}{I_i} = \frac{A_r^2}{A_i^2}$$

$(I \sim \omega^2 A^2)$ frequency ω conserved by reflexion and refraction

$$T = \frac{\text{Intensity of the transmitted wave}}{\text{Intensity of the incident wave}} = \frac{I_t}{I_i} = \frac{A_t^2}{A_i^2}$$

$R + T = 1$ From energy conservation

…after calculations:

$$R = \left(\frac{Z_1 - Z_2}{Z_1 + Z_2}\right)^2$$

$$T = \frac{4 Z_1 Z_2}{(Z_1 + Z_2)^2}$$

- R decreases when Z1 → Z2
- R large when Z1 and Z2 are very different

Medium 1	Medium 2	R	T
Air	Water	0.999	0.0001
Water	Steel	0.875	0.125
Air	Wood	0.9	0.1
Air	Curtain	0.2	0.8

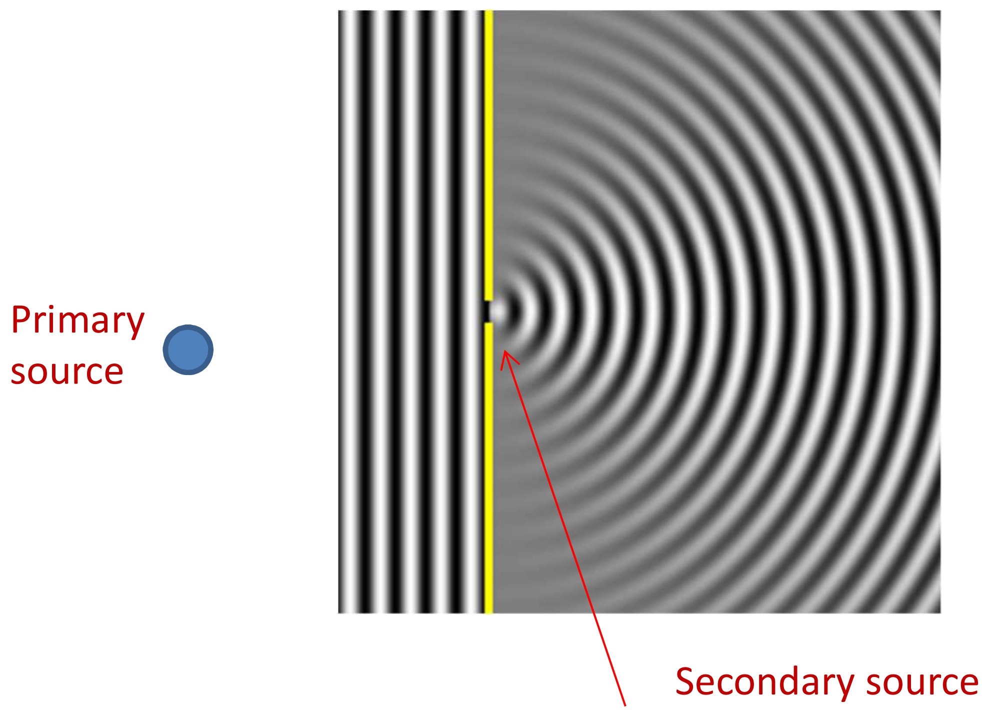
(2) Diffraction of waves

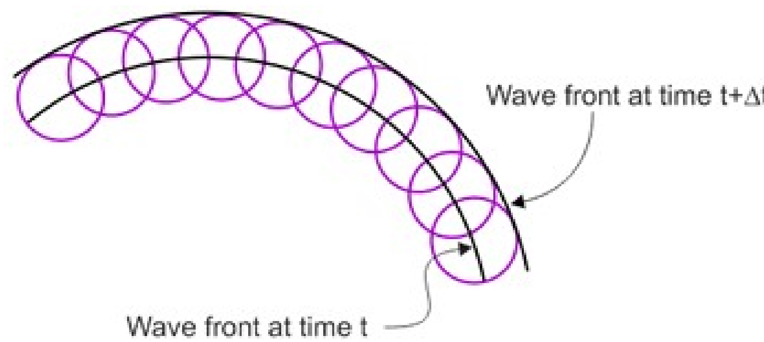
The phenomenon of diffraction consists in the penetration of waves in the geometrical shadow of small obstacles whose dimension is comparable with the wavelength of the respective wave.

The obstacle may be any object with specific shape (ex. Slit/Hole in opaque wall…).

The explanation and related properties based on **Huygens-Fresnel** principle:

every point on a wave-front is itself the source of (secondary) spherical wavelets





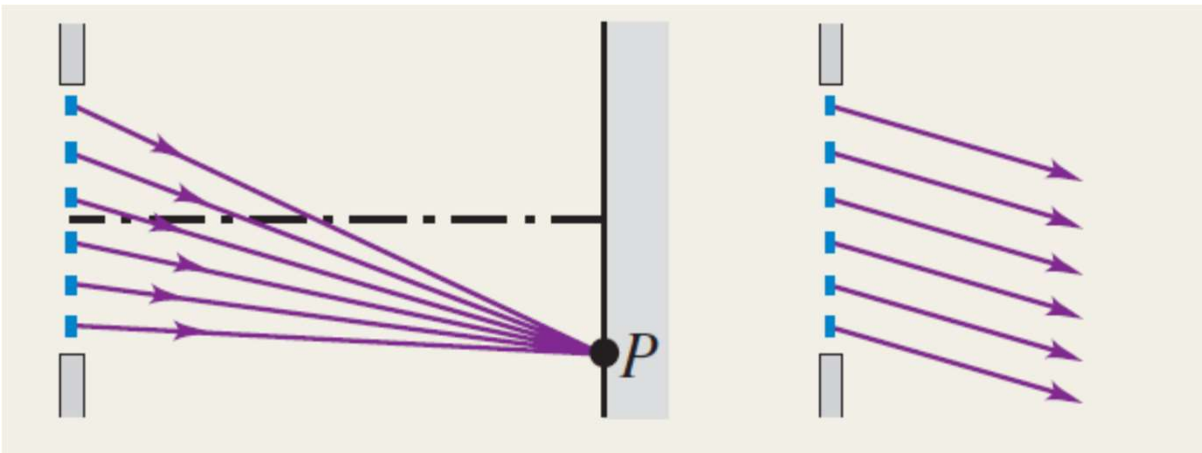
After a while (Δt), the new position of the wave front will be constituted by the surface tangent to all this secondary wave fronts.

Fresnel and Fraunhofer diffraction:

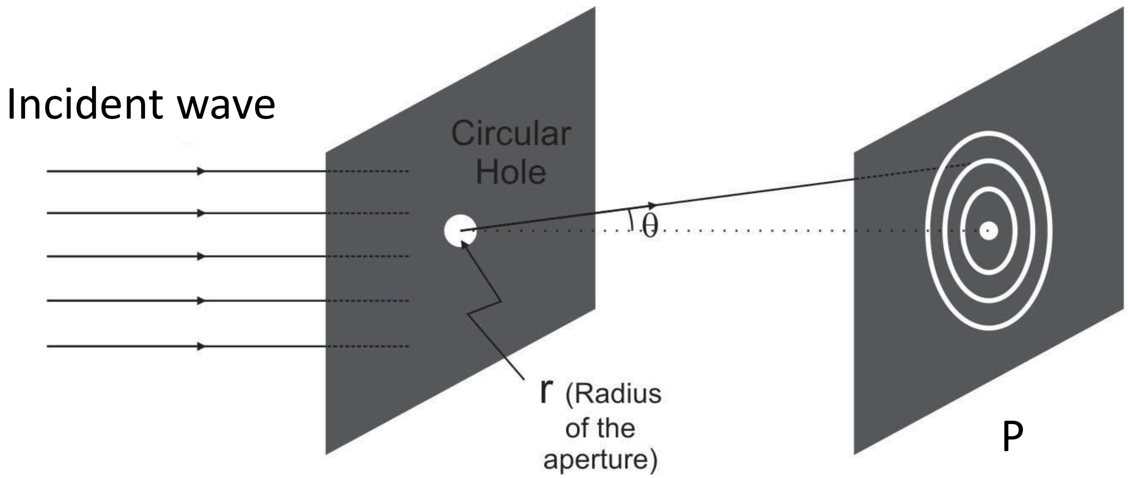
- Diffraction occurs when waves (light) pass through an aperture or around an edge.
- When the source and the observer are so far away from the obstructing surface that the outgoing rays can be considered parallel, it is called **Fraunhofer diffraction**.
- When the source or the observer is relatively close to the obstructing surface, it is **Fresnel diffraction**.

