ACOUSTICS II

Elements of wave optics

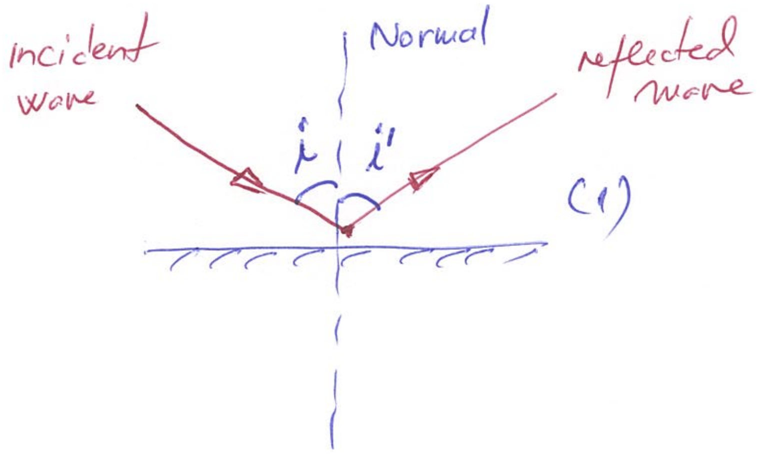
All the concepts adressed here, remain valid in case of light (electromagnetic wave)

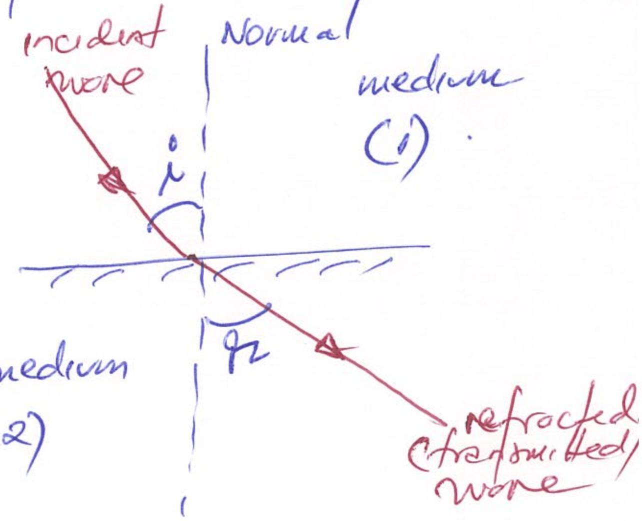
① Reflexion and refraction of (sound) waves

When a plane sound wave riches the boundary surface separating two elastical media, the incident wave will be splitted in two components:

→ reflected wave (returns in the same medium)

→ transmitted wave (passes in 2nd medium)





Acoustical impedance (Z)

Physical quantity describing the opposition of a medium against the propagation of sound waves through it.

$$Z = \rho v \quad ; \quad v = \sqrt{\frac{B}{\rho}}$$

$$[Z] = \frac{kg}{m^3} \cdot \frac{m}{s} = \frac{kg}{m^2 s} = \text{rayl} \quad (\text{Rayleigh})$$

aer	$Z = 430$ rayl	full brick	$2{,}4 \cdot 10^6$
expanded polystiren	$Z = 6 \cdot 10^3$ rayl	concrete	$7{,}2 \cdot 10^6$
		wood	$2{,}7 \cdot 10^6$

The reflection and refraction phenomena appear at the boundary between two media with different acoustical impedances Z_1, Z_2.

The principle of Fermat

The waves propagate between two points A and B along a path which requires the minimum time.

Upon geometrical considerations, based on the principle of Fermat one can deduce and enounce the reflection and refraction laws.

The reflection laws

1. The incident wave, the reflected wave and the normal at the surface are lying in the same plane.
2. The incidence angle is equal to the reflection angle. $\boxed{i = i'}$

The refraction laws

1. The incident wave, the refracted wave and the normal at the surface are lying in the same plane.
2. The angles of incidence and refraction satisfy the following equation

$$\boxed{\frac{\sin i}{\sin r} = \frac{v_1}{v_2}}$$

If one tries to quantify the amount of energy passed from the medium (1) to the medium (2) one needs to know the transmission (T) and reflection (R) coefficients.

$$\boxed{R = \frac{\text{Intensity of reflected wave}}{\text{Initial wave intensity (incident)}} = \frac{I_R}{I_i} = \frac{A_R^2}{A_i^2}}$$

after calculations: $R = \left(\frac{Z_1 - Z_2}{Z_1 + Z_2}\right)^2$

- R decreases when $Z_1 \to Z_2$
- large reflection coefficient when Z_1 largely different with respect to Z_2

Table transmission and reflection coefficients at the boundary (separation) between two media

Medium 1	Medium 2	R	T
air	water	0,9999	0,0001
water	steel	0.875	0.125
air	wood	0,9	0.1
air	curtin	0.2	0,8

Transmission coefficient (T)

$$\boxed{T = \frac{I_t}{I_i} = \frac{A_t^2}{A_i^2} = \frac{4Z_1Z_2}{(Z_1+Z_2)^2}}$$

