COURSE #2

DYNAMICS

Particles' dynamics

- → Newton's laws of motion
  forces, applying Newton's laws
- → Work, kinetic and potential energy
  Power. Conservation laws.
- → Momentum, impulse and collisions.
  Conservation laws.

1 Newton's laws of motion

Dynamics is a chapter of physics (mechanics) which studies the relationship between <u>the forces</u> acting on a body and <u>the movement</u> of the body.

"<u>dynamics</u>" comes from the Greek "<u>dinamis</u>" = power.

2 new concepts:
- → forces
- → mass

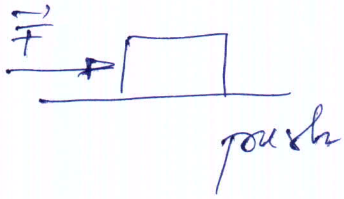
⇒ <u>principles of dynamics</u>
- stated out by Isaac Newton (1642-1727)
  ⇒ <u>Newton's laws of motion</u>

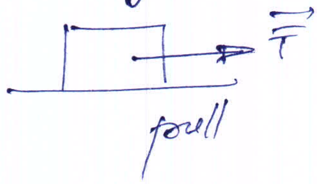
Newton did not derived the three laws but deduced them from analysis of multitude of experiments performed before him (especially by Galileo Galilei)

Newton's laws are the basis of the classical mechanics also called <u>Newtonian mechanics</u>.

Force and interaction

Scientifically a force is an <u>interaction</u> between two bodies or between a body and its environment





Newton 1st law

> A body either remains in rest or it keeps its movement in straight line with constant velocity if no net force acts on it.

$$\boxed{\vec{a} = 0 \quad \text{when} \quad \vec{F} = 0 \quad \text{or} \quad \sum \vec{F_i} = 0}$$

1st law is known as law of <u>inertia</u>.

Newton 2nd law

when the net force $\sum \vec{F_i} = \vec{F} \neq 0$

> A net force acting on a body causes the body to accelerate in the same direction by the net force.
>
> $$\vec{F} = \sum \vec{F_i} = m\vec{a}$$

The ratio: $\dfrac{|\vec{F}|}{|\vec{a}|} = ct$ regardless on the magnitude of the force

We call this ratio the <u>inertial mass</u> $\boxed{m = \dfrac{|\vec{F}|}{|\vec{a}|}}$

- mass is a measure of inertia (see 1st law)
- the unit of mass $[m]_{SI} = kg$
- the unit of Force $[F]_{SI} = N$ (newton)

$$1N = 1kg \cdot 1\,m/s^2$$

1N = the amount of a net force required to give an acceleration of 1 m/s² to a body with the mass of 1 kg.

- For multiple forces, the 2nd principle is:

$$\boxed{\sum \vec{F_i} = m\vec{a}}$$

If the movement is studied using a Cartesian coordinate system:

$$\vec{F} = F_x\vec{i} + F_y\vec{j} + F_z\vec{k}$$

$$\vec{a} = a_x\vec{i} + a_y\vec{j} + a_z\vec{k}$$

$$\Rightarrow \begin{cases} F_x = ma_x = m\dfrac{d^2x}{dt^2} \\ F_y = ma_y = m\dfrac{d^2y}{dt^2} \\ F_z = ma_z = m\dfrac{d^2z}{dt^2} \end{cases}$$

2nd law ⇔ set of 3 differential equations.

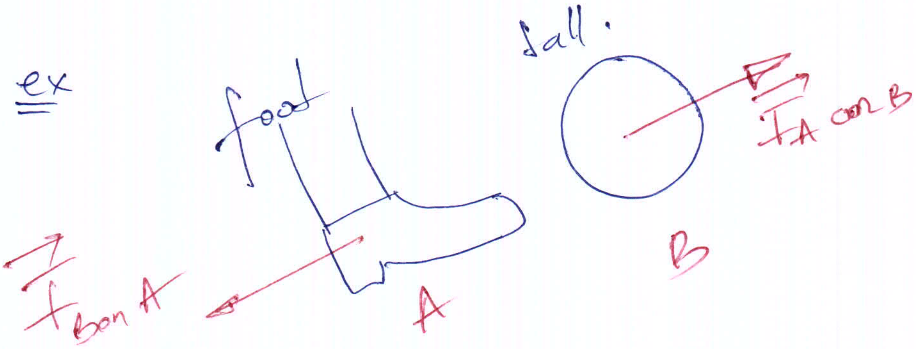
↳ decomposition of mechanical movement.

Newton's 3rd law (action-reaction)

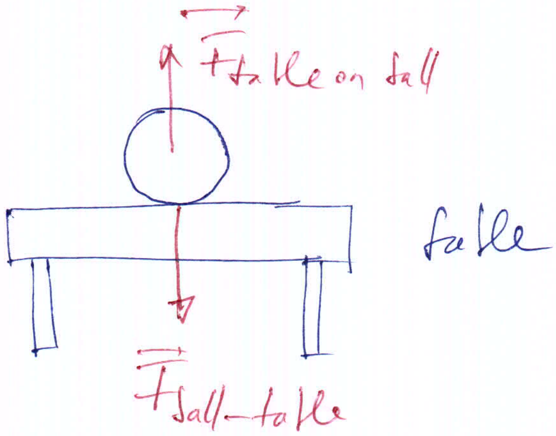
If a body A exerts a force on a body B (ACTION) then body B will exert a force on A (REACTION). These two forces are equal in magnitude but have opposite directions. These two forces act on different bodies

