FLUID MECHANICS

Fluids play a very important role in every day life. We drink them, breath them, swim in them. They circulate in our body and control our weather... Airplanes fly through them, boats float in them.

A fluid is any substance that can flow; we use this term for both liquid and gases.

Fluid → statics = study of fluids at rest in equilibrium situations

→ dynamics = study of fluids in motion

1 DENSITY

$$\boxed{\rho = \frac{m}{V}}$$

for homogeneous materials.

$$[\rho] = \frac{kg}{m^3}$$

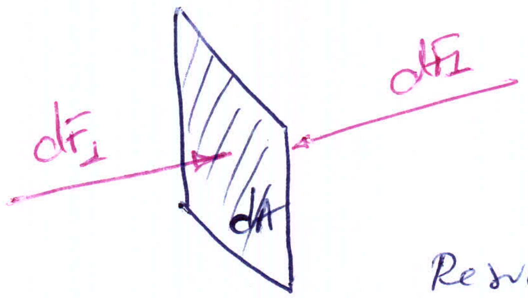
Density of some common substances

Material	ρ (kg/m³)	Material	ρ (kg/m³)
Air (1 atm, 20°C)	1,20	Concrete	$2 \cdot 10^3$
Ethanol	$0{,}81 \cdot 10^3$	Aluminium	$2{,}7 \cdot 10^3$
Ice	$0{,}92 \cdot 10^3$	Iron, steel	$7{,}8 \cdot 10^3$
Water	$1 \cdot 10^3$	Mercury	$13{,}6 \cdot 10^3$
Sea water	$1{,}03 \cdot 10^3$	Gold	$19{,}3 \cdot 10^3$
Blood	$1{,}06 \cdot 10^3$	Platinum	$21{,}4 \cdot 10^3$
		white dwarf star	10^{10}
		neutron star	10^{18}

2 PRESSURE IN A FLUID

When a fluid (liquid or gas) is at rest, it exerts a force perpendicular to any surface in contact with it

(physical origin: due to molecules composing the fluid colliding with their surroundings



a small surface dA within a fluid at rest.

Resulting force = 0

The fluid cannot exert any force parallel to the surface since that would cause the surface to accelerate.

We define the pressure p at that point as the normal force per unit area

$$\boxed{p = \frac{dF_\perp}{dA}}$$

If the pressure is the same on all points of surface within the total area A =>

$$\boxed{p = \frac{F_\perp}{A}}$$

$$[p]_{SI} = \frac{N}{m^2} = Pa \quad (\text{Pascal})$$

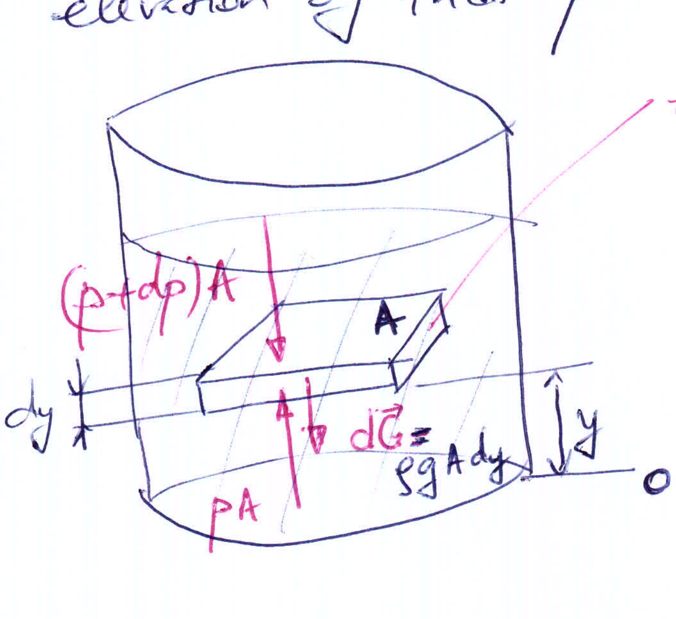
other units: $1 \text{ bar} = 10^5 \text{ Pa}$ ($1 \text{ mbar} = 100 \text{ Pa}$)