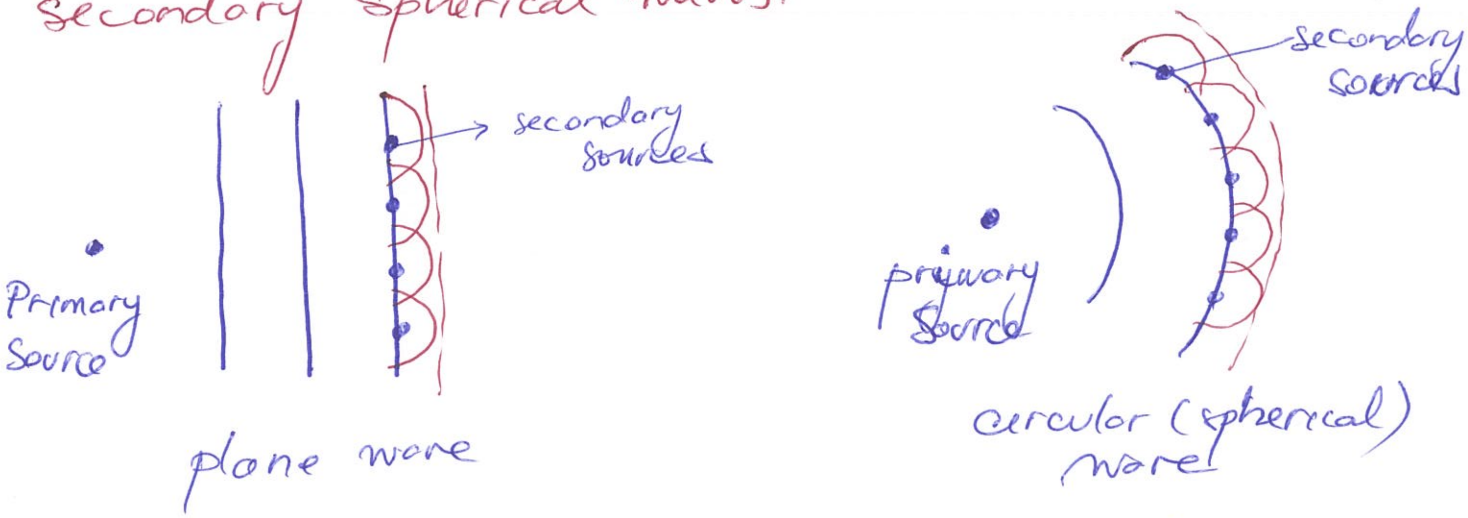
From energy conservation considerations:

$$\boxed{R + T = 1}$$

(2) DIFFRACTION OF WAVES

The phenomenon of diffraction consist in the penetration of waves in the geometrical shadow of the small obstacles, whose dimension is comparable with the wave length of the respective wave. The obstacle may be, for example, a paravane with a small slot (hole) or any small object with certain specific shape. The explanation of this phenomenon can be based, as well as all its related properties, upon the Huygens-Fresnel principle.

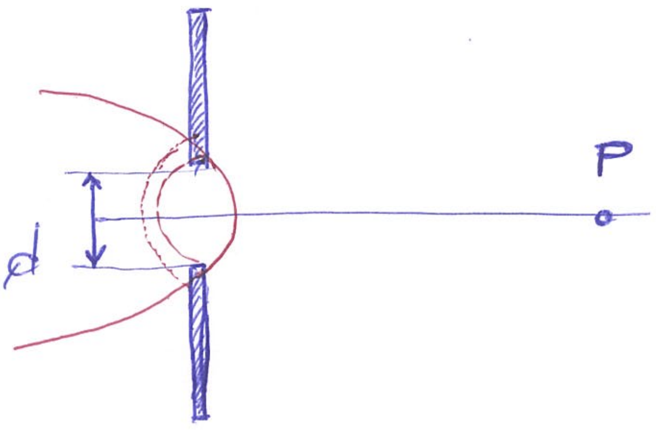
All the points belonging to the front of the wave can be considered as new point sources which produce secondary spherical waves.

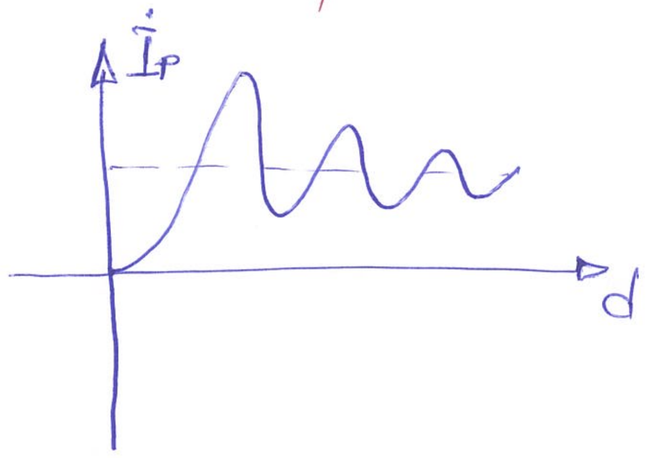


After a while, the new position of the wave front will be constituted by the surface tangent to all this secondary wave fronts (envelope).

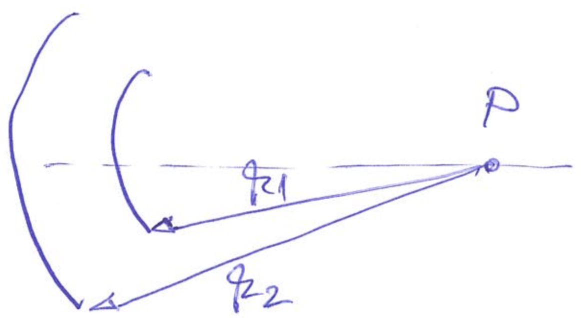
This can be demonstrated on the basis of the general theory of elasticity, starting from the observation that the initial perturbation propagating in a medium is replicated by all the points touched by the wave.

a) Fresnel difraction on a circular (hole) aperture.