$$\boxed{\vec{F}_{12} = -\vec{F}_{21}}$$

ex

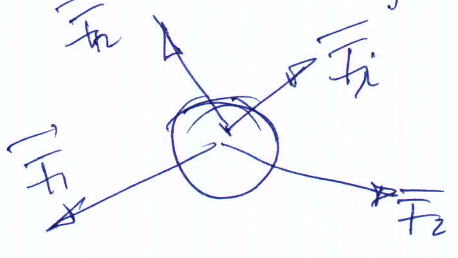
① 

$$\vec{F}_{A\to B} = -\vec{F}_{B-A}$$

② 

Superposition of forces

- many forces acting on a same body



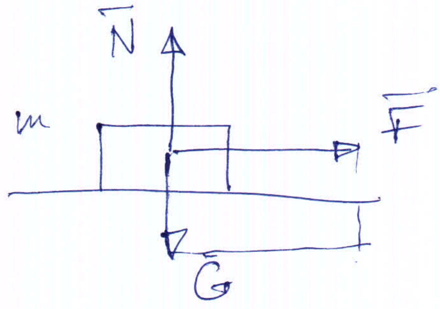
The total force

$$\sum \vec{F}_i = \vec{F}$$

if: $\sum \vec{F_i} = 0$ body in rest

$\sum \vec{F_i} \neq 0 \Rightarrow \boxed{\vec{a} = \frac{\sum \vec{F_i}}{m}}$

ex



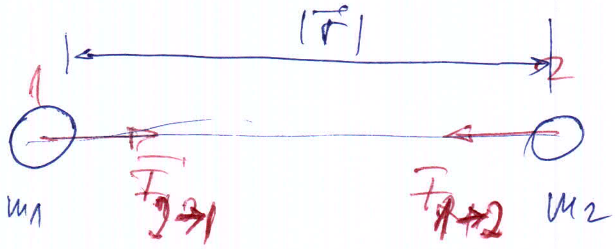
$\vec{F}_{rez} = \vec{F} + \vec{N} + \vec{G} = \vec{F}$

$\vec{N} + \vec{G} = 0$

Types of forces

(1) Gravitational force (gravity = weight) $\vec{G}$

law of universal atraction (Newton) 1698



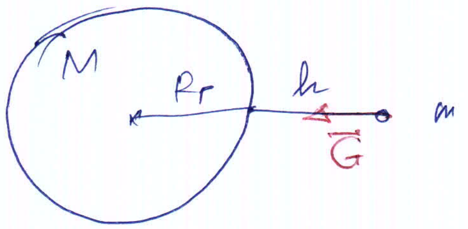
3rd principle. $\vec{F}_{1\to2} = -\vec{F}_{2\to1} = \vec{F}_g = \vec{G}$

Any material particle of the Universe attracts another particle with a force direct proportional with the masses product and inverse proportional with the square distance between the particles

$$\boxed{F_g = k\frac{m_1 m_2}{r^2}}$$

$K = 6,67\cdot10^{-11}\ Nm^2kg^{-2}$ universal atraction constant

Weight: the force with which a body is attracted by the Earth.



$$\boxed{G = k\frac{mM}{(R+h)^2}} =$$

m = body mass
M = Earth mass
R = Earth radius
h = altitude

If the body is located near the Earth's surface:
$h \ll R$

$$\Rightarrow G \simeq \underbrace{\frac{kM}{R^2}}_{g_0} m = m g$$

g ← gravitational acceleration or the intensity of the gravitational field

$$\begin{cases} \vec{G} = m\vec{g} \\ \vec{F_e} = q\vec{E} \end{cases}$$ analogy with electrostatics.

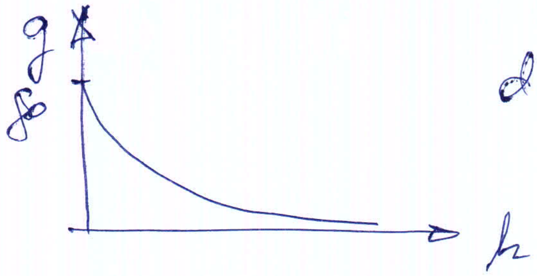
$\vec{E}$ ← electric field intensity

$$\left.\begin{array}{l} R = 6370\ km \\ M = 5{,}96 \cdot 10^{24} kg \end{array}\right\} \Rightarrow g_0 = 9{,}8\ m/s^2.$$

Variation with altitude

$$G = \frac{kMm}{(R_p+h)^2} = mg \Rightarrow g = \frac{KM}{(R+h)^2}$$

$$g = \frac{KM}{(R+h)^2} = \underbrace{\frac{KM}{R^2}}_{g_0} \cdot \frac{R^2}{(R+h)^2} = g_0\left(\frac{R}{R+h}\right)^2$$



decay with altitude

Variation with the planet mass

$\frac{M_{Earth}}{M_{moon}} \Longrightarrow$

$M_{Moon} = 7{,}4 \cdot 10^{22} \, Kg$

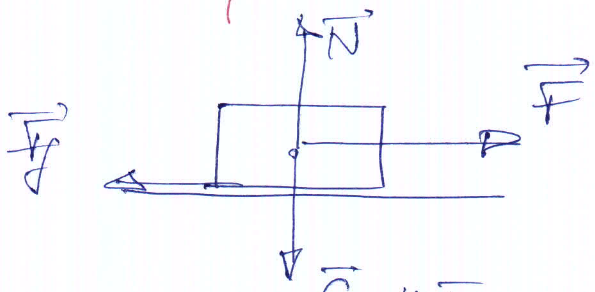
$R_{moon} = 1700 \, Km$

$$\boxed{g_{moon} = 1{,}62 \, m/s^2}$$

The weight of a mass m is

$$\frac{G_{Moon}}{G_{Earth}} = \frac{g_{Moon}}{g_{Earth}} = \frac{1{,}62}{9{,}8} = \underline{\underline{0{,}16}}$$

Friction forces



opposed to the movement

- static friction $\underline{F_s \leq \mu N}$
- dynamic (kinetic) $\underline{F_c = \mu N}$

μ = friction coefficient.

if $F < \mu N$ the body in rest

$F \geq \mu N$ the body moves with constant $\vec{a}$

Resistance forces appear when a body moves in a fluid (gas, liquid...)

$$\boxed{\vec{F_r} = -k\eta\vec{v}}$$

STOKES law

η = viscosity coeff. of the fluid!