$1 \text{ atm} = 1{,}013 \cdot 10^5 \text{ Pa}$

- **Atmospheric pressure** p_a is the pressure of the earth's atmosphere at the bottom of the sea
  → varies with weather and changes with altitude

- **Pressure, depth and Pascal law**

If the weight of the fluid would be neglected the pressure would be the same through all the volume.
However, often, the weight of the fluid is not neglectable.
Atmospheric pressure is less at high altitude than at sea level (why an airplane cabin has to be pressurized when flying at 11Km altitude).
When we dive into deep water our ears feel the increasing pressure rapidly with increasing depth.
We can derive a general relationship between the pressure p in any point of a fluid and the elevation of that point.



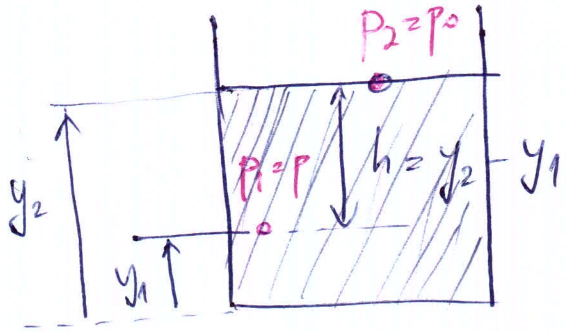
Element of fluid in rest (area A, thickness dy)

$$\sum F_y = 0 \Rightarrow pA - (p+dp)A - \rho g A\,dy = 0$$

$$\Rightarrow \frac{dp}{dy} = -\rho g$$

$$\Rightarrow \boxed{p_2 - p_1 = -\rho g (y_2 - y_1)}$$

We can rewrite this eq. in terms of depth below the surface (=) point 2 = surface ~~y_2~~
Point 1 any level within the fluid



$$p_2 = p_0$$
$$p_1 = p$$

$$\Rightarrow p_0 - p = -\rho g (y_2 - y_1) = -\rho g h$$

$$\Rightarrow \boxed{p = p_0 + \rho g h}$$

The pressure p at depth h is greater than the pressure at surface by an amount $\rho g h$

<u>Obs</u>: If p_0 increases (ex. using a piston fitting exactly in the cilinder, the pressure p increases exactly with the same amount.

$\Rightarrow$ **Pascal law**

Pressure applied to an enclosed fluid is transmitted undiminished to every portion of the fluid and the walls of containing vessel

1) ⇒ Application: hidraulic lift.

a piston with small cross sectional area A_1 exerts a force F_1 on the surface of a liquid (oil). This applied pressure $p = \frac{F_1}{A_1}$ is transmitted through the connecting pipe to a larger piston of area A_2

$$p = \frac{F_1}{A_1} = \frac{F_2}{A_2} \Rightarrow \boxed{F_2 = \frac{A_2}{A_1} F_1}$$

⇒ force multiplying device

ex: dentist chair, lifts and jacks, elevators, hidraulic brakes

2) • For gases, the assumption that ρ is uniform is realistic only over short vertical distances. In a room with a ceiling height of 3m filled with air of uniform density $\rho = 1,2\ kg/m^3$ ⇒

$$\rho g h = 1,2\ kg/m^3 \cdot 9,8\ m/s^2 \cdot 3 = 35\ Pa \simeq 0,00035\ atm$$

$$\ll 1\ atm.$$

But between sea and Everest (8882m), the density of air changes by a factor of 3, so we cannot use eq. $p = p_0 + \rho g h$. This is due to compression of air by the weight of the air column: the lower is the altitude the higher is the weight ⇒ higher ρ. In altitude, ρ decreases. Obs: Variation of g with altitude gives further correction.

• Liquids are incompressible ⇒ $\rho \simeq ct$ good approximation.

