ACOUSTICS

"Sound and hearing"

Acoustics deals with the study of mechanical waves in gases, liquids, solids.

Longitudinal waves in a medium (air) $\Rightarrow$ sound waves

Sound waves

Sound = longitudinal wave in a medium
(air, any gas, liquid, solid)

The simplest sond waves are sinusoidal waves with given:

- $\rightarrow$ frequency
- $\rightarrow$ amplitude
- $\rightarrow$ wavelength

The human ear is sensitive to waves with frequency:

- $f \in 20\,Hz - 20\,kHz$ $\Rightarrow$ AUDIBLE RANGE
- $f > 20\,kHz$ $\Rightarrow$ ULTRASONIC waves
- $f < 20\,Hz$ $\Rightarrow$ INFRASONIC waves

Sound waves travel in all direction from the source with an amplitude that depend on direction and distance from the source.

Idealized case: sound propagate along positive x direction only $\Rightarrow$ wave described by $\psi(x,t)$

$$\boxed{\psi(x,t) = A\cos(kx - \omega t)}$$

wave propagating along $+x$.

In a longitudinal sound wave, the particle displacement $\psi(x,t)$ is along the propagation direction.

Sound waves as pressure fluctuations

The human ear, microphones, ..., sense the pressure fluctuations. (put in motion the eardrum).

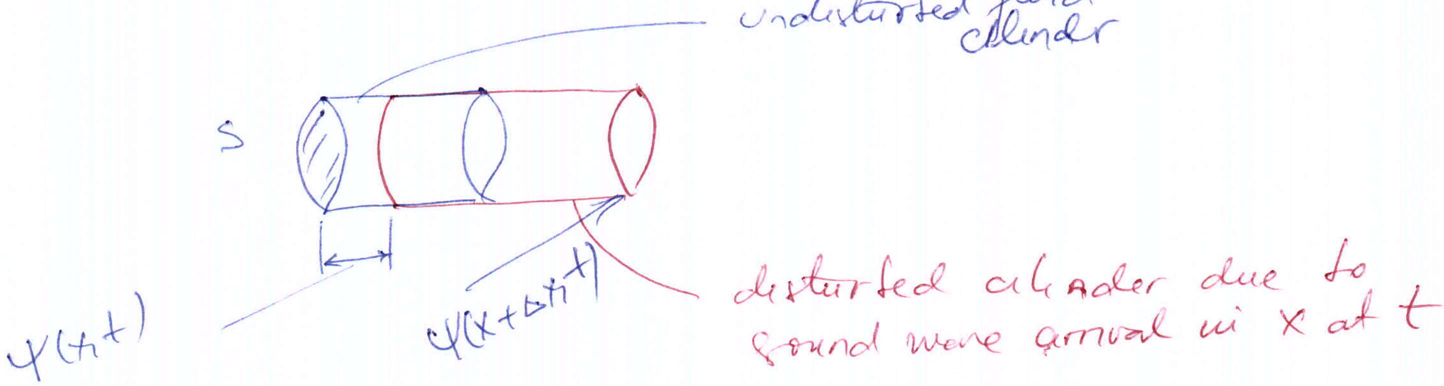
$\Rightarrow$ $p(x,t)$ represents the instantaneous pressure fluctuation in a sound wave at point x and time t

$\Rightarrow$ the amount by which the pressure differs from the average pressure p_a.

$\Rightarrow$ gauge pressure, it can be either positive or negative

$\Rightarrow$ the absolute pressure: $p_a + p(x,t)$.

Relation between $\psi(x,t)$ and $p(x,t)$



a sound wave displaces the left side of the cilinder with $\psi(x,t)$ and the right side by $\psi(x+\Delta x,t)$.

The bulk modulus:

$$B = -\Delta p \frac{1}{\frac{\Delta V}{V_0}} \Longrightarrow$$

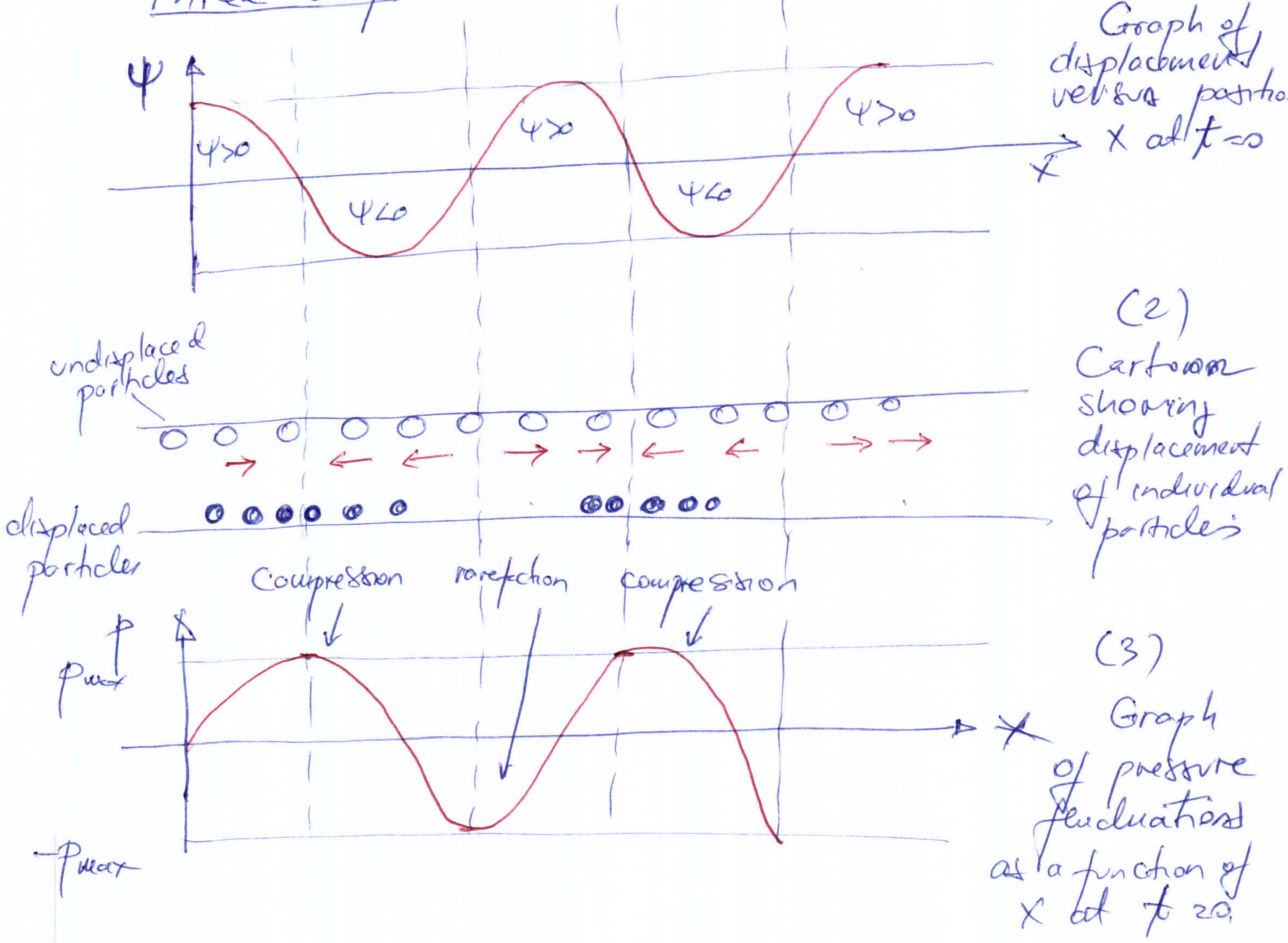
$$\frac{dV}{V} = \lim_{\Delta x \to 0} \frac{S[\psi(x+\Delta x,t) - \psi(x,t)]}{S\Delta x} = \frac{\partial \psi(x,t)}{\partial x}$$

$$p(x,t) = -B\,\frac{\partial \psi(x,t)}{\partial x}$$

For a sinusoidal wave: $\psi(x,t) = A\cos(kx-\omega t)$

$$\boxed{p(x,t) = BkA\,\sin(kx-\omega t)}$$

Three ways to describe a sound wave:



From: $p(x,t) = BkA\,\sin(kx-\omega t)$

$$\Rightarrow \boxed{p_{max} = BkA} \qquad k = \frac{2\pi}{\lambda} \Rightarrow \boxed{p_{max} = \frac{2\pi BA}{\lambda}}$$

- waves of shorter λ have larger pressure variation for a given amplitude (maxima and minima are squeezed close together).

Perception of sond waves

→ related to the perception by the listener.
For a given frequency, the greater the pressure amplitude of a sinusoidal sound wave, the greater the perceived loudness.

The relationship between pressure amplitude and loudness is complex and varies from one person to another.

The ear is not equally sensitive for all frequencies in the audible range.

Pitch of the sound → sounds are classified in:
[tonul]$_{RO}$ → low ($f \downarrow$)
→ high ($f \uparrow$)

~ frequency f

Spectral composition of sounds ⇒ timbre
[timbrul]$_{RO}$

Musical sounds have wave functions more complicated than simple sine function. => Fourrier analysis

→ [fundamental frequency] + [harmonics]

Two tones produced by different instrument may have same fundamental frequency but different harmonic content => they sound different
=> tone color, or TIMBRE

Noise => combination of all frequencies, not only frequencies which are multiple of fundamental

white noise = equal amount of all frequencies within audible range

Examples of white noise

sound of the wind, river water (cascade), hissing sound you make when pronouncing the conson "s".

Speed of sound waves

$$\boxed{v = \sqrt{\frac{B}{\rho}}}$$ in a fluid (B = bulk modulus)

$$\boxed{v = \sqrt{\frac{Y}{\rho}}}$$ in a solid (Y = Young modulus)

Table speed of sound in various media

	Material	Speed of sound (m/s)
Gases:	Air (20°C)	344 m/s
	He (20°C)	999 m/s
	H (20°C)	1330 m/s
Liquids	He (4K)	211
	Hg (20°C)	1451
	water (0°C)	1402
	water (20°C)	1482
	water (100°C)	1543
Solids	Al	6420
	Lead	1960
	steel	5941

Speed of sound in a gas (air)

$B = \gamma p_0$ $\qquad$ γ = ratio of heat capacities (1,4 air)

at the normal atmospheric pressure:

$p_0 = 1{,}013 \cdot 10^5\ Pa \Rightarrow$

$B = 1{,}40 \cdot 1{,}013 \cdot 10^5 = 1{,}42 \cdot 10^5\ Pa$

The gas density depends also on pressure $\Rightarrow$ $\frac{B}{\rho}$ does not depend on pressure but only on temperature

$$\boxed{v = \sqrt{\frac{\gamma R T}{M}}}$$

T = absolute temperature (K)

R = perfect gas constant = 8,314 J/molK

M = mol mass (mass of a mole of ideal gas).