Fresnel (near-field) diffraction

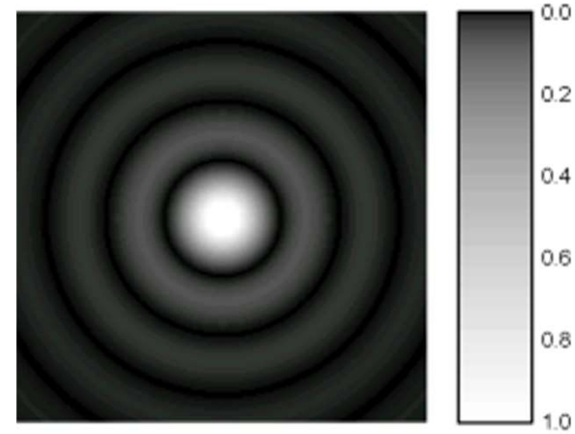
Fraunhofer (far field) diffraction

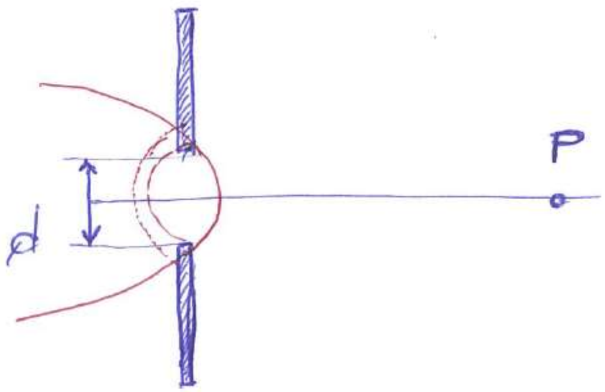


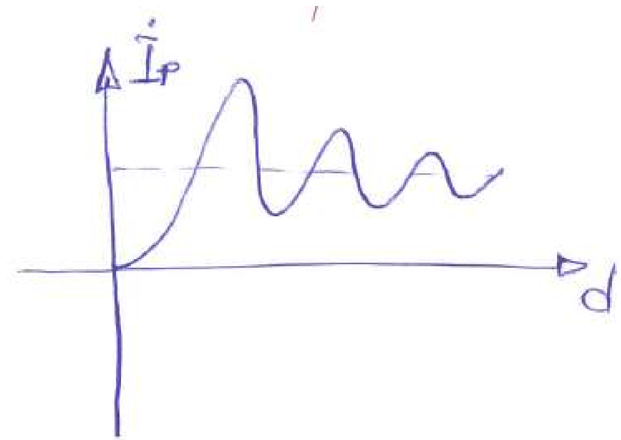
Fresnel diffraction on a circular aperture

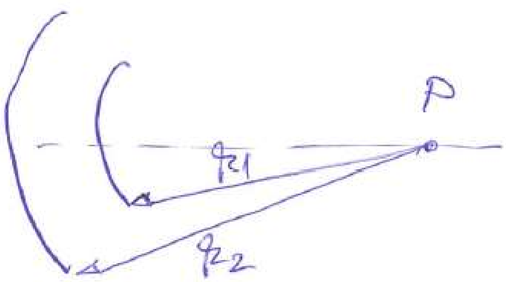


Diffraction pattern:
Airy disks



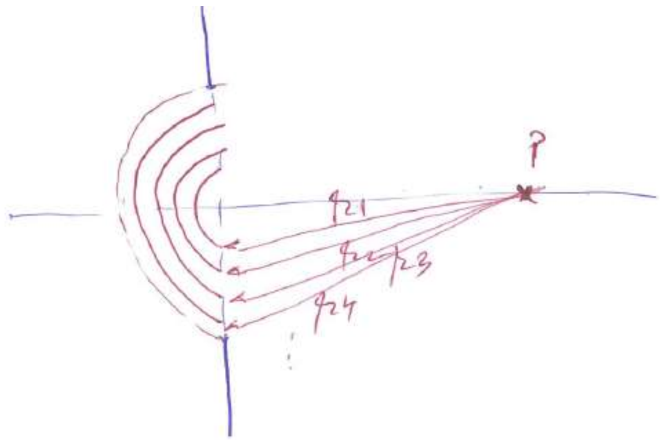






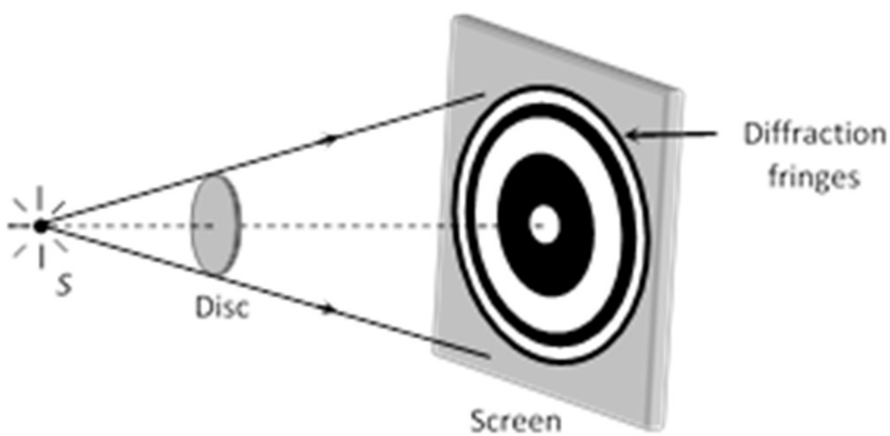
Fresnel zones → circles centered in P whose radius differs by $\frac{\lambda}{2}$

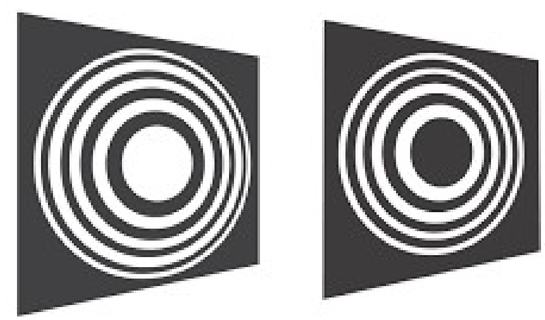
Due to the Fresnel diffraction on a circular hole in the point P we will obtain maximum or minim intensity depending if the hole opening lives unobstructed a pair or impair number of Fresnel zones.



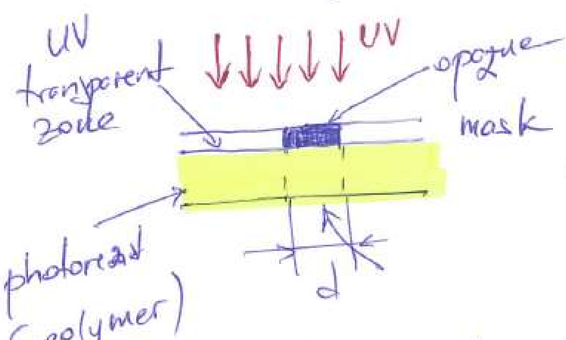
$$r_{i+1} - r_i = \frac{\lambda}{2}$$

Similar situation happens for circular obstruction with D~λ

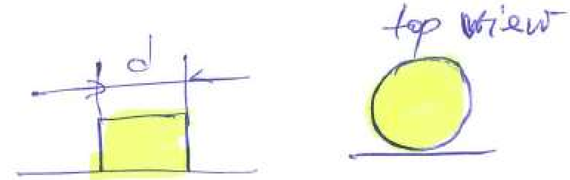




Limitation due to diffraction in optical lithography



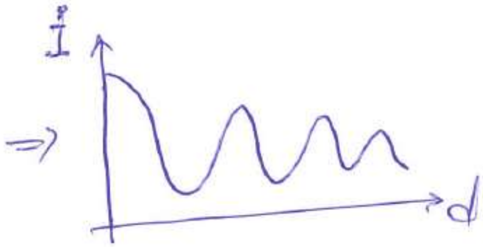
$\Longrightarrow$



mask pattern transposed on photoresist

UV-breaks bonds => soluble in developer of exposed zones

But when $d \sim \lambda$ =>



⇓ one can have maximum of exposure in the middle of the "opaque" zone of the mask