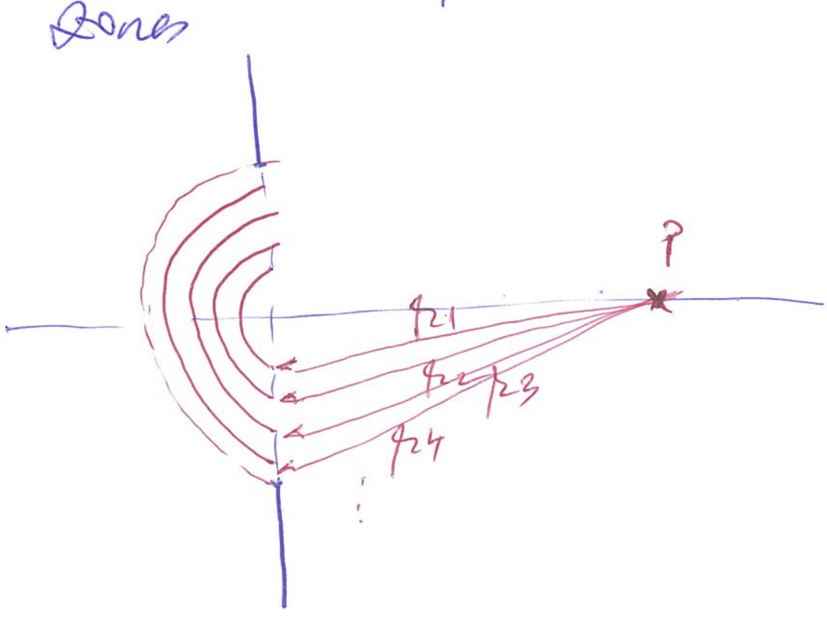
Fresnel zones → circles centered in P whose radius differs by $\frac{\lambda}{2}$



$$r_2 - r_1 = \frac{\lambda}{2}$$

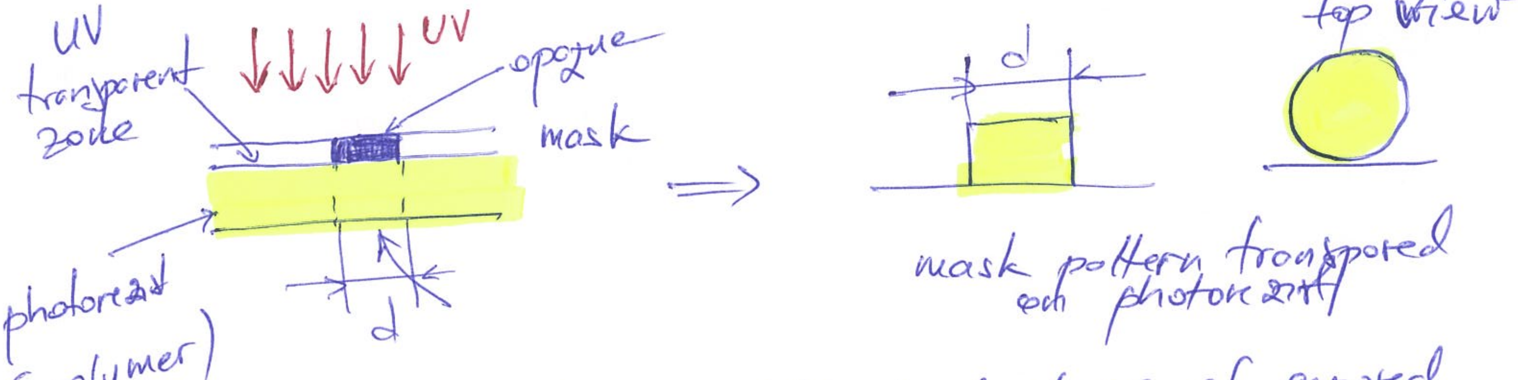
Due to the Fresnel diffraction on a circular hole in the point P we will obtain maximum or minimum intensity, depending if the hole opening live unobstructed a pair or an impair number of Fresnel zones

$$r_{i+1} - r_i = \frac{\lambda}{2}$$



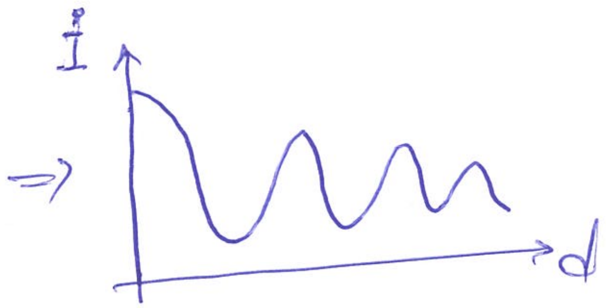
In an absolutelly similar way one can study the Fresnel diffraction by an obstacle of small dimensions. Here, the obstacle, as a function of dimensions, will obstruate the first Fresnel zones.

Obs: discussion on optical lithography when the mask pattern size becomes comparable to λ.