For any particular gas, γ, R, M are constant $\Rightarrow$

$$\boxed{v = \sqrt{T}}$$

Speed of sound in air

$T = 20°C = 293K$

The mean molecular mass of air (N_2/O_2) $= 28{,}8 \cdot 10^{-3}$ kg/mol

$\gamma = 1{,}4$

$\Rightarrow v = \sqrt{\frac{\gamma R T}{M}} = 344\ m/s$

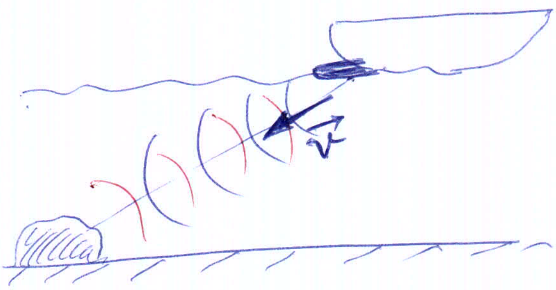
Using: $\lambda = \frac{v}{f}$ $\Rightarrow$

at 20°C $\lambda = 17\,m$ for $f = 20\,Hz$

$\lambda = 1{,}7\,cm$ for $f = 20\,kHz$

Applications

① Sonar waves



Sonar system uses underwater sound waves to detect and locate submerged objects

$$\lambda = \frac{v}{f} = \frac{\sqrt{B/\rho}}{f}$$

for $f = 262\,Hz$ $\quad \underline{\lambda = 5{,}65\,m}$

dolphins emit high-frequency sound waves ($f \simeq 100\,kHz$) and use echos for guidance and hunting.
The corresponding wavelength in water is:

$$\lambda = \frac{v}{f} = \frac{\sqrt{B/\rho}}{f} = \underline{1{,}48\,cm}$$

With this high frequency sonar they can sense objects that are roughly as small as the wavelength.

ultrasonic imaging

→ medical technique using exactly the sonar principle
sond waves of very high frequency (short wave length) called ultrasounds are scanned over the human body and the "echoes" from inner organes are used to create an image.

With ultrasound of $f = 5\,MHz = 5 \cdot 10^6\,Hz$,
λ in water, principal constituent of human body is
$\lambda = 0{,}3\,mm$ $\Rightarrow$ features as small as this can be discerned in images

→ ultrasounds are used also for the study of heart-valve action, detecting tumors, prenatal examinations

→ ultrasounds are more sensitive than X rays in distinguishing various types of tissues and don't have the radiation hazards associated with X ray imaging.

Sound intensity

Wave intensity I = average rate at which the energy is transferred per unit area across a surface which is perpendicular to the direction of propagation.

$$y(x,t) = A\cos(kx-\omega t)$$
$$p(x,t) = BkA\sin(kx-\omega t)$$

The particle velocity:

$$v_y = \frac{\partial y(x,t)}{\partial t} = \omega A\sin(kx-\omega t)$$

$$\frac{\text{The power}}{\text{unit area}} = \frac{\text{Force}}{\text{unit area}} \times \text{velocity} = \text{pressure} \times \text{velocity}$$

$$p(x,t)\, v_y(x,t) = BkA\sin(kx-\omega t)\cdot \omega A\sin(kx-\omega t)$$
$$= B\omega kA^2 \sin^2(kx-\omega t)$$

The intensity is, by definition, the time average of $p(x,t)\, v_y(x,t)$

$$\underset{\text{over a period } T}{\langle \sin^2(kx-\omega t)\rangle} = \frac{1}{2}$$

$$\Rightarrow I = \frac{1}{2} Bk\omega A^2$$

$$v^2 = \frac{B}{\rho}\ ; \quad \omega = vk$$

$$\Rightarrow \boxed{I = \frac{1}{2}\sqrt{\rho B}\ \omega^2 A^2}$$

intensity of a sinusoidal sound wave

This eq. shows why in a stereo system a low frequency woofer has to vibrate at much larger amplitude A than a high frequency tweeter to produce the same intensity of sound.

Intensity and pressure amplitude

$$p_{max} = BkA \implies A = \frac{p_{max}}{Bk}$$

$$I = \frac{1}{2}\sqrt{\rho B}\,\omega^2 A^2 = \frac{1}{2}\sqrt{\rho B}\cdot v^2 \cancel{k^2} \frac{p_{max}^2}{B^2 \cancel{k^2}}$$

$\omega = vk$

$$= \frac{1}{2}\sqrt{\rho B}\cdot \frac{B}{\rho}\frac{p_{max}^2}{B^2}$$

$$\boxed{I = \frac{1}{2}\frac{p_{max}^2}{\sqrt{\rho B}}}$$

intensity of a sinusoidal sound wave.

The total average power carried across a surface $\perp$ to the propagation direction by the sound is the product [intensity] $\times$ [area] (if I is constant over area)

Ex: The average total sound power emitted by:

→ a person speaking $\sim 10^{-5}$ W

→ loud speaker $\sim 10^{-2}$ W

→ total people of New-York speaking simultaneously (~ 8,3 billions)
~ 100 W ($\simeq 1$ electric bulb)

- If the sound source emits in <u>all directions equally</u> the intensity decreases with increasing r

$$\boxed{I \sim \frac{1}{r^2}}$$

- If the sound go predominantly in one direction (ex. shouting with putting hands around mouth) the decay is smaller than $1/r$.