=> doughnut shapes (circles with center holes)

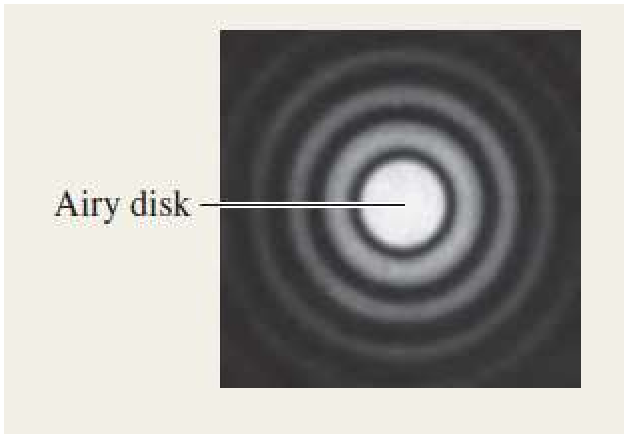
 instead of 

⇓

limitations of UV resolution (ultimate) by diffraction

Circular apertures and resolving power:

The diffraction pattern from a circular aperture of diameter D consists of a central bright spot, called the Airy disk, and a series of concentric dark and bright rings.



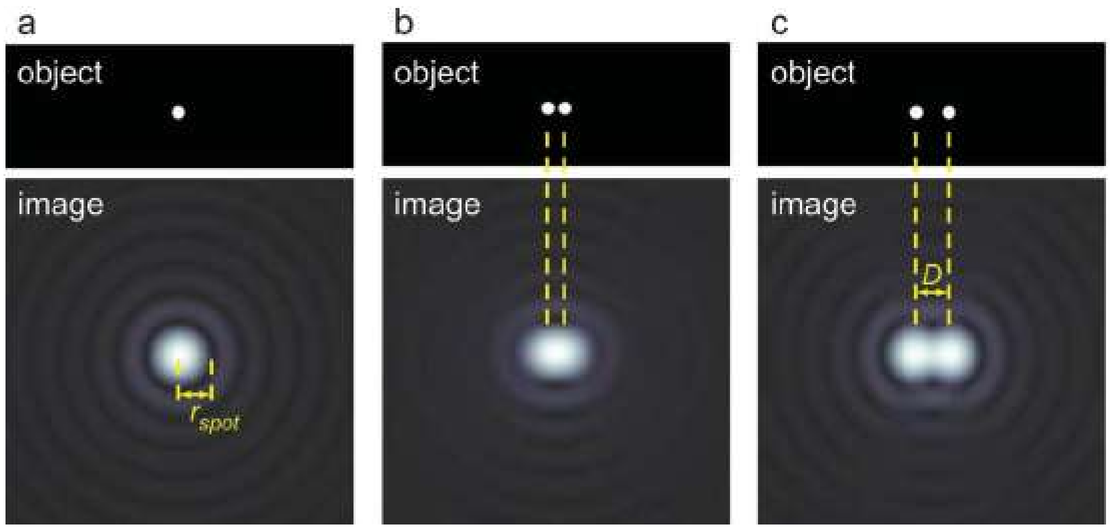
$$\sin\theta_1 = 1.22\frac{\lambda}{D} \qquad (*)$$

Eq. (*) gives the angular radius θ_1 of the first dark ring, equal to the angular size of the Airy disk.

Diffraction sets the ultimate limit on resolution (image sharpness) of optical instruments.

According to Rayleigh’s criterion, two point objects are just barely resolved when their angular separation θ is given by Eq. (*)

Conceptual-model illustration of the Rayleigh criterion
for the minimum resolvable distance, D , between two small point-like objects.

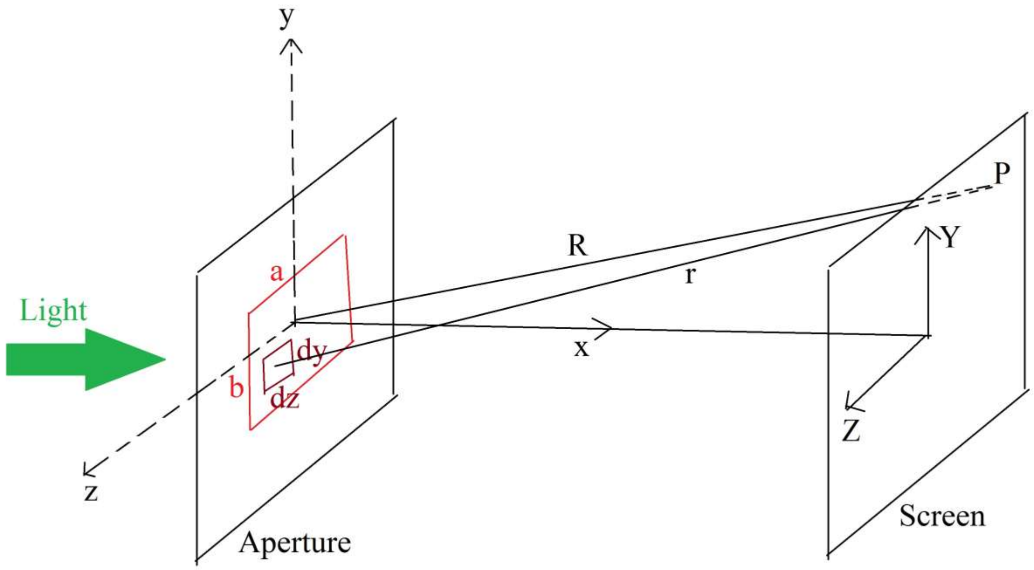


(a): The diffraction-limited predicted image (lower panel) of a small object (upper panel) consisting of a bright sphere.

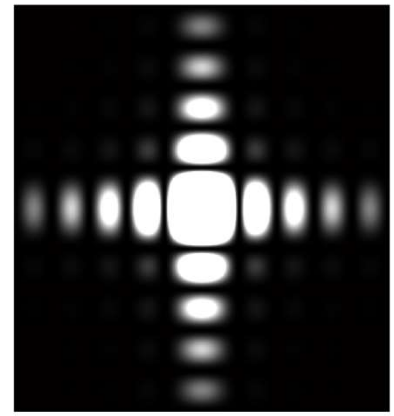
(b): The corresponding image of two such small objects, unresolved case.

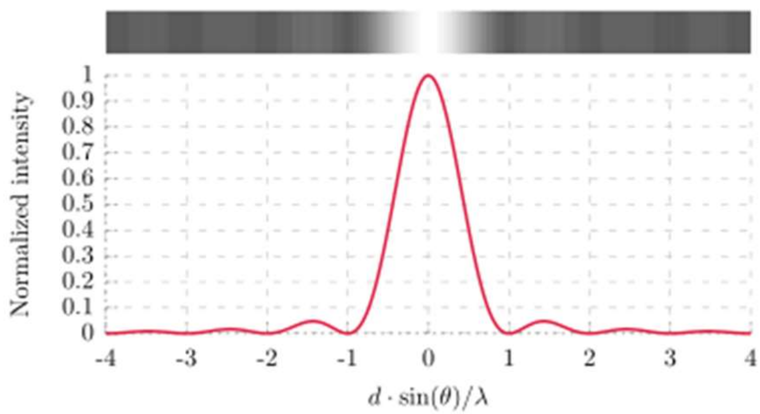
(c): Same as in (b) but the limit where the spacing, D , is just large enough for resolving the two small spheres. The radius of the imaged r_{spot} in (a) is equal to the minimum resolvable distance, D , in (c). The situation in (c) illustrates the Rayleigh criterion. This criterion is defined as when the first intensity minima from the center of the imaged spot from one of the objects coincides with the intensity maxima of the other object, i.e. , when D = r_{spot}.