K = const depending on the geometrical shape / dimension of the body.

$$[\eta]_{SI} = \frac{kg}{m\,s}.$$

spherical shape

$$\boxed{\vec{F}_{\vec{v}} = -6\pi\eta r\vec{v}}$$

ex:

$F_v = 6\pi\eta r v$

$\vec{G}_a = mg - F_A$

body falling in a fluid

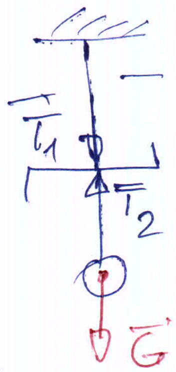
$F_A = \rho_e V_c g$ Archimedes force.

Applying Newton's law

(1) Newton's first law particles in equilibrium

$$\sum \vec{F} = 0 \iff \begin{cases} \sum F_x = 0 \\ \sum F_y = 0 \end{cases}$$

(1)

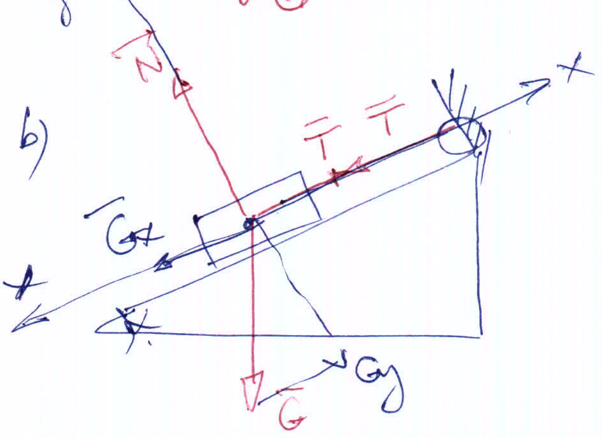


Equilibrium:

body: $-G + T_1 = 0$

rope: $T_1 - T_2 = 0$
(rigid rope keeps same length)

b)



ox: $T - G\sin\alpha = 0$

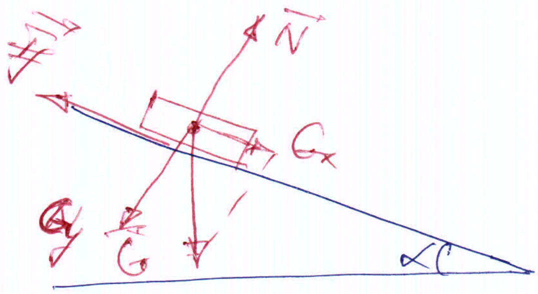
oy $N - G\cos\alpha = 0$

$$\Rightarrow \begin{cases} T = mg\sin\alpha \\ N = mg\cos\alpha \end{cases}$$

Newton's 2nd law

$$\sum \vec{F} = m\vec{a}$$

$$\begin{cases} \sum F_x = m a_x \\ \sum F_y = m a_y \end{cases}$$



inclined plane + friction

$$\begin{cases} \sum F_x = mg \sin\alpha - F_f = m a_x \\ \sum F_y = N - mg\cos\alpha = 0 \end{cases}$$

no mvt. along oy

and $F_f = \mu N$

$\Rightarrow mg \sin\alpha - \mu mg \cos\alpha = 0 \Rightarrow \boxed{\mu = \tan\alpha}$ measuring the friction coefficient.

$$\boxed{a_x = g(\sin\alpha - \mu\cos\alpha)}$$

Fundamental forces in nature (G.U.T. grand unification theory)

1. Gravitational
2. Electromagnetic (electric charged particles) + magnetic (magnetic particles)

atomic particles – gravitational forces 10^{35} smaller than electrostatics $\Rightarrow$ neglected

cosmological scale – gravitational interactions dominate

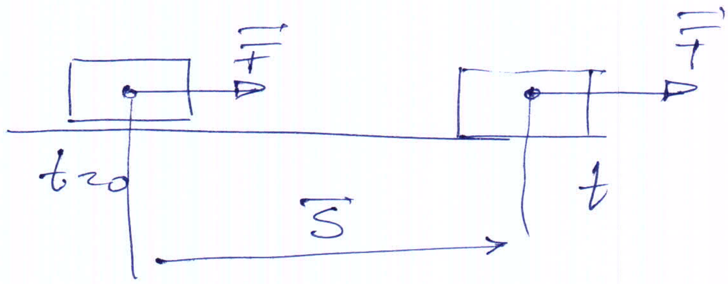
3. Strong interaction – responsible on nuclear bonding.
   – play important role in thermonuclear reactions $\Rightarrow$ power generation
4. Weak interactions : important at nucleus scale

ex. β decay $\quad n \to p + e + \tilde{\nu}$

WORK and ENERGY. Conservation laws

(1) Work

Force $\vec{F}$ $\xrightarrow[\text{2nd principle}]{}$ movement with $\vec{a}$



$$\boxed{W = \vec{F} \cdot \vec{S}}$$

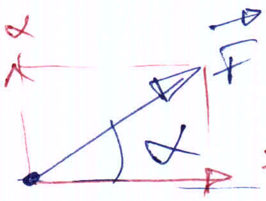
scalar product between vector force and vector displacement

W → scalar quantity

$[W]_{SI}$ = joule = 1 N · 1 m

Force acting with angle α with respect to displacement

$F_\perp = F \sin\alpha$ zero work!