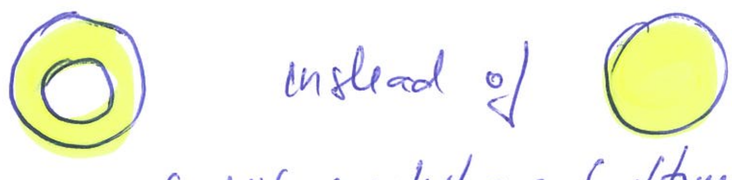
UV-breaks bounds $\Rightarrow$ soluble in developer of exposed zones

But when $d \sim \lambda$ $\Rightarrow$



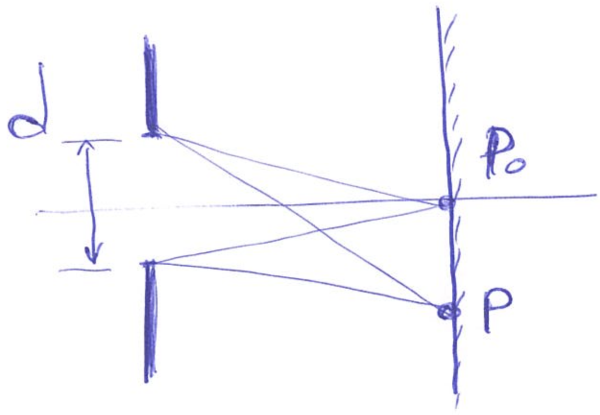
$\Downarrow$ one can have maximum of exposure in the middle of the "opaque" zone of the mask

$\Rightarrow$ doughnut shapes (circles with center holes)

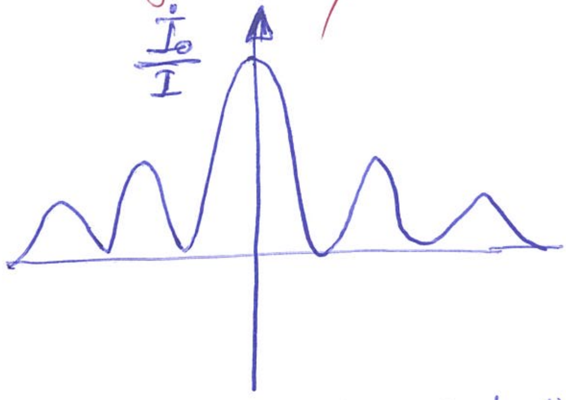


$\Downarrow$ limitations of UV resolution (ultimate) by diffraction

b) Fraunhofer diffraction – on a rectangular aperture



$\Longrightarrow$



intensity distribution curve

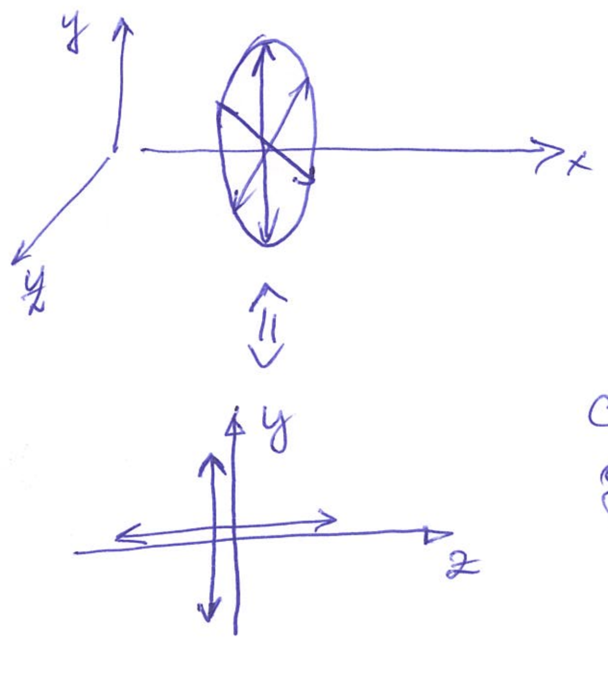
diffraction + interference

$\Rightarrow$ fringes parallel to the edges of the rectangular aperture

POLARIZATION OF WAVES

Polarization is a parameter of waves that specifies the geometrical orientation of oscillation. Electromagn. waves (e.g. light), sound waves in solids, gravitational waves exhibit multiple polarizations. On the other hand sound waves in gas and liquids oscillate along the propagation direction (longitudinal waves) and waves in a string are always transverse. In these cases we do not speak about polarization because the oscillation direction is not under question.

Wave polarization is an important property of waves in technological applications: optics, radio, microwaves, lasers, communications, ...

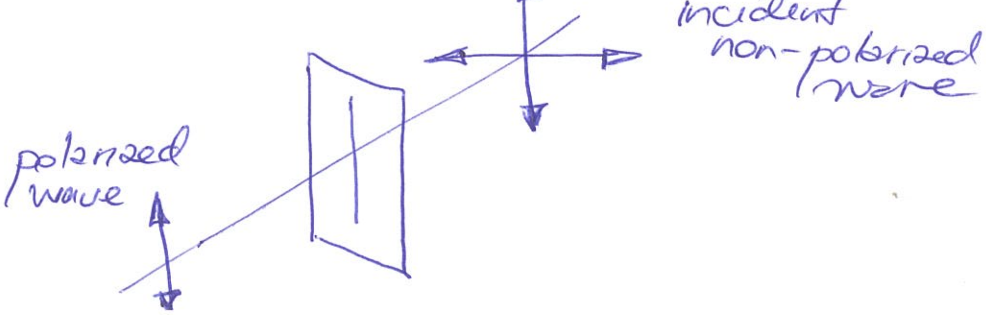


non-polarized wave

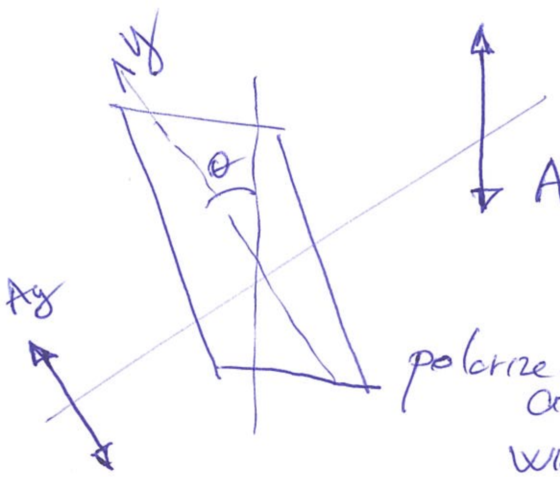
simplified representation

any randomly oscillating wave can be decomposed in a superposition of two perpendicular waves polarized along perpendicular directions