Fraunhofer diffraction on a rectangular aperture



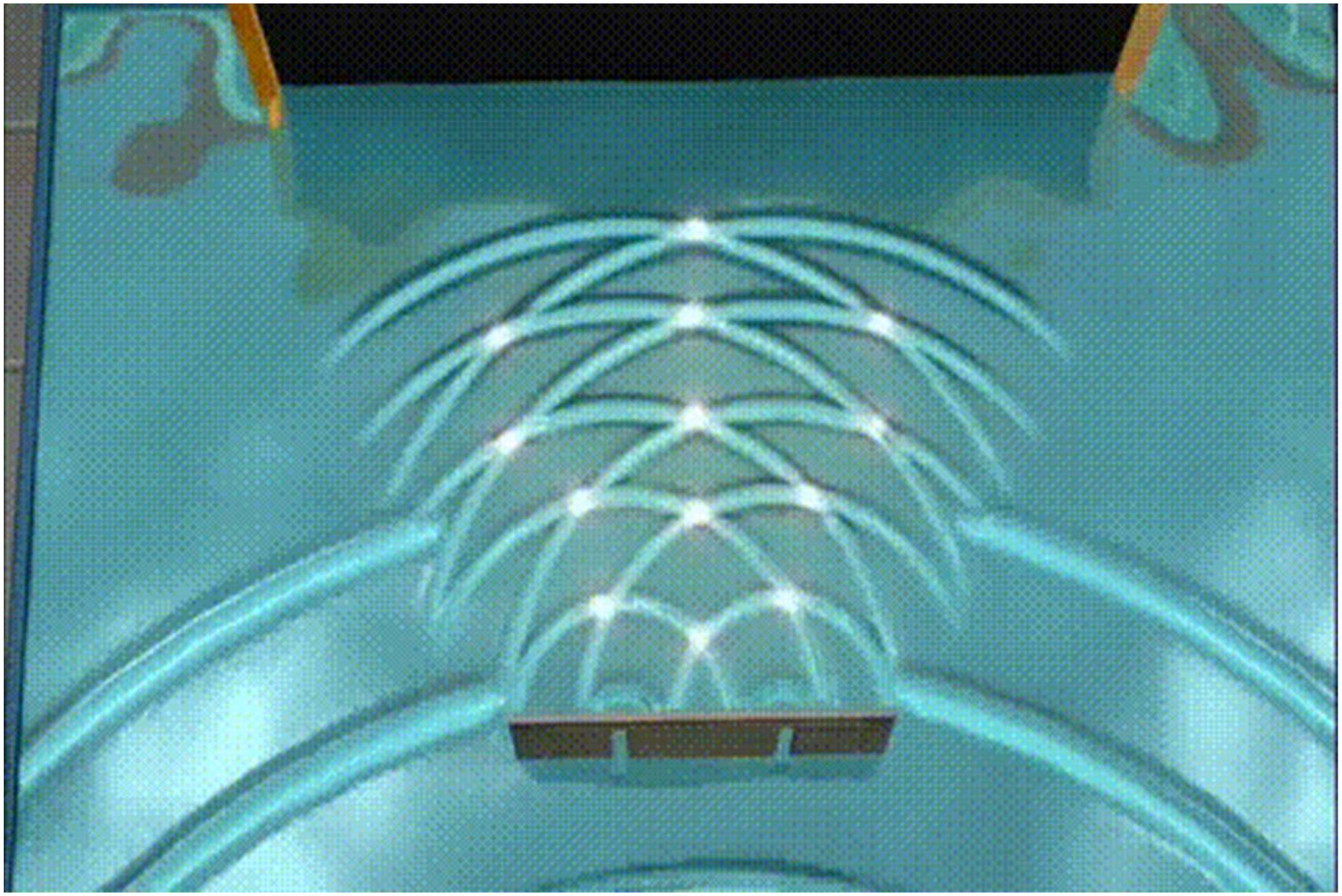
Fraunhofer diffraction pattern

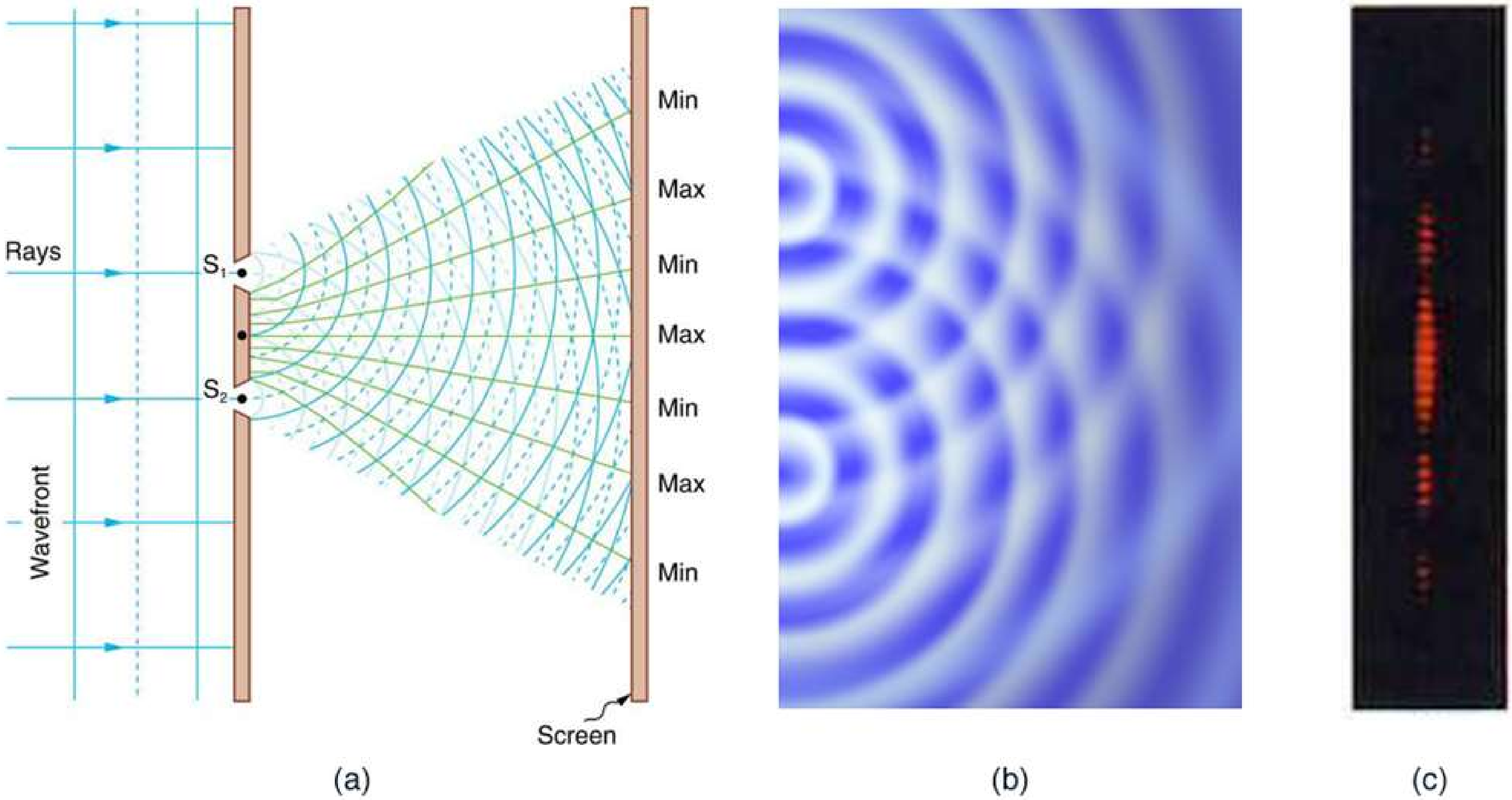




Fringes parallel to the edges of the rectangular aperture

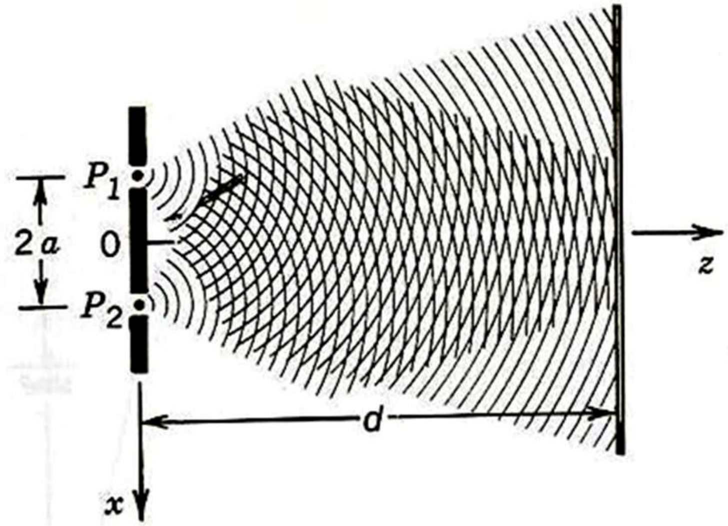
(3) Interference of waves

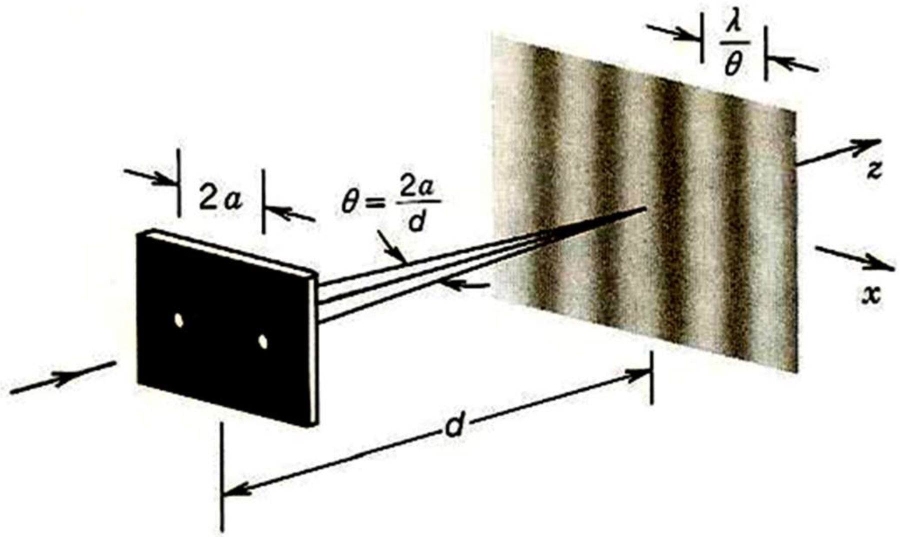


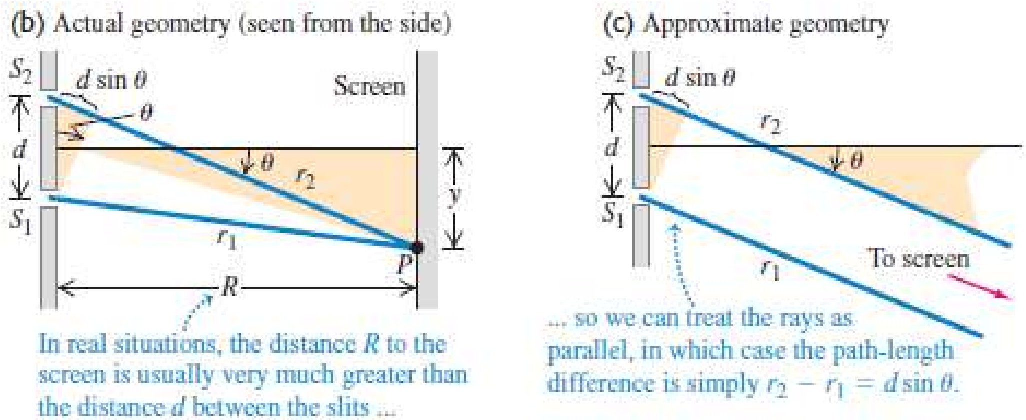

Rays
Wavefront
S_1
S_2
Min
Max
Min
Max
Min
Max
Min
Screen
(a)
(b)
(c)

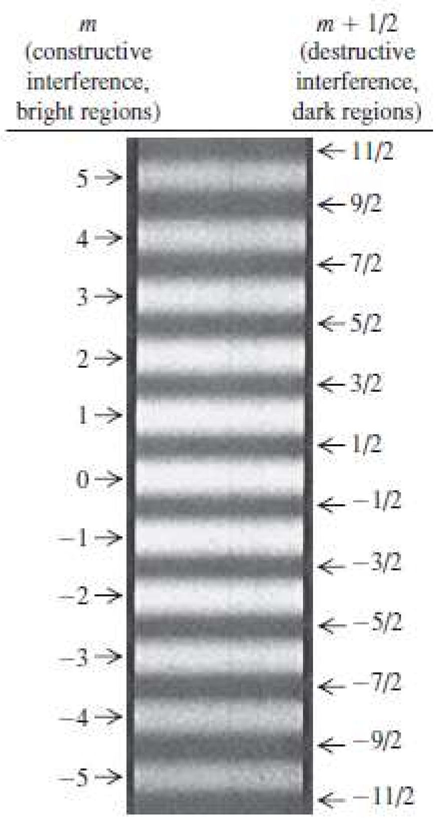
Young two slit experiment

Thomas Young
(1773 - 1829)









$$d\sin\theta = m\lambda \qquad (m = 0, \pm1, \pm2, \ldots)$$ (constructive interference, two slits)

$$d\sin\theta = \left(m + \tfrac{1}{2}\right)\lambda \qquad (m = 0, \pm1, \pm2, \ldots)$$ (destructive interference, two slits)

y_m the distance from the center of the pattern (θ=0) to the center of the m^{th} bright band. θm is the corresponding angle.

$$y_m = R\tan\theta_m$$ Small angles $$y_m = R\sin\theta_m$$