$F_\parallel = F\cos\alpha$ does work

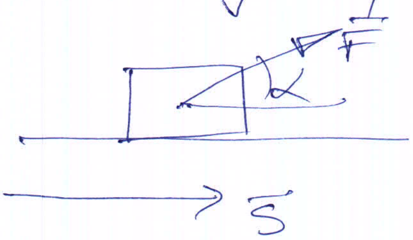
$$W = \vec{F} \cdot \vec{S} = F\,S\cos\alpha = F_\parallel\,S$$

Sign of work

a) $\vec{F}$ has a component on the direction of displacement

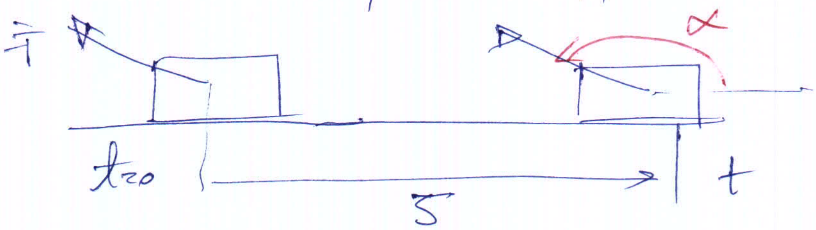
$$W = \vec{F} \cdot \vec{S} = FS\cos\alpha > 0$$

positive work



acceleration

(b) $\vec{F}$ has a component opposite to the displacement



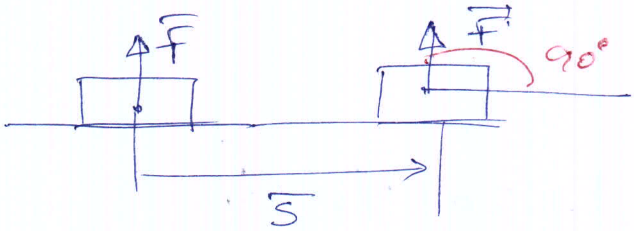
$\alpha > \pi/2$

$\Rightarrow W = \vec{F}\cdot\vec{S} = FS\cos\alpha < 0$ negative work

$\Rightarrow$ deceleration

c) Force $\vec{F} \perp \vec{S}$ (direction of motion)

$W = \vec{F}\cdot\vec{S} = FS\cos 90^\circ = 0$ the force does no work.



Total work

when many forces act on the body:

$\vec{F}_{tot} = \sum_i \vec{F}_i$

$W_{tot} = \vec{F}\cdot\vec{S} = \sum_i \vec{F}_i\cdot\vec{S} = \sum_i W_i$

$$\boxed{W_{tot} = \sum_i W_i}$$

(2) Kinetic energy

$$\boxed{K = \frac{1}{2} m v^2}$$

Scalar quantity

depends on the particle mass and speed. and not on direction of motion.

$K_i = \frac{1}{2} m v_i^2$ initial kinetic energy

$K_f = \frac{1}{2} m v_f^2$ final kinetic energy.

$$\boxed{W_{Tot} = \vec{F} \cdot \vec{s} = K_f - K_i = \Delta K}$$

work–energy theorem

The work done by a net force on a particle equals the change in particle kinetic energy.

Discussion

$W_{tot} > 0$ $\quad \Delta K > 0 \quad \Rightarrow K_f > K_i$ — kinetic energy increases $\Rightarrow$ speed increases

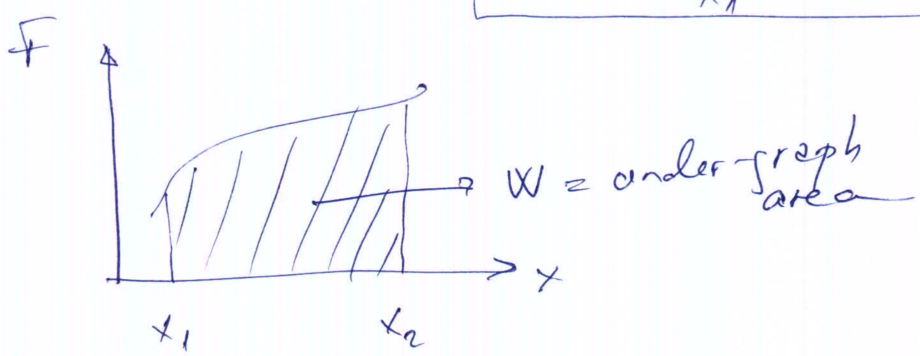
$W_{tot} = 0$ $\quad \Delta K = 0 \quad K_f = K_i$ — constant kinetic energy, constant speed.

$W_{tot} < 0$ $\quad \Delta K < 0 \quad K_f < K_i$ — kinetic energy decreases $\Rightarrow$ decreasing speed

$[K]_{SI} = [W]_{SI} = $ joule

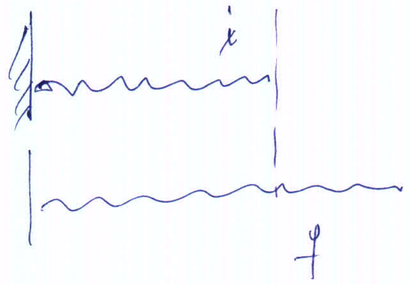
Work and energy with varying forces

$F = F(x) \Rightarrow$
$$\boxed{W = \int_{x_1}^{x_2} F(x)\,dx}$$
integral form.