A wave can be polarized by crossing a system called polarizer.



Intensity of transmitted polarized wave



linearly polarized wave

A = amplitude of oscillation

polarizer with polarization axis making angle θ with initial polarization direction

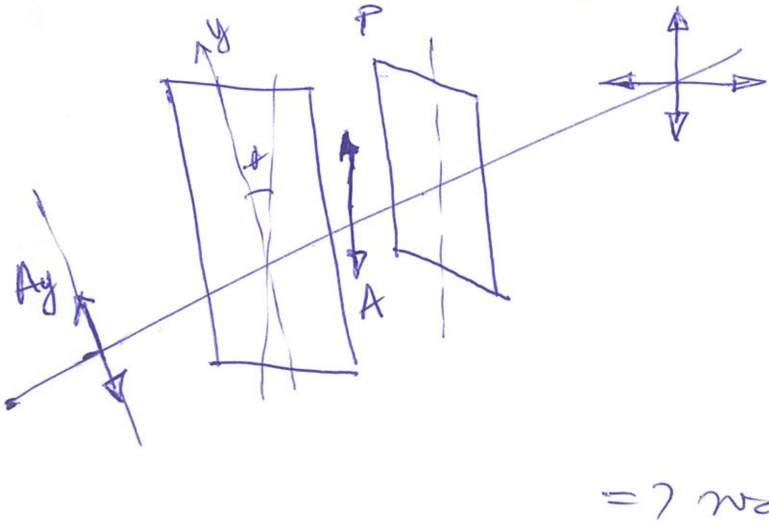
$$\boxed{A_y = A\cos\theta}$$

only the component ∥ with the polarizer axis is transmitted

=> The transmitted intensity:

$$\boxed{I \sim A_y^2 = I_0\cos^2\theta}$$

Polarizer – analyzer system



If $\theta = \pi/2$

=> $\boxed{I = 0}$

(P and A have perpendicular polarization axes)

=> wave not transmitted.

Obs: Concepts valid for any type of waves (e.g. light – electromagnetic waves).

3 Atenuation of sond waves

At the interfaces between propagation media the sound waves are: reflected, refracted, dispersed, adsorbed, attenuated..

The atenuation is produced by:
→ absorbtion
→ dispersion
→ reflexion

When the sounds propagate in an elastical medium which is infinite, the attenuation of their intensity may be related to the redistribution of the sound energy over larger and larger volumes (case of spherical waves) or due to the absorbtion
→ geometrical causes
→ energetical causes.

When the sound crosses separating walls, they can atenuate via sucessive reflexions on the surface of the media constituting those composite walls.

1 Geometrical atenuation

The wave function describing the propagation of the spherical waves in homogenous and isotropical media is:

$$\boxed{\psi(r,t) = \frac{A}{r}\cos(kr - \omega t)}$$

obs: This is different with respect to the plane wave $\psi(x,t) = A\cos(kx - \omega t)$ where $A = ct.$

$\Rightarrow A(r) = \frac{A_0}{r}$ $\Rightarrow$ reduction of the amplitude with the distance

$\Rightarrow$ reduction of the intensity:

$I \sim \omega^2 A^2$; $A = \frac{A_0}{r}$ $\Rightarrow$ $I \searrow \frac{1}{r^2}$

In case of plane waves, if one neglects the absorbtion the wave intensity remains constant.

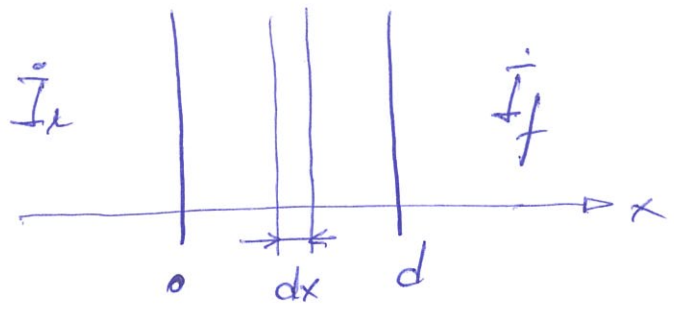
② Atenuation by absorbtion

In their propagation, the sound wave induce to the particles of the propagating elastic medium an oscillatory motion. Due to non-conservative dissipative forces (friction, viscosity) these oscillations will be damped, the initial energy being dissipated in terms of heat. The magnitude of the energetical damping depends on:

→ the nature of the elastic medium itself

→ the frequency of the elastical wave.