• The $1/r^2$ decay does not apply indoor due to multiple reflection on walls

$\Rightarrow$ intelligent architecture room design to provide

$$\dot{I} = ct$$

The decibel scale

Because the ear is sensitive over a broad range of intensities, a <u>logarithmic intensity scale</u> is used

The <u>sound intensity level:</u> β of a sound wave is defined by the equation:

$$\boxed{\beta = (10\ db) \log \frac{\dot{I}}{I_0}}$$

$[\beta]$ = decibels

$\dot{I}_0 = 10^{-12} \frac{W}{m^2}$ = reference intensity
$\approx$ treshold of human hearing at 1 kHz

$1 dB = \frac{1}{10}$ bell (Alex. Graham Bell - inventor of telephone)

if $\dot{I} = 10^{-12} \frac{W}{m^2} \Rightarrow \beta = 0\ db$

$\dot{I} = 1 \frac{W}{m^2} \Rightarrow \beta = 120\ db$

maximum noise allowed in a working place by security reasons: $< 75\ dB \Rightarrow$ use special protection headphones

ex
- $\to$ military jet aircraft 30 m away $\Rightarrow$ 140 dB $\quad 10^2\ W/m^2$
- $\to$ pain treshold $\Rightarrow$ 120 dB $\quad 1\ W/m^2$
- $\to$ average whisper $\Rightarrow$ 20 dB $\quad 10^{-10}\ W/m^2$
- $\to$ ruste of leaves $\Rightarrow$ 10 dB $\quad 10^{-11}\ W/m^2$

STANDING SOUND WAVES AND NORMAL MODES

When longitudinal (sound) waves propagate in a fluid in a pipe with finite length, the waves are reflected at the ends in the same way that transverse waves in a string are reflected at its end.

The superposition of waves traveling in different directions (opposite) ⟹ standing waves:

Standing wave in a pipe (normal modes) can be used to create sounds in air ⟺ operating principle of human voice, musical instruments pipe organs.

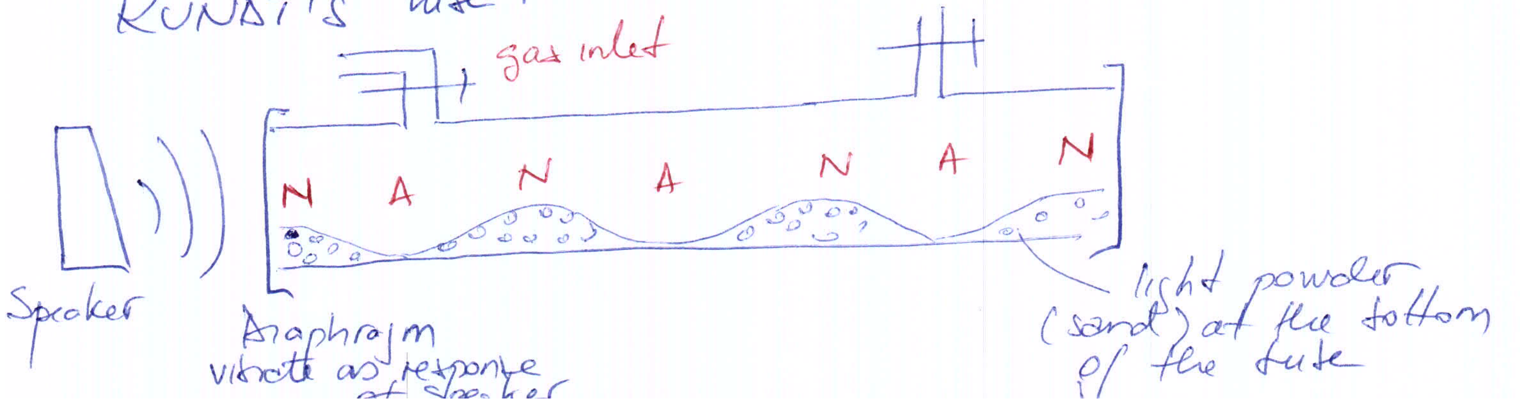
Transverse waves are commonly described in terms of displacement of the particle. Sound waves are described either in terms of particle displacement or pressure fluctuations. ⟹

displacement nodes	⟶	pressure antinodes
displacement antinodes	⟶	pressure nodes

$y = A\cos(kx - \omega t)$ $\qquad$ $p(x,t) = BkA \sin(kx - \omega t)$

Example 1

We can demonstrate standing sound wave using the KUNDT'S tube.



As we vary the frequency of the sound, we pass through frequencies at which the amplitude of the standing waves is large enough to allow the powder to be sweeped along the tube at those parts where the gas is in motion. The powder collects at the displacement nodes (where the gas is not moving)

Adjacent nodes are separated by $\lambda/2$, which can be measured.

Given the wave length, one can measure the wave speed:

$$v = \lambda f$$ f - known from the oscillator

Example 2

The sound of science.

When a sound wave is directed at a wall, it interfere with the reflected wave to create a standing wave. $\Rightarrow$ nodes and antinodes for the displacement

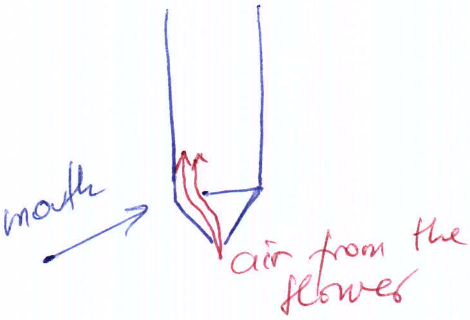
The ear detects the pressure variations in the air. You hear no sound at pressure nodes (displacement antinode).

The wall is a displacement node, the distance to adjacent antinode is $\lambda/4$ and node $\lambda/2$.

$\Rightarrow$ in displacement antinode ($\lambda/4$ distance from the wall) the ear hears no sound (pressure node).

Organ pipes, wind/flowing instruments

organ pipes → air supplied by a blower at a gauge pressure $\sim 10^3\,Pa$ ($10^{-2}\,atm$) to the bottom end of the pipe.



The column of air in the pipe is set in vibration ⇒ series of possible normal modes as in the stretched string.