$$\Rightarrow \quad y_m = R\frac{m\lambda}{d}$$ (constructive interference in Young's experiment)

We can measure R, d and the positions of y_m of the bright fringes,

$\Rightarrow$ experiment provides a direct measurement of the wavelength λ

$\Rightarrow$ Young's experiment: the first direct measurement of wavelengths of light.

(4) Polarization of waves

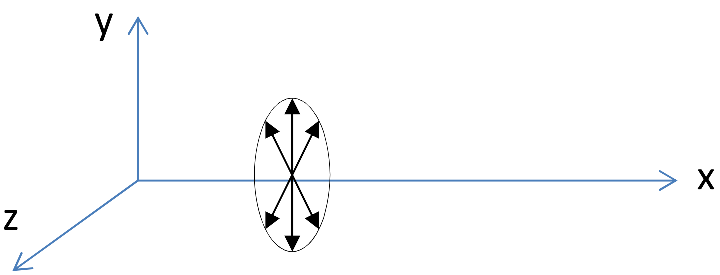
Polarization is a property applying to transverse waves that specifies the geometrical orientation of the oscillations.

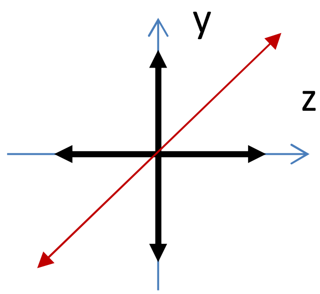
Sound waves in gases and liquids are pure longitudinal waves, waves in a string are always transverse
=> here we do not speak about polarization, the oscillation direction is well established

Electromagnetic waves (e.g light), sound waves in solids, gravitational waves exhibit multiple polarizations.

Wave polarization: important property of waves in technological applications: optics, radio, microwave, lasers, communications…