Absolute pressure and gauge pressure

If the pressure inside a car tire is equal to the atmospheric pressure, the tire is flat. The pressure has to be greater that the atmospheric pressure to support the car $\Rightarrow$ the significant quantity is the difference between inside and outside pressures.

When we say that the pressure in a car tire is $2{,}2 \cdot 10^5$ Pa we mean that it is greater than the atmospheric pressure with that amount.
($1{,}01 \cdot 10^5$ Pa)

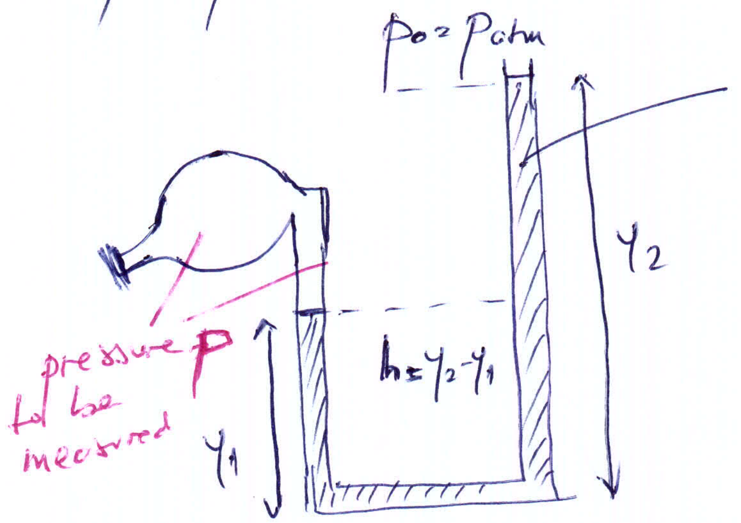
$\Rightarrow$ the total pressure in the tire is then:

$$10^5 \cdot (2{,}2 + 1{,}01) = 3{,}21 \cdot 10^5 \text{ Pa}.$$

The excess pressure above the atmospheric pressure is called GAUGE PRESSURE and the total pressure is called absolute pressure.