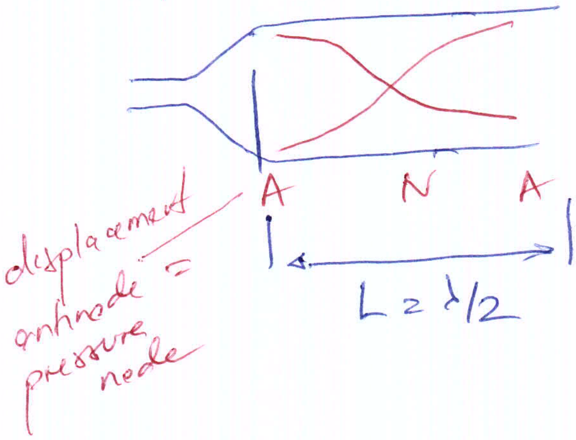
The <u>mouth of the pipe</u> allways acts as an <u>opened end</u>

⇒ pressure node = <u>displacement antinode</u>

The other end can be <u>opened</u> or <u>closed</u>

⇒ – opened pipe
– closed pipe.

Opened pipe



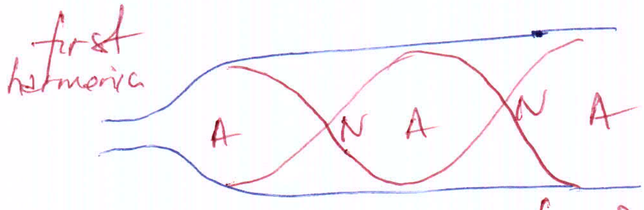
<u>fundamental</u>

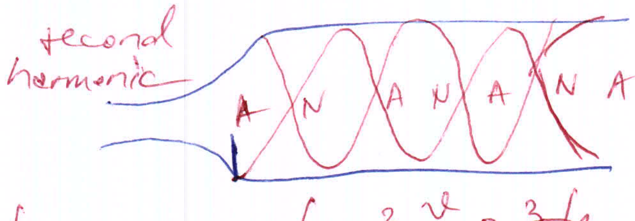
$\lambda/2 = L$ lenght of the pipe

$\Rightarrow f = \frac{v}{\lambda}$;

$$\boxed{f_1 = \frac{v}{2L}}$$

<u>harmonics</u> : $\boxed{f_n = n\frac{v}{2L}}$ $n = 2, 3, \ldots$





Stopped pipe $\Rightarrow$ a pipe opened at one end and closed off the other end

opened end $\Leftrightarrow$ displacement antinode (pressure node)

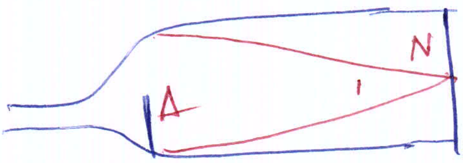
closed end $\Leftrightarrow$ displacement node (pressure AN)

The distance between a displacement A and N is $\lambda/4$

$\Rightarrow$ fundamental $L = \lambda/4 \Rightarrow f_1 = \frac{v}{\lambda} = \frac{v}{4L}$

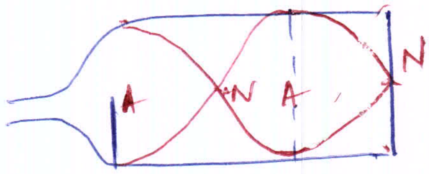
$$\boxed{f_1 = \frac{v}{4L}}$$

harmonics : $f_n = n\frac{v}{4L}$



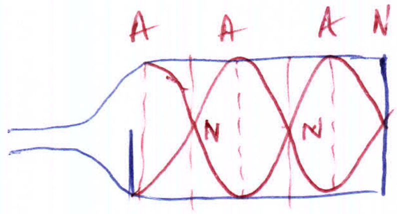
$L = \frac{\lambda}{4}$

fundamental



$L = 3\frac{\lambda}{4}$

third harmonic



$L = 5\frac{\lambda}{4}$

fifth harmonics

only odd-harmonics are possible.

Obs : → in an organ pipe in use, several modes are allways present at once $\Rightarrow$ the motion of the air is a superposition of these modes (see analogy with the plucked string).

$\Rightarrow$ harmonic content $\Rightarrow$ timber

→ from $f = n\frac{v}{2L}$ or $f = n\frac{v}{4L}$ $\Rightarrow f \sim v$

but $v \sim \sqrt{T}$ $\Rightarrow$ $\boxed{f \uparrow \text{ when } T \uparrow}$ $\Rightarrow$ it is important to have all the pipes from organ at the same temperature otherwise the sound seems to be out of tone.

RESONANCE AND SOUND

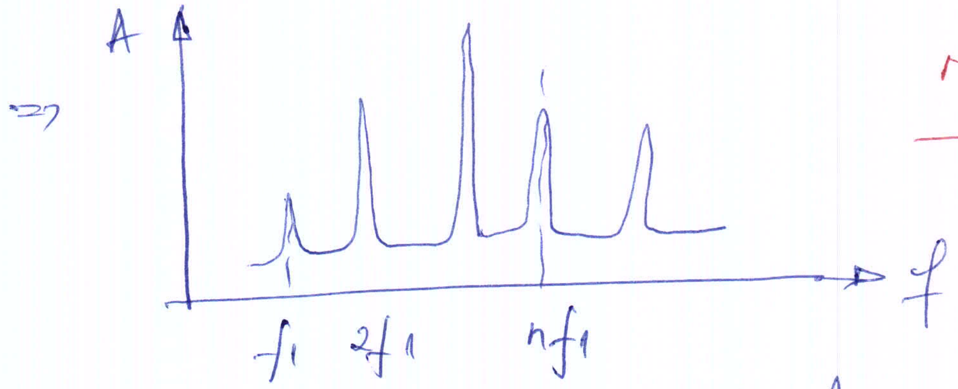
Mechanical systems have normal modes of oscillations. In each mode, each particle of the system oscillote with SHM at the same frequency of the mode.

Air columns in pipes or stratched strings have infinite series of normal modes.