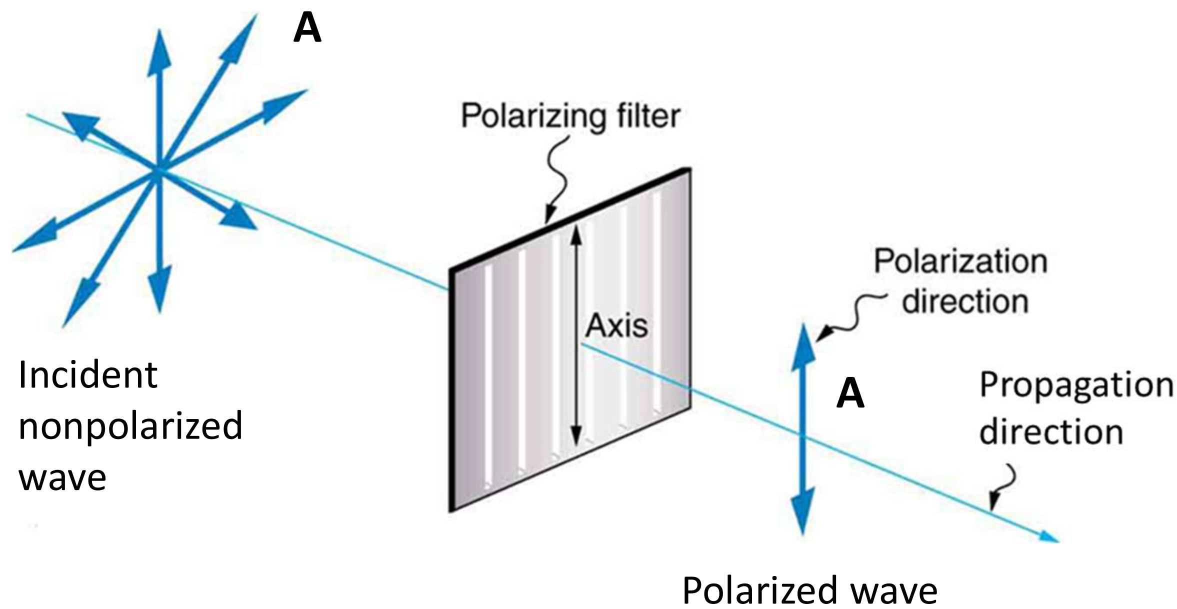
Simplified representation:
non-polarized wave





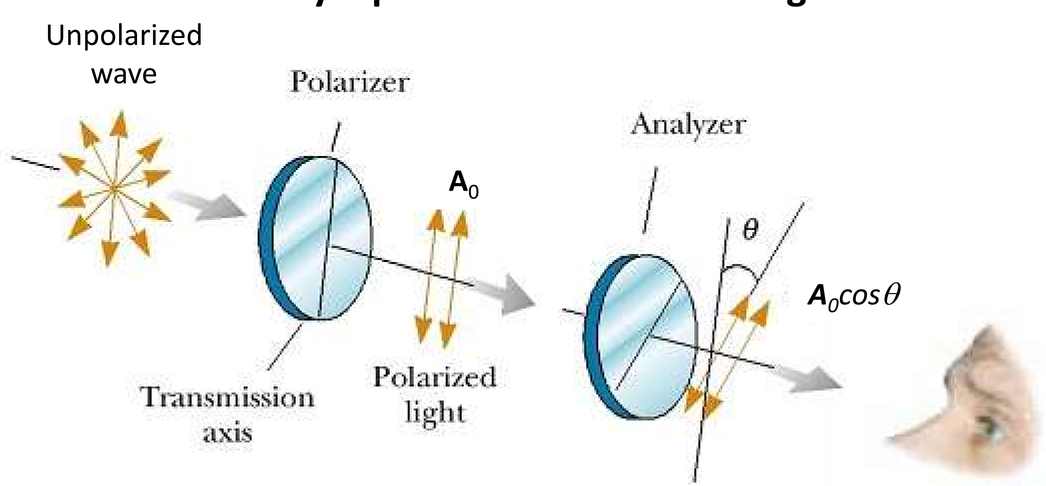
Any randomly oscillating wave can be decomposed as a *superposition* of two perpendicular waves polarized along perpendicular directions

A wave can be polarized by crossing a system called **polarizer**



Malus law: Polarizer analyzer system

transmission by a polarizer oriented at angle θ



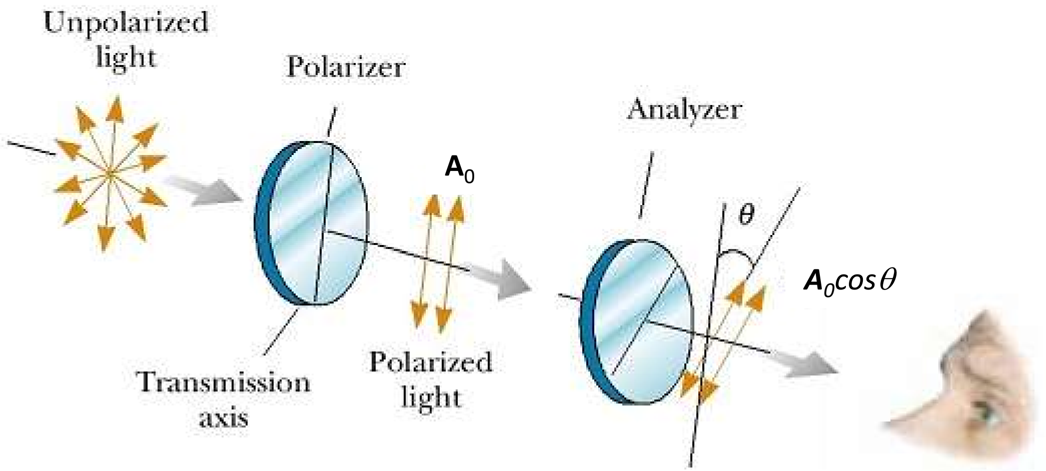
A=amplitude of oscillation

$A_\theta = \boldsymbol{A_0} \cos\theta$

Only the component of the oscillation parallel with the polarizer axis I s transmitted

The transmitted intensity:

$$I_\theta = I_0 \cos^2\theta$$



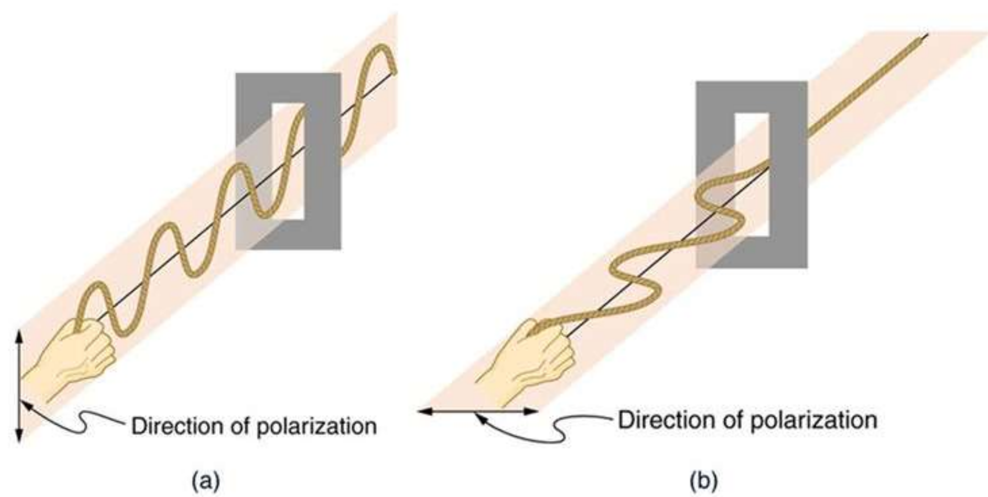
The transmitted intensity:

$I_\theta = I_0 \cos^2\theta$

If $\theta = \pi/2$, **I =0**
P and A have perpendicular axes
=> wave is not transmitted

Ex. Eye glasses with polarization filters

The concepts are valid for any type of waves: mechanical waves, electromagnetic waves (e.g. light)…



The transverse oscillations in one rope are in a vertical plane, and those in the other rope are in a horizontal plane. The first is said to be vertically polarized, and the other is said to be horizontally polarized. Vertical slits pass vertically polarized waves and block horizontally polarized waves.