$s = x_2 - x_1$

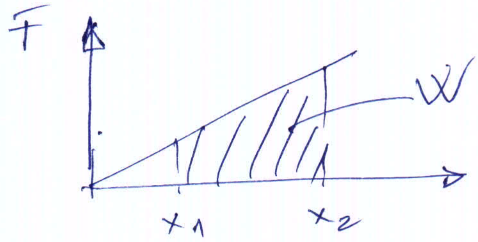
Ex : strethed / compressed string:



$$\boxed{F(x) = kx}$$

Hooke's law of elongation

$$W = \int_{x_1}^{x_2} F(x)\,dx = \int_{x_1}^{x_2} kx\,dx = \frac{k x_2^2}{2} - \frac{k x_1^2}{2}.$$



Power = rate of doing work

$$\boxed{P_{av} = \frac{\Delta W}{\Delta t}}$$ — average power

$$\boxed{P = \frac{dW}{dt}}$$ → instantaneous power

$$[P]_{SI} = \frac{J}{s} = \text{Watt } (W)$$

multiples : $1\,kW = 10^3\,W$
$1\,MW = 10^6\,W$

$1\,HP$ = horse-power = $\underline{746\,W}$

Relation power-velocity

$\vec{F}$ acting on body ⇒ displacement $\vec{s}$

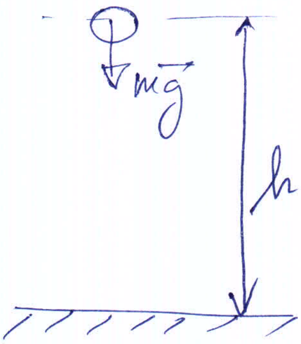
$$W = \vec{F}\cdot\vec{s} \qquad P = \frac{dW}{dt} = \frac{d(\vec{F}\cdot\vec{s})}{dt} \underset{\text{ct force}}{=} \vec{F}\cdot\frac{d\vec{s}}{dt} = \vec{F}\cdot\vec{v}$$

$$\Rightarrow \boxed{P = \vec{F}\cdot\vec{v}}$$

Potential energy

Gravitational

$$\boxed{U_g = mgh}$$



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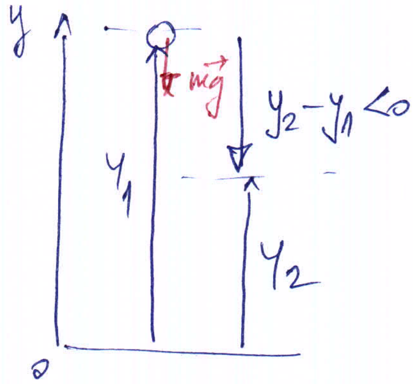


work done by gravity

$\Delta S = y_2 - y_1 < 0$

$$W_{grav} = \vec{G} \cdot \vec{\Delta s} = -mg(y_2 - y_1)$$
$$= mg(y_1 - y_2) > 0$$

$$U_{grav} = mgy \Rightarrow \boxed{W_{grav} = U_{grav_1} - U_{grav_2} = -\Delta U_{grav}}$$

$$[U_{grav}]_{SI} = \text{joule}.$$

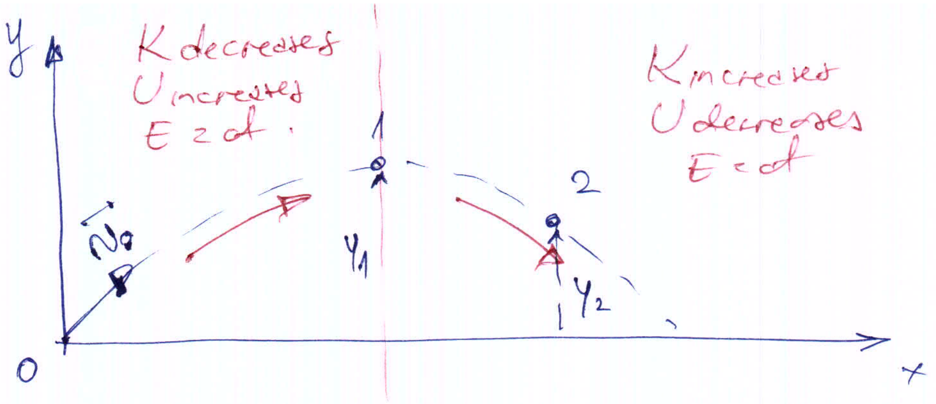
Conservation of mechanical energy (gravitational forces only)

$$\left.\begin{aligned} W_G &= \Delta K = K_2 - K_1 \\ W_G &= -\Delta U_g = U_1 - U_2 \end{aligned}\right\} \Rightarrow \boxed{K_1 + U_1 = K_2 + U_2}$$

$$E_{T_1} = E_{T_2}$$

$$\boxed{\frac{1}{2} m_1 v_1^2 + mgy_1 = \frac{1}{2} m_2 v_2^2 + mgy_2}$$

total mechanical energy is constant.



$$0: \quad E = \frac{mv_0^2}{2} + mg\overset{0}{y_0} = \frac{mv_0^2}{2}$$

$$1: \quad E = \frac{mv_x^2}{2} + mg\,y_{max} = \frac{mv_0^2}{2}\cos^2\alpha + mg\,y_{max}$$

$(v_y = 0)$

conservation of total mechanical energy:

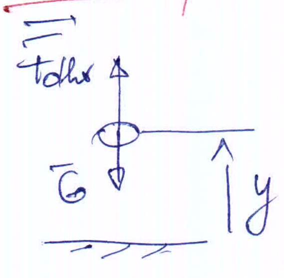
$$E_0 = E_1$$

$$\Rightarrow \frac{mv_0^2}{2} = \frac{m}{2}v_0^2\cos^2\alpha + mg\,y_{max}$$

$$\Rightarrow \boxed{y_{max} = \frac{v_0^2\sin^2\alpha}{2g}}$$

similar with expression obtained by applying 2nd principle of dynamics.