Pressure gauges

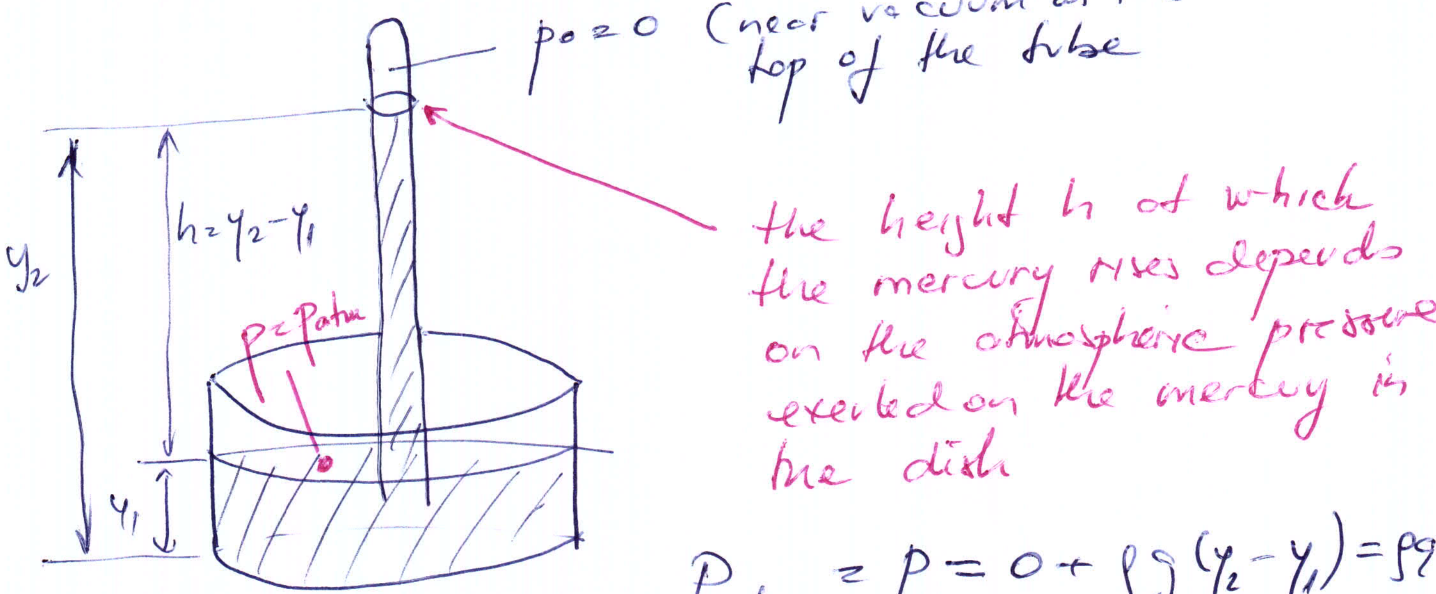
1) Open tube manometer



$$p + \rho g y_1 = p_{atm} + \rho g y_2$$

$$\Rightarrow \boxed{p - p_{atm} = \rho g (y_2 - y_1) = \rho g h}$$

Mercury barometer



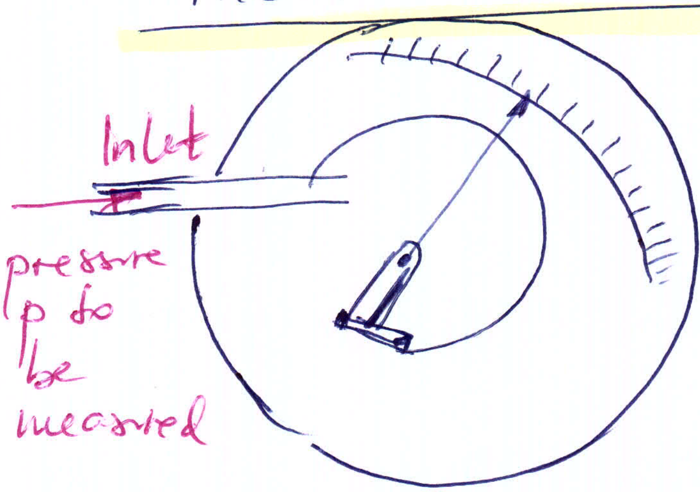
$$p_{atm} = p = 0 + \rho g (y_2 - y_1) = \rho g h$$

$\Rightarrow \boxed{p_{atm} = \rho g h}$ the barometer reads the atmospheric pressure directly from the height of the mercury column.

$\Rightarrow$ pressure expressed in mm col Hg
mmHg.

A pressure of 1 mm col Hg is 1 Torr after Evanghelista Torricelli, inventor of the mercury barometer.

Mechanical barometer



Changes in the inlet pressure cause the tube to coil or uncoil which moves the pointer

$\downarrow$ callibration

pressure value.

Other types of barometers

→ piezoelectric ($V_{piezo} \sim \Delta V \sim p$)

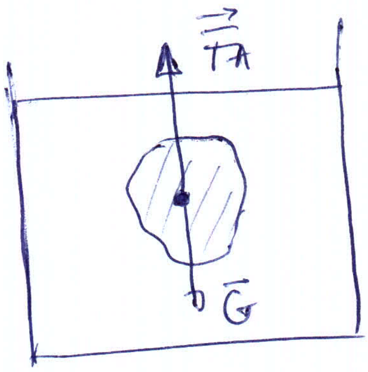
③ BUOYANCY

– familiar phenomenon: a body immersed in water seems to weigh less than in air. When the body is less dense than the fluid it floats.

ex: human body floats in the water, a helium balloon floats in the air.