Suppose we apply a periodically varying force to the system that oscillates => forced oscillations with the frequency of the driving force.

In the forced oscillations regime the RESONANCE occurs when the frequency of the driving force is equal to the one of a normal mode.

=>



resonance curve

⚠ Close to f_i, A increases, at f_i the A is maximum.

If there were be no dissipation mechanism, a driving force at a normal mode frequency would continue to add energy to the system, and the amplitude would increase indeffinitelly. However, in reality dissipation occurs and, therefore, the amplitude of the resonant peaks is finite due to the damping.

Examples: The "Sound of the ocean" produced when we put the ear close to a sea shell is due to the resonance of the air (normal modes) inside the sea-shell => shape dependent

BEATS

We analyzed the interference when two waves with the same frequency superpose.

Now, let's consider the case when 2 waves with slightly different frequency but equal amplitude interfere

$$y_1 = A \sin (kx + \omega_1 t) \Rightarrow A \sin \omega_1 t = A \sin 2\pi f_1 t$$

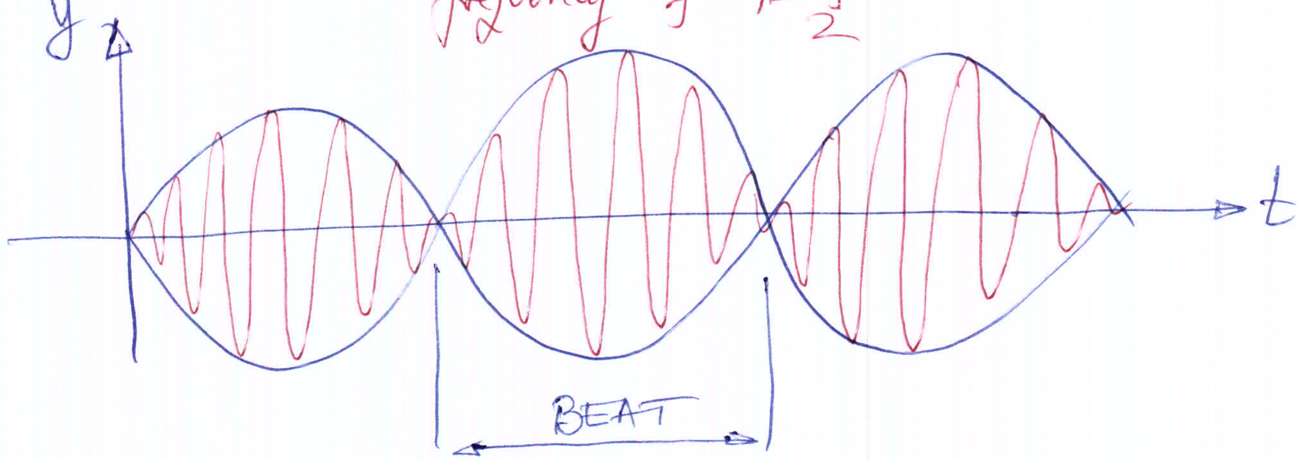
$$y_2 = A \sin (kx - \omega_2 t) \Rightarrow A \sin (-\omega_2 t) = -A \sin 2\pi f_2 t$$

at $x = 0$

$$y = y_1 + y_2 = A \left[\sin(2\pi f_1 t) - \sin(2\pi f_2 t) \right]$$

$$\sin a - \sin b = 2 \sin \frac{a-b}{2} \cos \frac{a+b}{2}$$

$$\Rightarrow \boxed{y(t) = \underbrace{\left(2A \sin \frac{1}{2} 2\pi (f_1 - f_2) t\right)}_{\text{slow variation amplitude with frequency } f = \frac{f_1 - f_2}{2}} \underbrace{\cos \frac{1}{2} (2\pi)(f_1 + f_2) t}_{\text{rapid variation with } \frac{f_1 + f_2}{2}}}$$



The amplitude variation causes variation of loudness called beats.

The amplitude factor

$$A' = 2A \sin\left(2\pi \frac{f_1 - f_2}{2} t\right)$$

-7-

The sound intensity $\sim A^2$, and the frequency of $\sin^2\left(2\pi \frac{f_1-f_2}{2} t\right)$ is $f_{beat} = 2\,\frac{f_1-f_2}{2} = f_1-f_2$