When forces other than gravity do work



$$\boxed{E_2 - E_1 = W_{other}} \Leftrightarrow$$

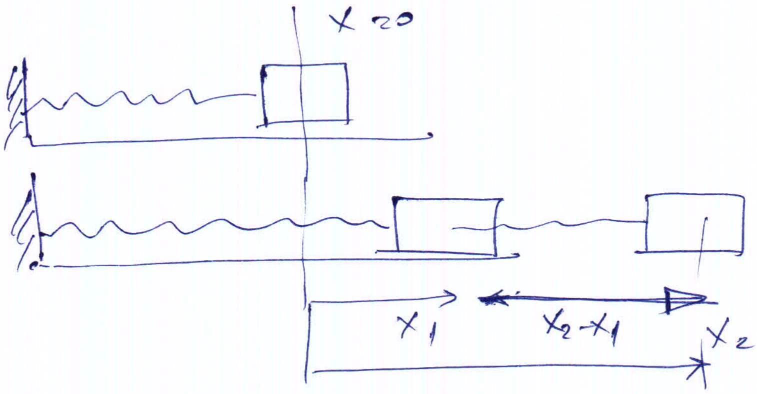
$$\boxed{\left(\tfrac{1}{2}m_2v_2^2 + mgy_2\right) - \left(\tfrac{1}{2}m_1v_1^2 + mgy_1\right) = W_{other}}$$

$W_{other} > 0 \Rightarrow E_2 - E_1 > 0 \Rightarrow E$ increases.

$W_{other} < 0 \Rightarrow E_2 - E_1 < 0 \Rightarrow E$ decreases

$W_{other} = 0 \Rightarrow E = ct.$ energy conservation.

Elastic potential energy



$$\boxed{U = \frac{Kx^2}{2}}$$

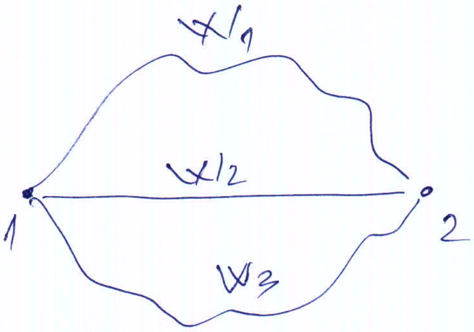
$$\boxed{\frac{1}{2} m v_1^2 + \frac{K x_1^2}{2} = \frac{1}{2} m v_2^2 + \frac{K x_2^2}{2}}$$

Total mechanical energy is conserved.

Conservative – non conservative forces

conservative force $\Rightarrow$ force that allows the two way conversion between kinetic and potential energy

ex gravitational forces, electric forces, elastic force.



$\Rightarrow W_1 = W_2 = W_3$

for conservative forces the work does not depend on path.

The work of conservative forces has some properties

1. W can be expressed as a difference between an initial and a final potential energy function

$$W = U_i - U_f$$

2. It is reversible
3. It is independent on the path, depends only on initial and final point
4. When initial and final point are the same, the work is zero.
5. Mechanical energy is conserved $\boxed{E = K + U = ct}$

Non-conservative forces

ex: friction forces / dissipative forces

$\Rightarrow$ mechanical energy not conserved

$$\boxed{E_2 - E_1 = W_{non\text{-}cons.}}$$

cannot be represented in terms of

The law of conservation of energy

friction / dissipative forces does negative work

non-conservative forces can-not be expressed in terms of potential energy. But, we can express the effect of these forces in terms of other energy than kinetic or potential $\Rightarrow$ internal energy of the system

ex when a car with locked brakes stops the tires and the road surface become heater $\Rightarrow$ temperature increases

$\Rightarrow$ internal energy increases

$$\Delta U_{int} = U_2 - U_1 = -W_{other}$$

but $\quad E_2 - E_1 = \Delta K + \Delta U = W_{other}$

$$\Rightarrow \quad \boxed{\Delta K + \Delta U + \Delta U_{int} = 0}$$

general form of energy conservation law

In a given process, the kinetic, the potential, the internal energies may change, but the sum of these changes is always zero.

- Energy is never created or distroyed, it only changes form.
- The relationship of internal energy to the temperature changes, heat and work is the subject of physics area called thermodynamics.

Force and potential energy

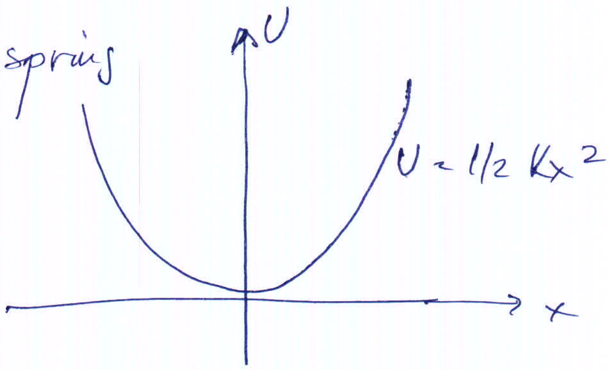
For conservative forces, the force can be deduced from a potential energy:

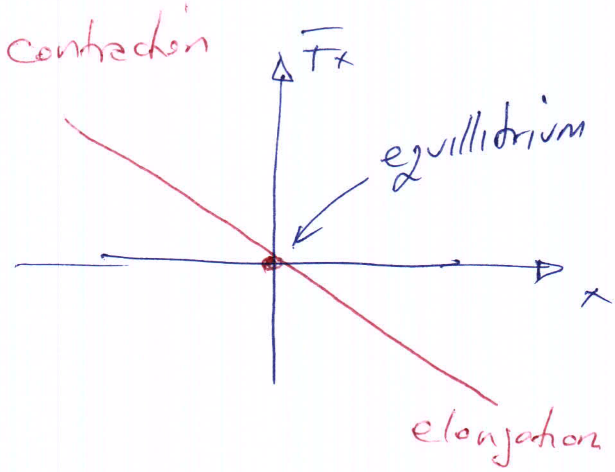
$$\vec{F} = -\nabla U \qquad \boxed{U = U(x,y,z)}$$

$$\vec{F}(x,y,z) = -\frac{dU}{dx}\vec{i} - \frac{dU}{dy}\vec{j} - \frac{dU}{dz}\vec{k}$$