Archimede's principle

When a body is immersed in a fluid (partially or totally immersed) it is pushed outward by the fluid with a force equal to the weight of the fluid displaced by the body.



$$\boxed{F_A = \rho_l V_c g}$$

apparent weight:

$$F_a = G - F_A = mg - \rho_l V_c g$$

$$= \rho_c V_c g - \rho_l V_c g$$

$$F_a = (\rho_c - \rho_l) V_c g$$

If $\rho_c = \rho_l \Rightarrow F_a = 0$

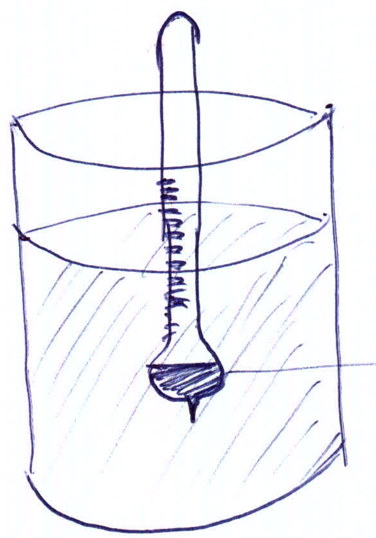
Ex: fish flesh denser then watter, yet the fish can float submerged due to gas (air) filled cavity within its body ⇒ fish "average" density equal to the on of the watter.

A body with $\rho < \rho_e$ can float partially submerged at the free upper surface of the fluid. The greater is the density of the fluid (liquid), the less submerged the body is. Ex: swimming is sea (salted) watter, $\rho = 1030\ kg/m^3$, your body floats higher than in water ($1000\ kg/m^3$)

Practical example

$\boxed{\text{The hydrometer}}$ – used to measure the density of liquids.

The calibrated float sinks into the fluid till the weight of the fluid it displaces is equal to its weight. The hydrometer floats higher in denser fluids than in less dense fluids and a scale enables to read, via a calibration, the fluid density.



the weight at the bottom makes the scale float upright.

4 FLUID FLOW

Complex problem but can be simplified within the ideal fluid model

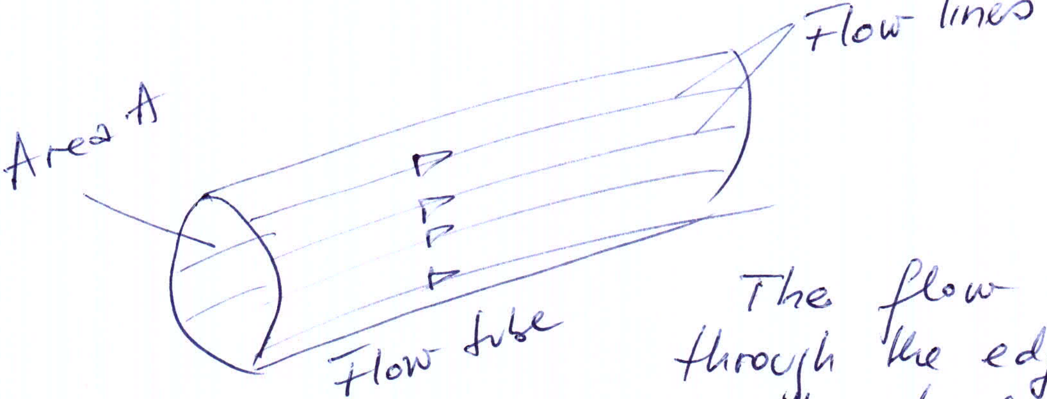
→ incompressible ⇒ ρ cannot change

→ has no internal friction ⇒ no viscosity

ex: liquids.

The path of an individual particle in a fluid is called flow line. If the overall flow pattern does not change in time, the flow is called steady flow.

Streamline = a curve whose tangent at any point is the direction of the fluid velocity in that point. When the flow pattern changes in time, the streamlines do not coincide with the flow lines. We will consider only steady flow situations for which flow lines and streamlines are identical



The flow lines passing through the edge of an imaginary element of area A form a flow-tube.