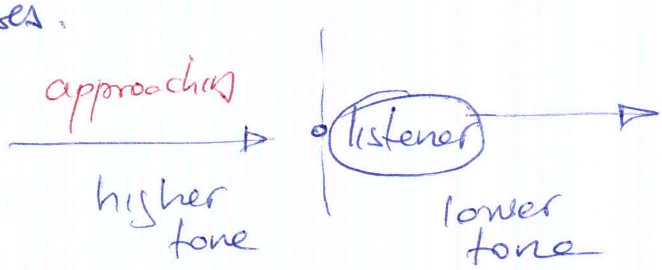
Applications

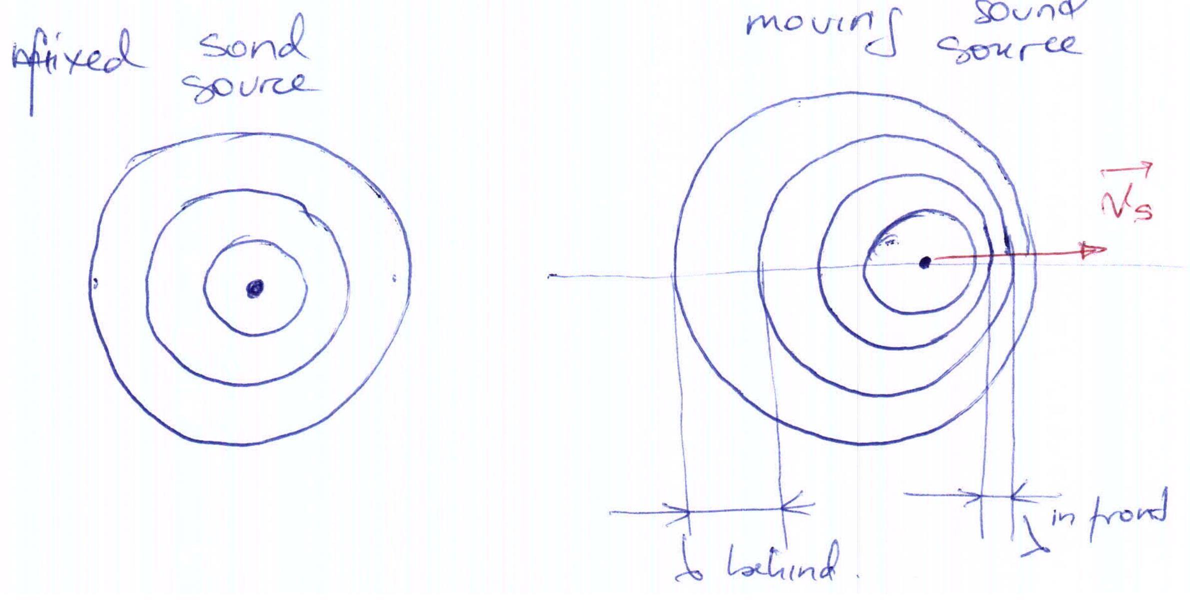
→ The engines of a multiengine plane have to be synchronized so that their sound do not create anoying beats. One some planes, this is done electronically, on others this is done manually by the pilote, as like tuning a piano

→ In modern radars the beats phenomena is used to measure the doppler shift of waves ~ velocity of the moving scanned object. (see later the doppler effect).

THE DOPPLER EFFECT

We noticed that when a car approaches with its horn sounding the pitch seems to drop when the car passes.





$$\lambda^{\text{in-front}} = \frac{v - v_s}{f_s}$$ compressed waves

f_s = frequeny of the sond emitted by the source

v_s = speed of the moving source

v = speed of the sound in air.

$$\lambda_{\text{behind}} = \frac{v + v_s}{f_s}$$ stretched out waves

A listener (in rest) will hear a sound with the frequency:

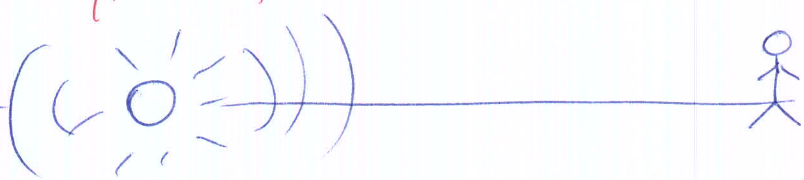
$$f_L = \frac{v}{\lambda^{\text{in-front}}} \quad \text{or} \quad \frac{v}{\lambda^{\text{behind}}}.$$

$$\Rightarrow \boxed{f_L = f_s \frac{v}{v \mp v_s} = f_s \frac{1}{1 \mp \frac{v_s}{v}}}$$

in front (approaching source) $\longrightarrow f_L = f_s \dfrac{1}{1 - \frac{v_s}{v}}$ higher frequency

behind (moving away source) $\Rightarrow f_L = f_s \dfrac{1}{1 + \frac{v_s}{v}}$ lower frequeny.

Doppler effect for electromagnetic waves $\boxed{v = c}$

sorce  obsener

$\Longrightarrow$ relativistic approach

Approaching source $\boxed{f = \sqrt{\frac{c+u}{c-u}}\, f_0}$ $\boxed{f > f_0}$

blue-shift ($\lambda < \lambda_0$)

where c = speed of light (electromagn wave)

u = speed of the source

Moving away source $\boxed{f = \sqrt{\frac{c-u}{c+u}}\, f_0}$ $\boxed{f < f_0}$

red-shift ($\lambda > \lambda_0$)

The red-shift of the light emitted by galaxies in univers demonstrate the expansion of the Universe (moving-away source case).

Application Doppler-radar

A rador device is monted on the side window of a police car to check other's car's speed. The electromagnetic wave emitted by the device is reflected by a moving car which acts as a moving source. Therefore, the wave reflected back to the device is Doppler shifted in frequency.

The transmitted and reflected signal are combined to produce beats and the speed can be computed from the frequency of the beats.

Similar techniques ("Doppler radar") are used to measure wind velocities in the athmosphere.

Doppler echo-cardiography => procedure that uses ultrasounds technology to examine the heart and blood vessels.

An echo-cardiogram uses high frequency sound waves to create an image of the hearth while the use of the Doppler technology allows determination of the speed of the blood and its direction using the Doppler effect. Velocity measurements allow investigation of the cardiac ~~valves~~ areas and functions

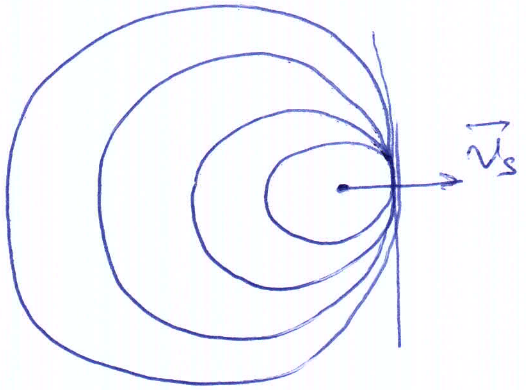
SHOCK WAVES

When an airplane flies with a speed higher than the speed of the sound in the air one can hear sonic booms.