$\boxed{\text{Ex}}$ 1D case

→ spring





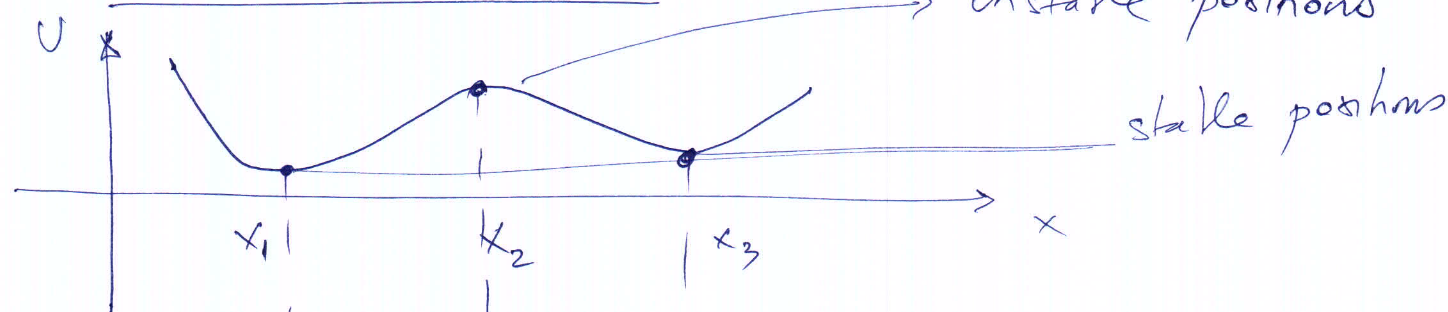
$$F = -\frac{dU}{dx} = -Kx$$

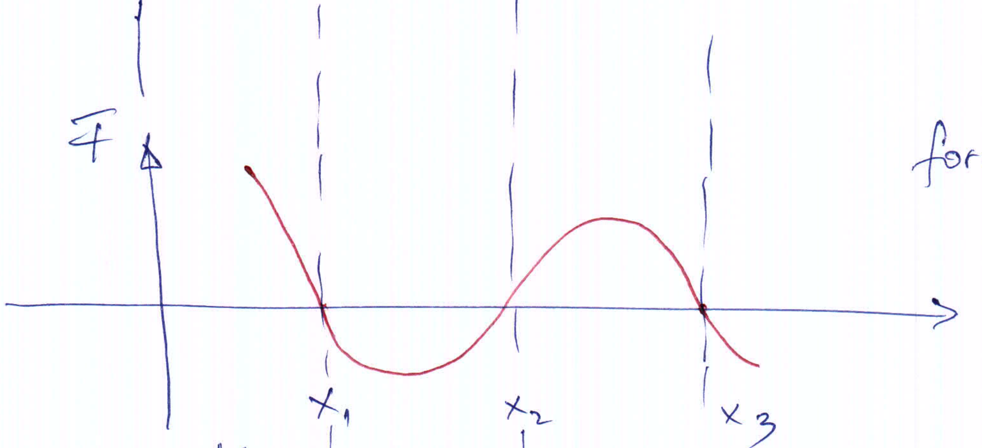
equillibrium $\quad F = 0$ (1st principle) $\Rightarrow$

$$\begin{cases} \dfrac{dU}{dx} = 0 \Rightarrow \\ \dfrac{d^2U}{dx^2} > 0 \end{cases} \quad \boxed{U = \text{minimum}}$$

(stationary point)

General case:





$$\text{force} = -\frac{dU}{dx}$$

$x < x_1$	$x_1 < x < x_2$	$x_2 < x < x_3$	$x > x_3$
$\frac{dU}{dx} < 0$	$\frac{dU}{dx} > 0$	$\frac{dU}{dx} < 0$	$\frac{dU}{dx} > 0$
$F_x > 0$	$F_x < 0$	$F_x > 0$	$F_x < 0$

MOMENTUM, IMPULSE, COLLISIONS

There are many problems involving forces that cannot be answered by directly applying Newton's 2nd law $\sum \vec{F} = m\vec{a}$. For instance, problems involving collisions between bodies.

$\Rightarrow$ new approach $\Rightarrow$ new concepts

- momentum
- impulse

\+ new conservation law $\Leftrightarrow$ momentum conservation

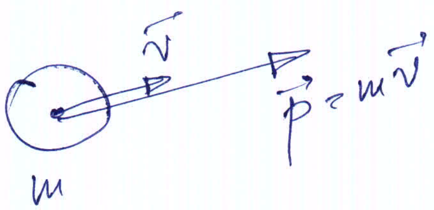
① Momentum and impulse

Newton's 2nd law in terms of momentum

$$\sum \vec{F} = m\vec{a} = m\frac{d\vec{v}}{dt} = \frac{d}{dt}(m\vec{v})$$

$m = ct$ (non-relativistic approx)

$$\boxed{\vec{p} = m\vec{v}} = \text{MOMENTUM (linear momentum)}$$



$\vec{p}$ = vector quantity, same orientation as $\vec{v}$

$$[p]_{SI} = kg\frac{m}{s} = N\cdot s$$

Let's consider a constant force $\vec{F} = \sum \vec{F_i}$ acting on a particle in the interval $\Delta t = t_1 - t_2$.

The <u>impulse</u> of the net force is defined as:

$$\boxed{\vec{J} = \sum \vec{F} \Delta t = \sum \vec{F} (t_2 - t_1)}$$

→ vector quantity, same direction as $\sum \vec{F}$

$$|\vec{J}|_{SI} = N\,s = kg\,\frac{m}{s^2} \cdot s = kg\, m/s$$

From 2nd law of Newton =>

$$\sum \vec{F} = \frac{d\vec{p}}{dt} \Rightarrow \qquad \sum \vec{F} = \frac{\Delta \vec{p}}{\Delta t} = \frac{\vec{p}_2 - \vec{p}_1}{t_2 - t_1}$$

$$\Rightarrow \boxed{\sum \vec{F} (t_2 - t_1) = \vec{p}_2 - \vec{p}_1} \Rightarrow$$