The compressions and elongations of different parts of the medium induced by the wave produce local variations of temperature. If the frequency of the wave is high or their amplitude is low the phenomenon is adiabatic ($Q = ct$). However, in case of intense sounds with large oscillations of amplitude there are also energy loses by thermal conduction and radiation. A medium where the wave energy is lost by heat is called dissipative medium.



$$\frac{dI}{I} = -\mu dx \Rightarrow$$

$$\int \frac{dI}{dI} = -\int_0^d \mu dx$$

$$\Rightarrow \boxed{I_f = I_i e^{-\mu d}}$$ the atenuation law by absorbtion

μ = absorbtion coefficient.

For couples of materials:

$$\boxed{\mu = \alpha \omega^2}$$

α = constant dependent of the medium

air : $\alpha = 4 \cdot 10^{-13}\ s^2/m$

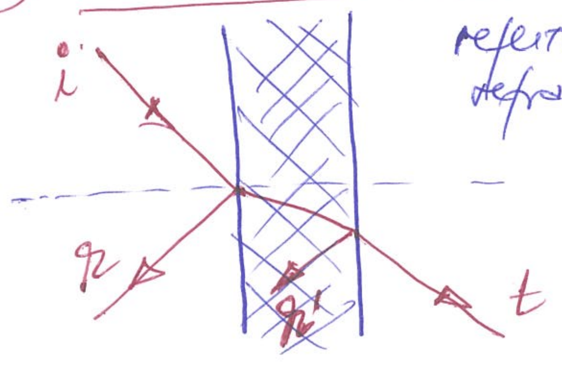
the atenuation through absorbtion is larger for the high frequency sounds then lower frequency sounds.

If on the way of the sound waves there is a wall, the total sound atenuation is:

→ atenuation through the absorbtion in the wall

+ atenuation by reflection on the two surfaces of the wall.

③ Sound atenuation through separating walls



reflections + refraction

Rigid piston model

under the sound wave pressure of the sound arriving at angle θ, the wall can move as a rigid piston

One can calculate a transmission coefficient:

$$\boxed{T = \frac{1}{1+\left(\frac{\pi m' \nu \cos\theta}{Z_0}\right)^2}} \qquad (*)$$

m' = mass of the unit surface

$Z_0 = \rho_0 v_0$ = the acoustical impedance of the air

At normal incidence ($\theta = 0$) $\Rightarrow$

$$\boxed{T \cong \frac{1}{1+\left(\frac{\pi m' \nu}{Z_0}\right)^2} \simeq \left(\frac{Z_0}{\pi m' \nu}\right)^2}$$

(Mc. Lauren) $\frac{\pi m'}{Z_0} \gg 1$

the law of masses

The transmission coefficient is larger for smaller unit masses of the wall (light materials) and at low frequencies

<u>Obs</u>: In case of the reflecting walls of a room, the transmission coefficient represents an average over all the orientations of the wall $\Rightarrow$ eq (*) has to be averaged over all the values between 0 and π.

In practice, instead of the transmission coefficient one uses the <u>acoustical atenuation index</u>:

$$\boxed{A = 10 \log \frac{1}{T}} \qquad [dB]$$

In case of the above discussed separating wall:

$$A = 10 \log \frac{1}{T} = 10 \log \left(\frac{\pi m' \nu}{Z_0} \right)^2$$

$$\boxed{A = 20 \log \frac{\pi m' \nu}{Z_0}}$$

the acoustical atenuation index depends logarithmically on the mass of the unit surface of the wall

when doubling the wall mass (i.e. doubling thickness)

$$A' = 20 \log \frac{\pi\, 2m' \nu}{Z_0} = 20 \log \frac{\pi m' \nu}{Z_0} + 20 \log 2 = A + 6\,dB$$

$$\log 2 \simeq 0{,}301$$

⇒ When doubling the wall mass we do not double the sound atenuation, but only an enhancement by 6 dB.

⇒ the solution to obtain a significant sound atenuation is not the mass enhancement but the use of multilayered walls ⇒ atenuation by multiple reflections.

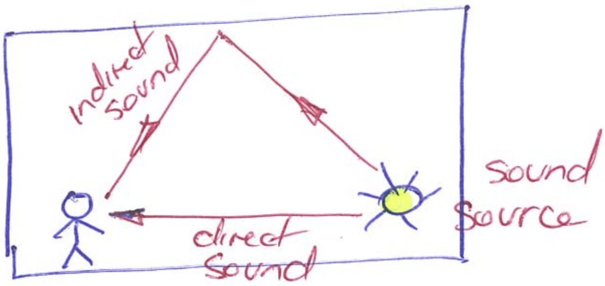
Obs

- The above mentioned and discussed equations are valid for infinite extended walls. In real world, the rigid piston approach is no longer valid due to the finite dimensions of the walls. Therefore, the walls should be modelled as membranes fixed at extremities.

- In practice, to calculate the atenuation of sound one use empirical or semi-empirical formulas.

4 Reverberation of sounds

In a closed space (room), besides the direct sond, coming from the emitting source towards the listener, the listener will also detect the sound waves successively reflected by the walls. One can wander: how much time the sound can be still heard after the emitting source stopped to produce sounds? This time is called <u>REVERBERATION TIME</u> and it is a characteristic of the room.



More precisely, the reverberation time is defined as the time after which the sound intensity level of the sounds in the room is reduced by 60dB, or, in terms of intensity, the sound intensity in the closed space decays by 10^6 times.