The motion of the plane in the air produces sound. If the planes moves with v_s, and $v_s < v$ (speed of sound in air) the waves in front are crowded together with

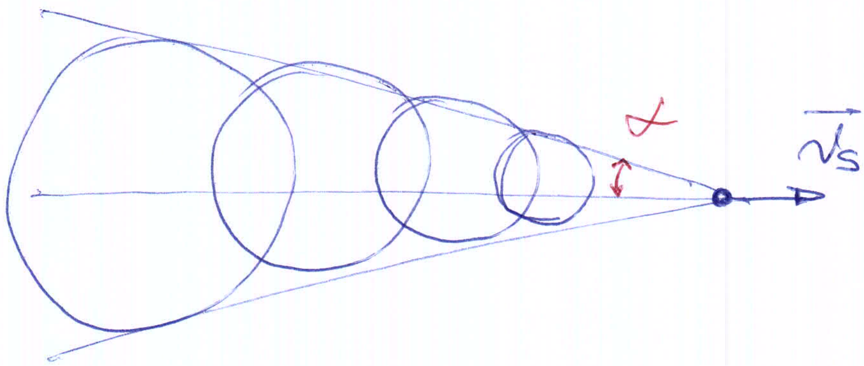
$$\lambda^{in\text{-}front} = \frac{v - v_s}{f_s}$$

when $v \to v_s \Rightarrow \lambda \to 0$



when $v_s > v \Rightarrow$ source of sound is supersonic, the eq. deduced for Doppler effect are no longer valid.

The wave fronts remain behind the emitting source.



Circular crests interfere constructivelly at positions along a line called <u>shock wave line</u> where the resulting amplitude is very large:

$$\boxed{\sin\alpha = \frac{v}{v_s}} \qquad \boxed{v_s \to v \Rightarrow \sin\alpha \simeq 1 \quad \alpha \simeq \pi/2}$$

v_s = source speed relative to the air

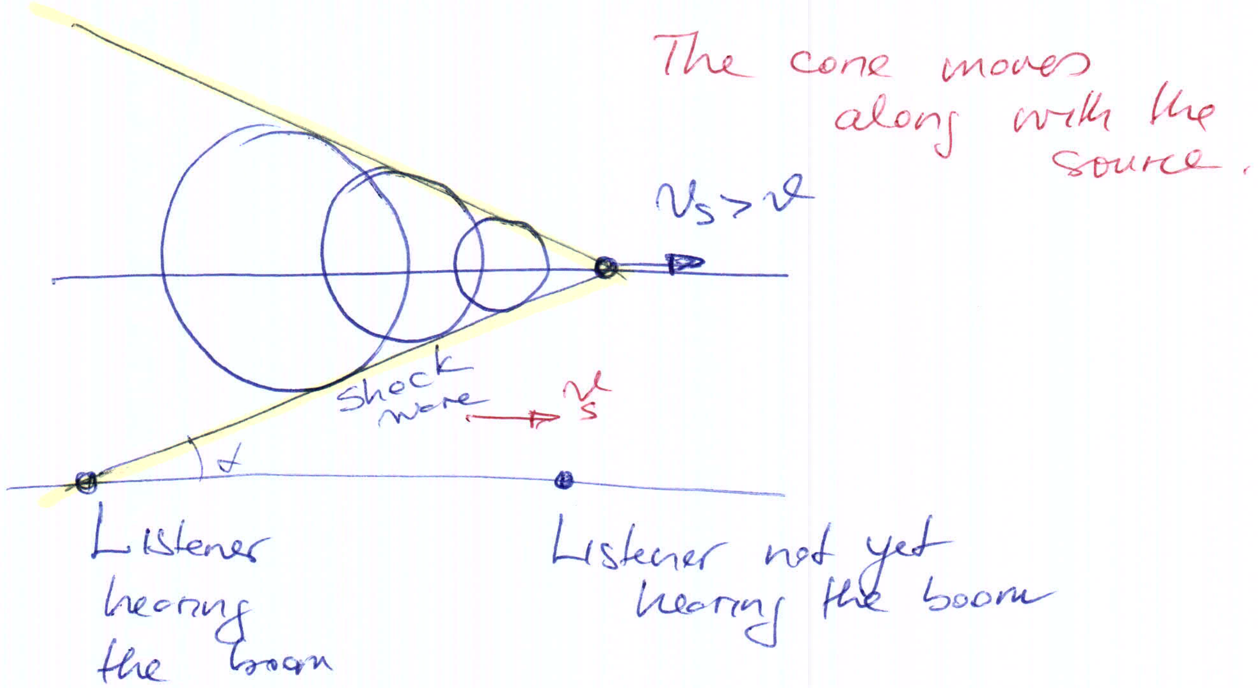
the ratio $\boxed{\frac{v_s}{v} = \frac{1}{\sin\alpha}}$ = <u>MACH NUMBER</u>

MACH NUMBER >1 $\Rightarrow$ <u>SUPERSONIC SPEED</u>

Hystorically, the first person breaking the sound barrier was Cpt. <u>Chuck Yeager</u> of US Air Force flying the Bell X-1 at Mach 1,06 in Oct. 14/1947.

<u>shock waves are 3D</u> $\Rightarrow$ they form a cone around the plane direction.

They produce a pressure variation of about 20 Pa for a Concorde flying at 12000 m, when the shock wave arrives at the Earth's surface. In front of the shock wave there is no sound. Inside the cone, a stationary listener would hear the Doppler shifted sound of the plane moving away.



Shock waves are produced continuously by any object that moves in air at $v_s > v$ (supersonic speed). The listener hears the boom when the shock wave arrives to the listener's place.

Other examples of sonic booms

→ cracking noise of a bullet

→ cracking noise of the tip of a circus whip

are due to their supersonic motion.