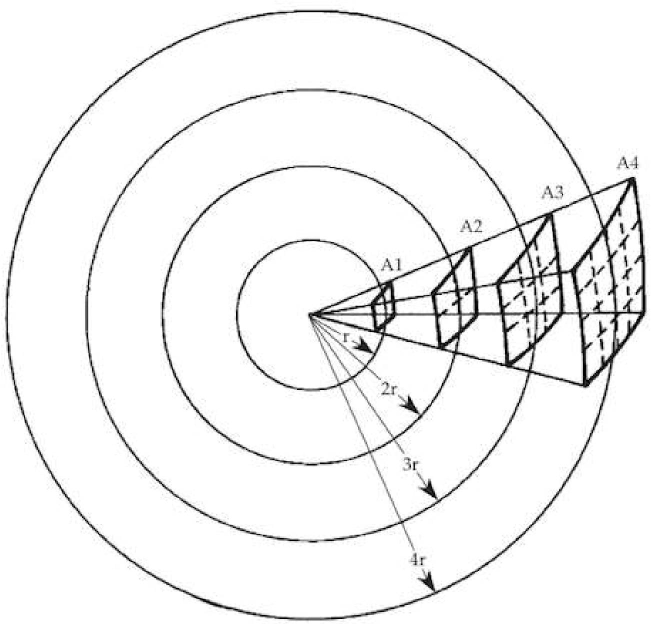
(5) Attenuation of (sound) waves

At the interface (boundary) between propagation media the sound waves are:
reflected, refracted, dispersed, adsorbed => attenuated…

When a sound propagated in an elastic infinite medium, the attenuation of intensity is produced by:

- **Geometric attenuation**: redistribution of energy on larger and larger areas
- **Absorbtion** : related to dissipation (propagation of damped oscillations)
- **Reflexion:** at interfaces in inhomogeneous/composite elastic medium

1/ Geometric attenuation



Redistribution of energy on larger and larger spherical areas

=> $I \sim I_0/r^2$

the wave function describing the propagation of the **spherical waves** in homogeneous and isotropic medium is:

$$y(r,t) = \frac{A}{r}\cos(kx - \omega t)$$

Different with respect to the **plane wave** function:

$$y(r,t) = A\cos(kx - \omega t)$$

For a plane wave if one neglects the absorption, the intensity remains constant.

For a spherical wave $A(r)=A_0/r$ => $I \sim \omega^2 A^2$ -> decays in $1/r^2$

2/ Attenuation by absorption

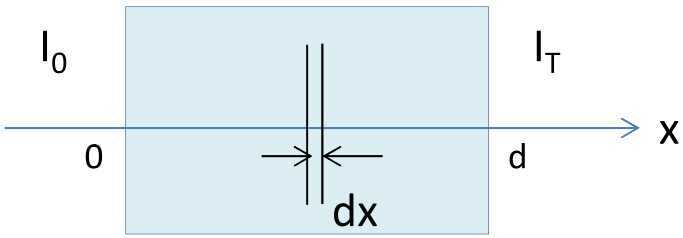
In their propagation, the sound wave induce oscillating motion to the particles of the medium.

Due to non-conservative/dissipative forces (friction, viscosity), these oscillations are damped, the initial energy being dissipated in terms of heat.

The magnitude of the energetic damping depends on:

- ❑ the nature of the elastic medium.
- ❑ the frequency of the elastic wave.

In the ultrasonic regime, at large frequencies and/or small amplitudes the phenomenon is adiabatic (Q=ct). However, in case of intense sounds with large oscillation amplitude there are energy losses by thermal conduction and radiation. A medium where the wave energy is lost by heat is called **dissipative medium**.



μ = the absorption coefficient

$$\frac{dI}{I} = -\mu dx \qquad \Rightarrow \int_{I_0}^{I} \frac{dI}{I} = -\int_0^d \mu dx$$

$$I_T = I_0 e^{-\mu d}$$

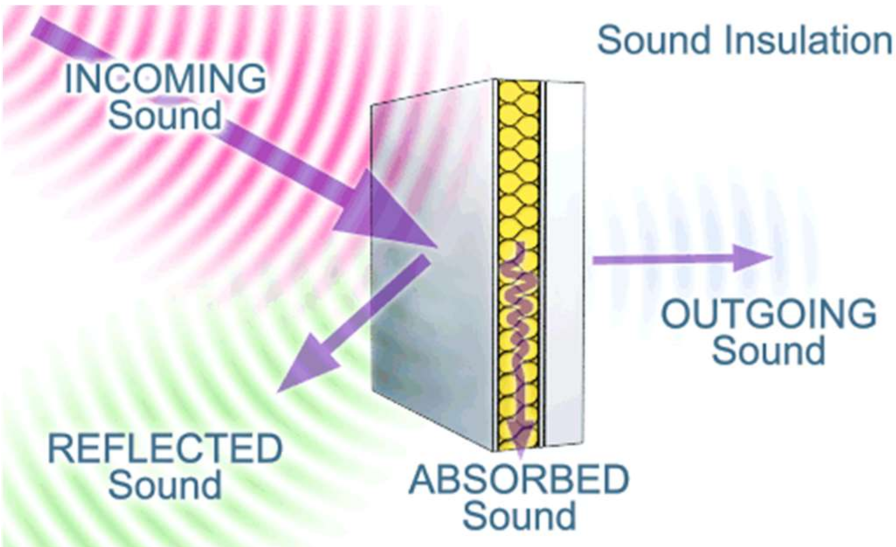
For a couple of materials: $\mu \sim \alpha \omega^2$

α =constant dependent on the medium

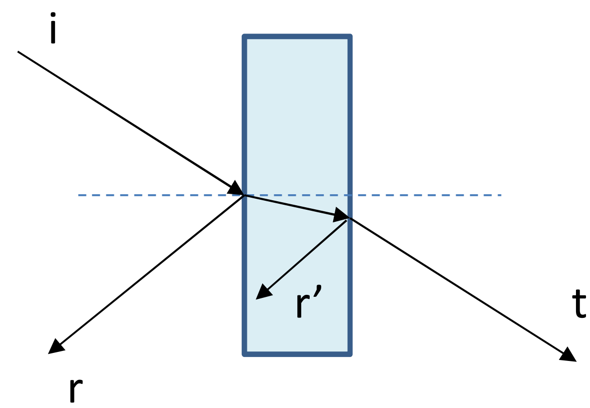
air: $\alpha = 4\ 10^{-13}\ s^2/m$

Sound attenuation by absorption increases with increasing frequency.

3/ Attenuation by reflexion at separating walls



mechanism



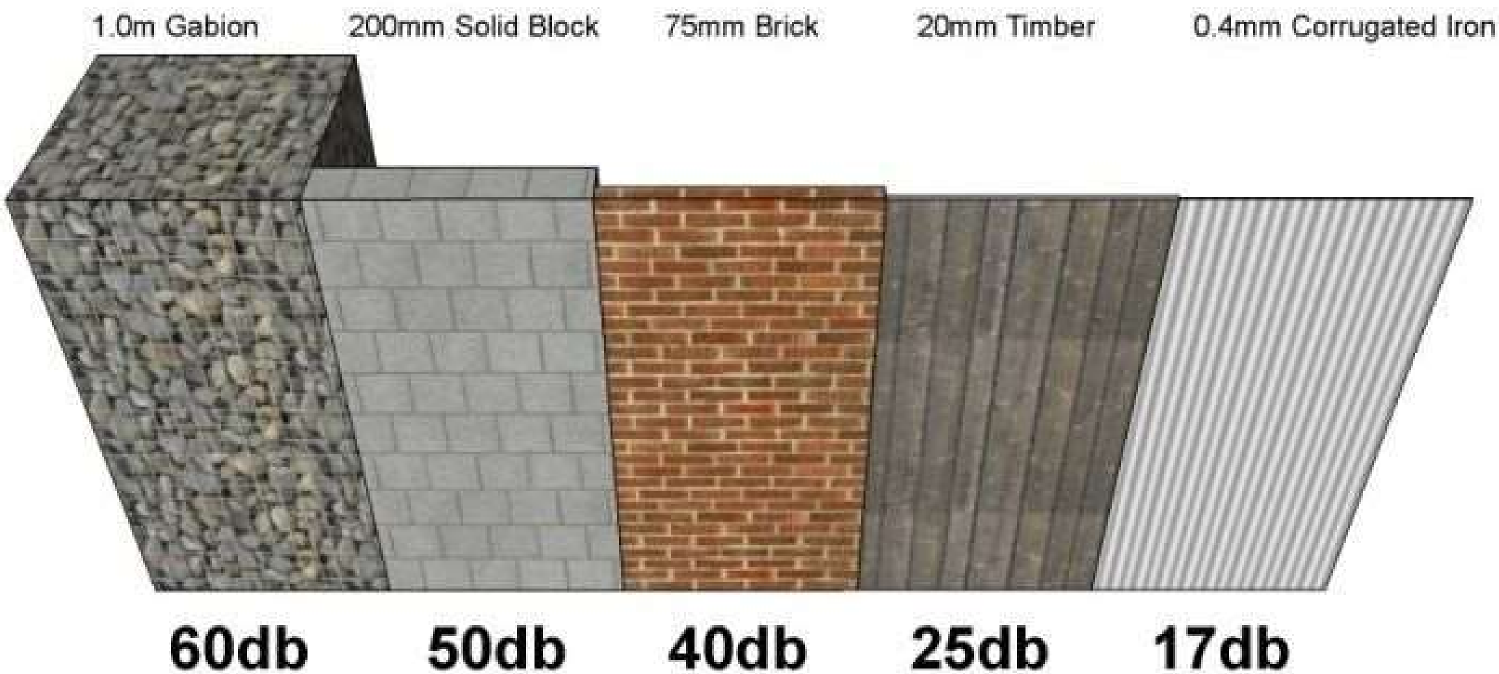
Reflexion + refraction at each interface
=> Transmission t decays with # of reflexions