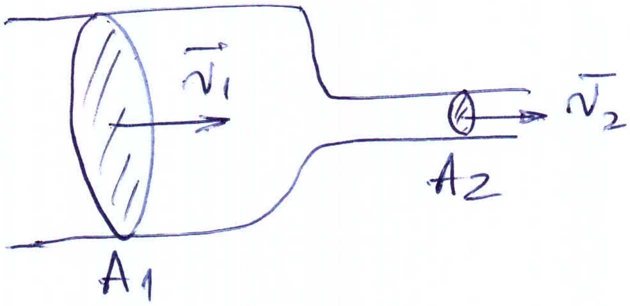
Regimes of flow:

LAMINARY ⟶ steady flow

TURBULENT FLOW ⟶ irregular, chaotic, no steady flow pattern! (continuous changement)

The Continuity equation

Consequence of the mass conservation during fluid flow.



$dm_1 = dm_2$; $\rho A_1 v_1 dt = \rho A_2 v_2 dt$

$$\Rightarrow \boxed{A_1 v_1 = A_2 v_2}$$ continuity equation, incompressible fluid.

* valid in different branches of physics (ex. electricity) and real life!

One can generalize this equation, in case of compressible fluid: $\Rightarrow \rho_1 \neq \rho_2$

$$\Rightarrow \boxed{\rho_1 A_1 v_1 = \rho_2 A_2 v_2}$$

5 BERNOULLI EQUATION

According to the continuity equation the speed of fluid can vary along the paths of fluid. The pressure can also vary, depends on height, as in the static situation.

The law describing this leads to the Bernoulli equation, which can be deduced from energy-work theorem.

$$\boxed{p_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = p_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2}$$

$\Updownarrow$

$$\boxed{p + \rho g y + \frac{1}{2}\rho v^2 = ct}$$

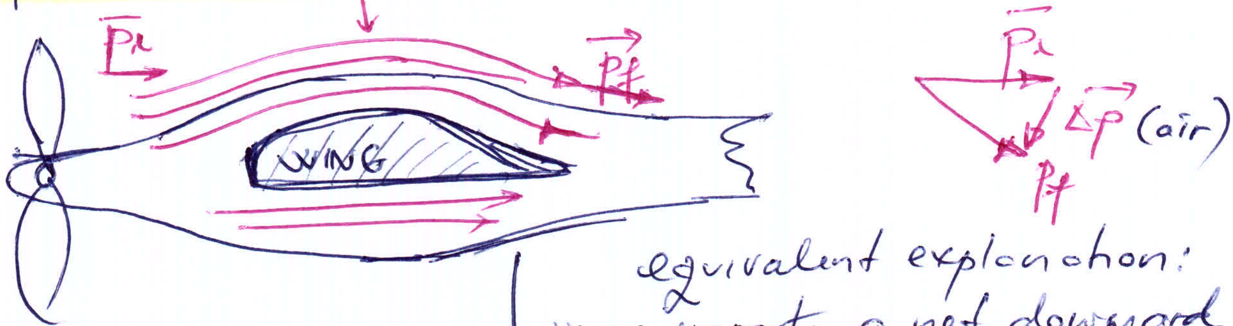
Bernoulli law

STATIC PRESSURE — POSITION PRESSURE — DYNAMIC PRESSURE

⚠ valid for ideal fluid with no viscosity

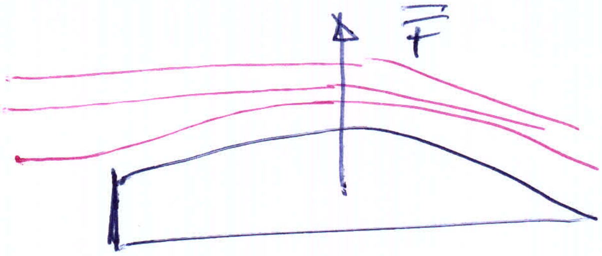
Obs: when the fluid is not moving this eq. reduces to the one known for the fluid in rest $\underline{p + \rho g y = ct}$.