One can mathematically demonstrate that the energy density inside of the chamber after a time t following the stop of the emitting source, satisfyes the equation:

$$\boxed{w = w_0 e^{\frac{\alpha S c}{4V}}}$$

V = the chamber volume

S = surface of the walls

$c = 340\ m/s$ = the speed of sound in the air

α = absorbtion coefficient of the walls of the chamber

$\alpha = 0{,}015$ for the concrete

$\alpha = 0{,}8$ for felt [pâslă]

One can demonstrate that the reverberation time satisfies the equation:

$$\boxed{\tau_R = 0{,}16 \frac{V}{\alpha S}}$$

In case of many surfaces of different absorption coefficient α_i, the reverberation time will write:

$$\boxed{\tau_R = 0{,}16 \frac{V}{\sum_i \alpha_i S_i}}$$

the formula of WALLACE

valid in case of a small size empty chamber.

In case of large size chambers one has to use more complex equations, deduced semi-empirically. The reverberation time can be measured experimentally and subsequently be compared to theoretical predictions.

The reverberation times are very importants for the acoustics of a chamber, hall, etc. A ~~reverberation~~ time which would be too long or too short will lead to a poor (bad) acoustics of the chamber. The optimum value of τ_R depends on the destination and the volume of the chamber.

<u>Obs</u>: The reverberation phenomenon applies also to electromagnetic waves $\Longrightarrow$ electromagnetic reverberation chambers

$\Longleftrightarrow$ <u>cavity resonator with high Q factor</u>

$\Longleftrightarrow$ standing E.M. waves...

ULTRASOUNDS / ULTRA-ACOUSTICS

The ultrasounds are mechanical vibrations with a frequency higher than 20 KHz (in the range 20 kHz → 10 GHz)

They can be produced using either mechanical generators (up to 200 kHz) but also electro-acoustical generators

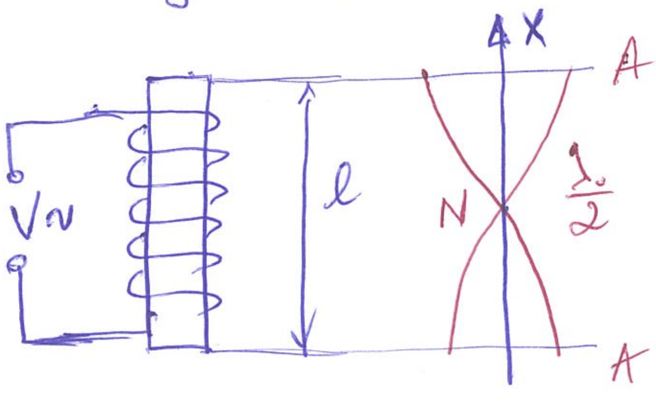
- For medium frequencies to produce ultrasounds one uses the magneto-striction effect, produced by a magnetic field on ferromagnetic materials. (Fe, Co, Ni, alloys...).

- For high frequencies one uses the electro-striction phenomenon or inverse piezo-electric effect. This effect appears in case of the crystal quartz placed in external electric field.

Due to the short wavelength of ultrasounds, typically they don't experience diffraction phenomena, therefore their propagation can be described by laws specific to the geometrical optics of reflection, refraction,...

① The magneto-strictive generator

The ferromagnetic materials (Fe, Co, Ni, alloys...) present the property to be deformed when introduced in external magnetic fields ⟹ magneto-striction effect.



An alternative magnetic field (produced by an alternative voltage V which leads to alternative current I), produces oscillations of the free ends of a ferromagnetic rod.

The frequency of these oscillations of the ends of the rod is controlled via the frequency of the current that produces the magnetic field.

The oscillation's amplitude is maximum if the frequency of the applied current is at resonance, e.g. at the extremities of the rod we have antinodes (A).

$$\Rightarrow \frac{\lambda_0}{2} = l \qquad \lambda = vT = \frac{v}{f}$$

$$v = \sqrt{\frac{E}{\rho}}$$

$$\Rightarrow \boxed{f_0 = \frac{1}{2l}\sqrt{\frac{E}{\rho}}}$$

E = longitudinal elasticity modulus

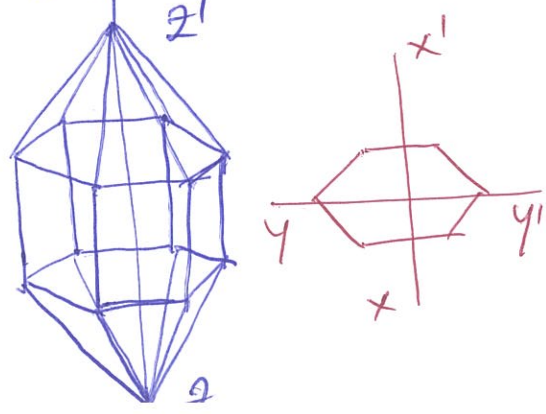
ρ = linear mass density of the rod.