=> Multilayered/Composite walls: sound barriers/acoustic walls

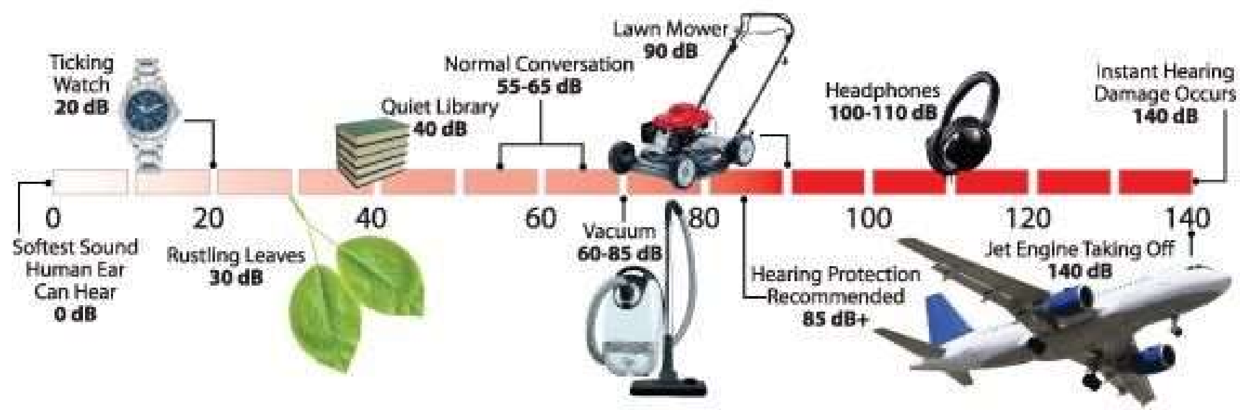
Acoustic attenuation index

$$A = 10 \log \frac{1}{T} \text{ (dB)}$$

T = transmission coefficient



Noise from heavy trucks is around 85db. The denser the wall the more noise it stops, a 100% solid timber fe stop 25db of noise, the remaining sounds passes through. The gabion noise barrier wall stops the most direc noise.

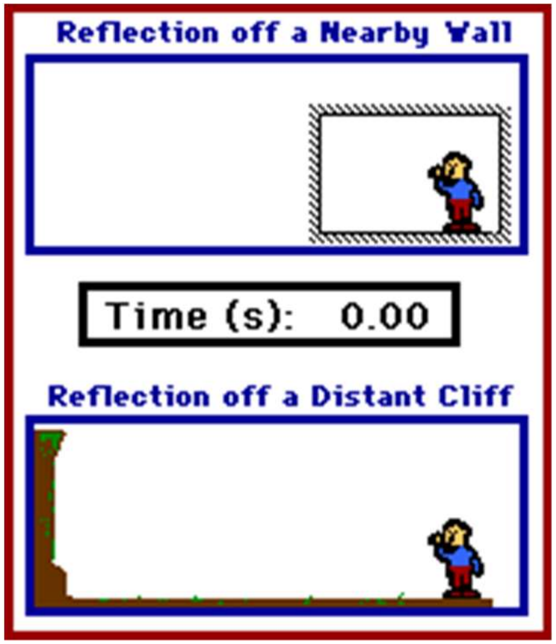


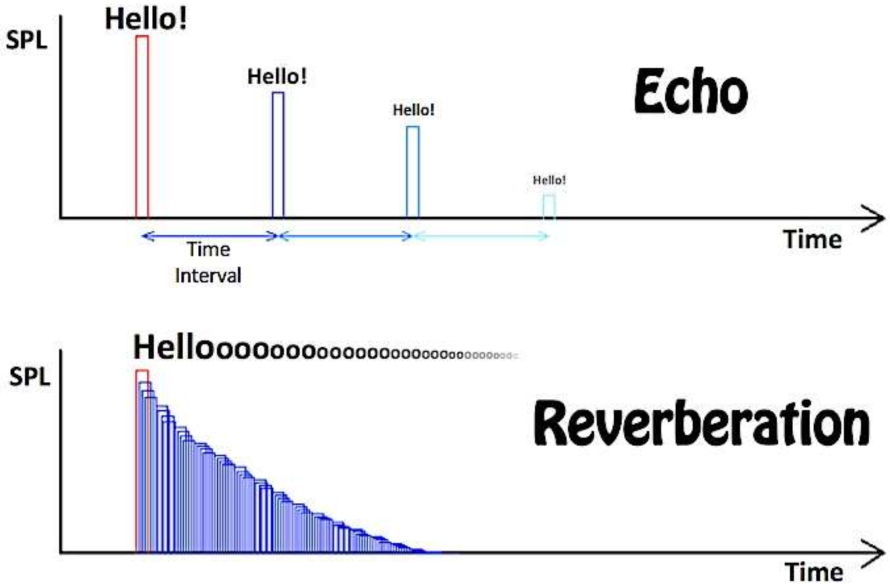
(6) Sound reverberation

Echo or Reverberation?

In acoustics, an **echo** is a *sound reflection*, arriving at the listener some time after the direct sound. Ordinarily, echo is a sound phenomenon related to an outdoor environment. It is simply a slightly delayed repetition of a sound.

However, when indoors, sound reflections are so numerous they end up being merged with the original sound. As a result, when so many reflections arrive at a listener that they cannot be distinguished from each other, the proper term is reverberation.

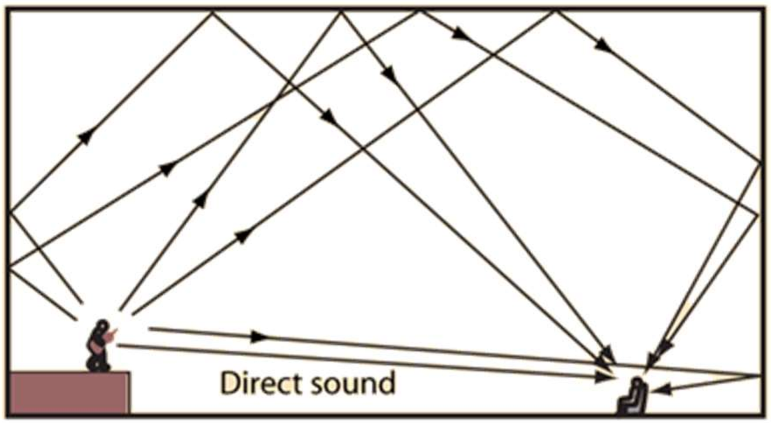




The inability to hear the distinct repetitions is what distinguishes reverberation from multiple echoes.

Reverberation, in acoustics, is the persistence of sound after a sound is produced.

created when a sound or signal is reflected causing a large number of reflections to build up and then decay as the sound is absorbed by the surfaces of objects in the space – which could include furniture, people, and air. This is most noticeable when the sound source stops but the reflections continue, decreasing in amplitude, until they reach zero amplitude.



Reverberation is frequency dependent:

the length of the decay, or **reverberation time**, receives special consideration in the architectural design of spaces which need to have specific reverberation times to achieve optimum performance for their intended activity.

- *In comparison to a distinct echo* that is a minimum of 50 to 100 ms after the initial sound, reverberation is the occurrence of reflections that arrive in less than approximately 50 ms. As time passes, the amplitude of the reflections gradually reduces to zero.
- Reverberation is not limited to indoor spaces as it exists in forests and other outdoor environments where reflection exists.
- Reverberation occurs naturally when a person sings, talks, or plays an instrument acoustically in a hall or performance space with sound-reflective surfaces.

Reverberation time: The time it takes for a signal to drop by 60 dB.

Reverberation phenomena applies also to electromagnetic waves => *electromagnetic reverberation chambers* <=> *cavity resonators* with high Q factors => standing EM waves