Lift on the airplane wing



Flow lines are crowded together above the wing => flow speed is higher => static pressure is lower => upward force

equivalent explanation: wing imparts a net downward momentum to the air, so reaction force is upward.



$$p + \rho\frac{v^2}{2} + \rho g z = ct$$

$$v \uparrow \Rightarrow p \downarrow$$

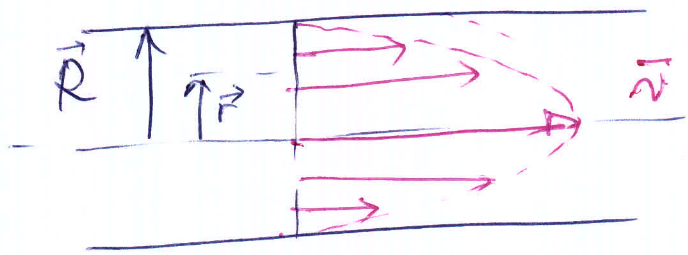
6 VISCOSITY AND TURBULENCE

Viscosity ⇒ internal friction in a fluid opposes to the motion of one fluid portion relative to another.

⇒ depends on fluid nature (ex. water viscosity < oil viscosity < honey viscosity

⇒ depends on temperature. This temperature variation has to be minimised for fluids used in lubrification (oils, vaseline)

⇒ has an effect on fluid flowing through pipes, i.e. velocity profile.



$\vec{v} = 0$ on walls $(r = R)$
→ $\vec{v}_{max}$ in the center.

The motion is like a lot of concentric tubes sliding relative one another, with the central tube moving fastest and the outer tube at rest.

Viscous forces between tubes oppose moving
⇒ to keep the flow going we have to push with a back pressure on the tube higher than the pressure at front.

⇒ squeezing a tooth paste tube, ketchup tube to keep the fluid coming out from the container.

Turbulence

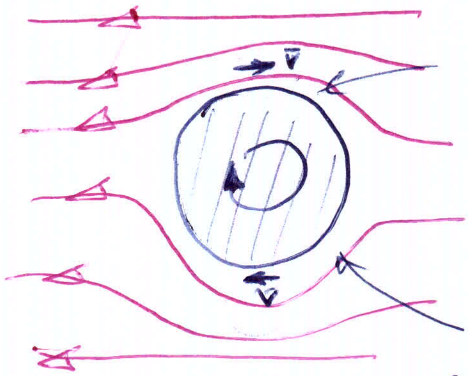
When the speed of a flowing fluid exceeds a certain limit, the flow is no longer laminar. Instead, the flow pattern become irregular and extremelly complex, it changes continuously with time, there is no steady-state pattern. This irregular, chaotic flow is called turbulence.

Ex. Flow of water from a faucet is (a) laminar at low speeds and turbulent at high speeds. (b)

Turbulences are induced by irregularities (deffects), roughness of the pipe, etc... They are damped out at low speeds but conserved and amplified at larger speeds.

Examples of fluid dynamics applied in real life

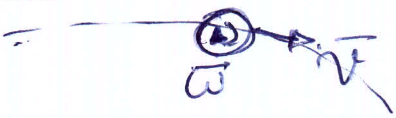
① moving of a spinning ball



on one side, the ball slows the air creating a region of high pressure

at the other side, the ball speed the air => creating a region of low pressure

=> curved trajectory



The resultant force point in the direction of the low pressure side.

ex : footbal, tennis, baseball, ...

=> no spinning ball ($\vec{\omega} = 0$) => linear trajectory

② turbulences during air-plane flight

air flows regularly in such-called jet-streams with certain speed which can be used to increase the plane relative speed => carburant economy. The lenght of air jet streams, distribution of orientation, speed of air may vary => turbulences felt as shaking of plane, trajectory deviation, changes of altitude, etc....