One can see that:

$f_0 \nearrow$ if $\mu \searrow$ and $l \searrow$. From physical reasons the length l cannot be reduced infinitelly. This induces a high frequency limitation for the ultrasounds produced by this type of generator. The harmonics, even if they have higher frequencies, have the disadvantage of lower amplitude

② The piezo-electric generators

→ is based on the fact that a quartz crystal (SiO_2 with single crystal structure) changes its volume when placed in an external electric field (e.g. inside of a plane capacitor)



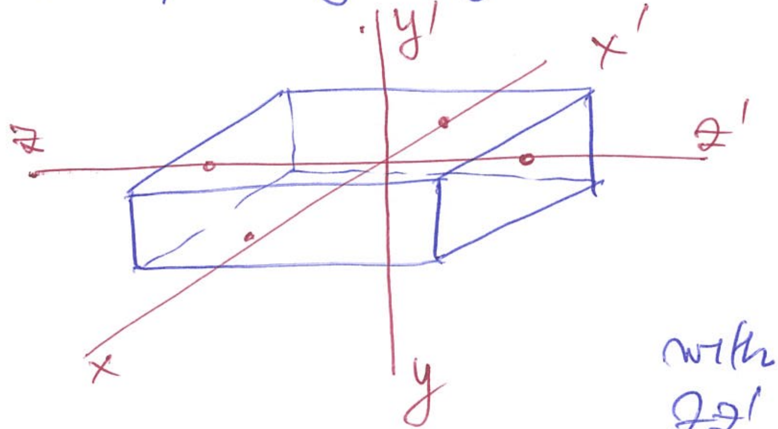
⇒ INVERSE PIEZOELECTRIC EFFECT

zz' = optical symmetry axis

xx' = mechanical symmetry axis

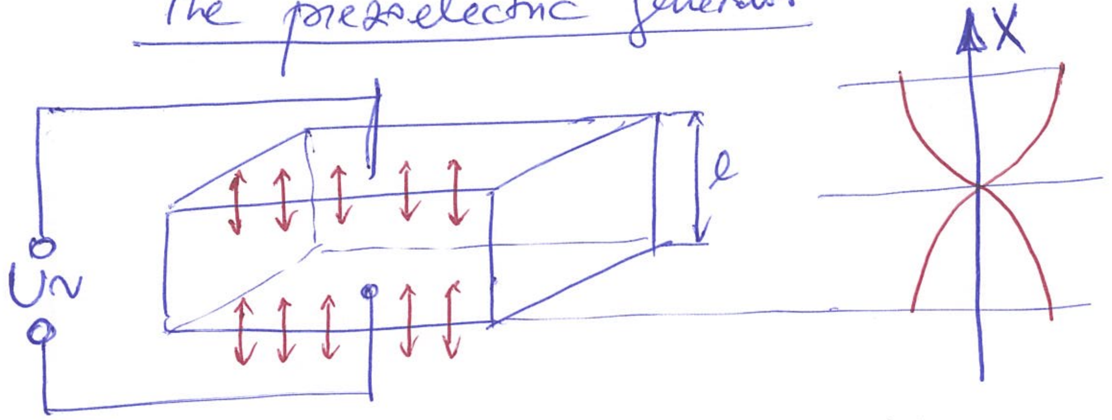
yy' = electrical symmetry axis

To provide piezo-electric properties, a crystal of quartz has to be cut from the original crystal in the following way:



The facets on which electrical charges appear will suffer mechanical deformations one parallel with the optical symetry axis zz' and perpendicular on the electrical axes yy'.

The piezoelectric generator



$$\frac{\lambda}{2} = l$$

If the electric field $E = \frac{U}{d}$ is variable with a certain frequency, sound waves can be produced with the same frequency.

Here again, the above discussion concerning the magnetostrictive generator concerning antinodes at extremities remain valid

$$\Rightarrow \boxed{f = \frac{1}{2l}\sqrt{\frac{E}{\rho}}}$$

l = the thickness

Because, from technological point of view, it much easier to cut the quartz slices very narrow (small l) than build feromagnetic bars with short l ⇒ the ultrasound frequency produced through electro-strictive effect can be significantly larger than the one of those produced by magneto-striction.

Phenomena specific to ultrasounds

Beyond conventional phenomena common to any elastic wave in case of the ultrasounds they are some specific phenomena, such as the cavitation.

The cavitation

is a specific phenomenon that appears during the propagation of sound waves with high energy in liquid media. The sound propagation takes place though compressions and expansion (successive) of the medium where the sounds travels in. In case of ultrasounds of high energies, during the expansions, usually in places where there are particles in suspension, air bubbles, vapors – the liquid can be fractured. In the respective places holes appear (cavities) towards which dissolved gases from liquid may be forwarded. In a next stage, corresponding to compression, the cavity reduces its volume and the pressure inside increases up to thousands of atmospheres (IMPLOSION).

The process of cavity formation is assisted by a local temperature enhancement (up to 5000 K) and even electrical discharges.

⚠ 6000 K = the temperature at the basis of the Sun's chromosphere.

Due to this huge pressure, the cavities break out producing intense hydraulic shocks. These effects are widely used in technics for mechanical engineering (cleaning in ultrasonic baths, drilling, polishing).

Applications of ultrasounds

The ultrasounds have multiple applications in technics, medicine, navigation, ... Having in view the way in which ultrasounds are implicated in different processes, their applications are classified as:

→ passive applications
→ active applications

] upon the way that the structure and the properties of the propagation medium are modified or not.

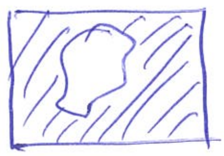
Active applications

→ solid materials engineering (polishing, cutting, welding...)
→ enhancement of speed of reaction for chemical processes
→ distruction of viruses, bacteria, micro-organisms
→ surgery / medicine (breaking of kidney stones)
→ dispersion of substances, sedimentation, filtering, emulsioning, extraction, crystalisation

Passive applications

⇒ the ultrasonic beams are used to obtain information about quality, shape, dimensions of investigated materials

→ ultrasonic defectoscopy



defects or inclusions imaged in inhomogeneous samples

→ echography

→ ultrasonic microscope (Sokolov) allows to get zoomed images of defects in samples

→ the sonar

→ the ultrasonic tomography.