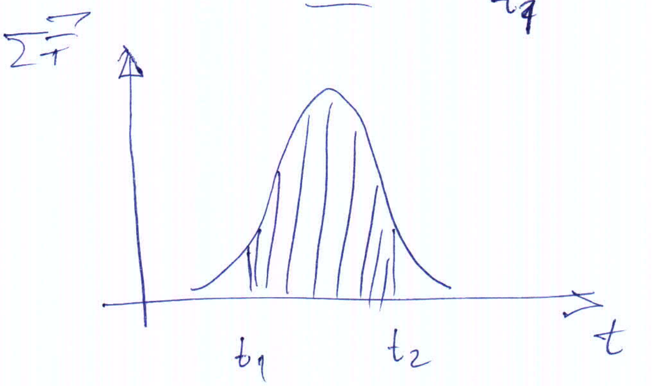
$$\boxed{\vec{J} = \vec{p}_2 - \vec{p}_1}$$

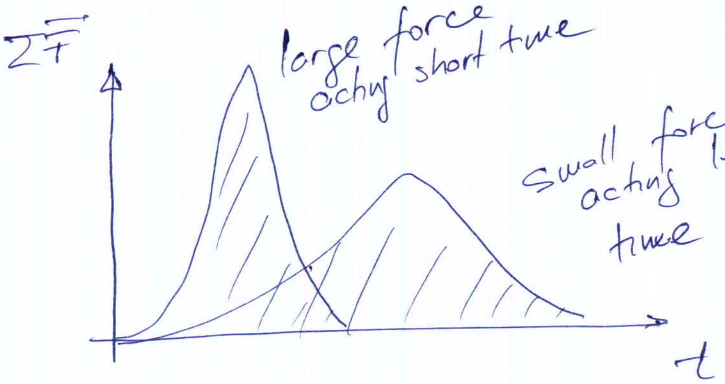
impulse - momentum theorem.

The change of momentum of a particle equals the impulse of the net force acting on the particle on an interval of time Δt.

For <u>time-dependent</u> forces:

$$\boxed{\vec{J} = \int_{t_1}^{t_2} \sum \vec{F}(t)\, dt} = \int_{p_1}^{p_2} d\vec{p} = \vec{p}_2 - \vec{p}_1$$





similar under-graph areas

$\Rightarrow$ similar impulse

$\Leftrightarrow$ similar $\boxed{\Delta\vec{p} = \vec{p}_2 - \vec{p}_1}$

② Conservation of momentum

interacting bodies.

3rd principle $\vec{F}_{BA} = -\vec{F}_{AB}$

Concepts:

internal forces = forces between particles of the system

external forces = forces exerted by objects outside of systems

isolated system = external forces = 0

$$\left.\begin{aligned} \vec{F}_{B\to A} &= \frac{d\vec{p}_A}{dt} \\ \vec{F}_{A\to B} &= \frac{d\vec{p}_B}{dt} \\ \vec{F}_{B\to A} &= -\vec{F}_{A\to B} \end{aligned}\right\} \Rightarrow \frac{d}{dt}(\vec{p}_A + \vec{p}_B) = 0$$

$$\Rightarrow \boxed{\vec{p}_A + \vec{p}_B = ct}$$

$$\boxed{\frac{d\vec{P}}{dt} = 0}$$

principle of conservation of momentum

* direct consequence of Newton 3rd law.

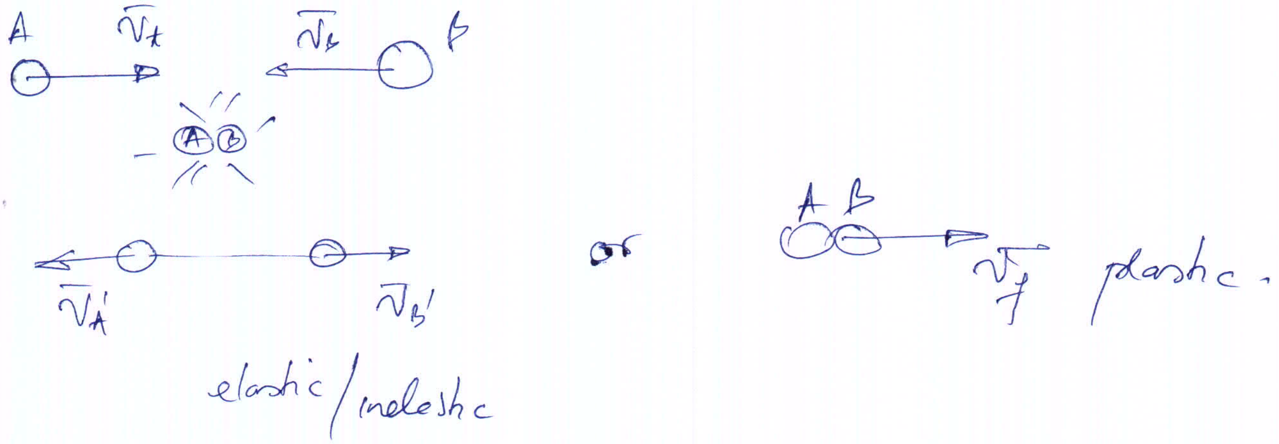
Ellastic and inellastic collisions

If the forces between bodies are conservative, no mechanical energy is lost or gained during collision, so the total energy of the system is the same before and after collision.

Elastic colission = collision with conservation of kinetic energy

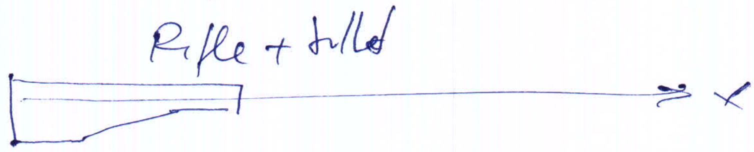
Inellastic collision = collision in which the total kinetic energy after collision is less than before.

completelly inellastic collision = collision in which bodies have same final velocity (ex. bullet shot in a body)

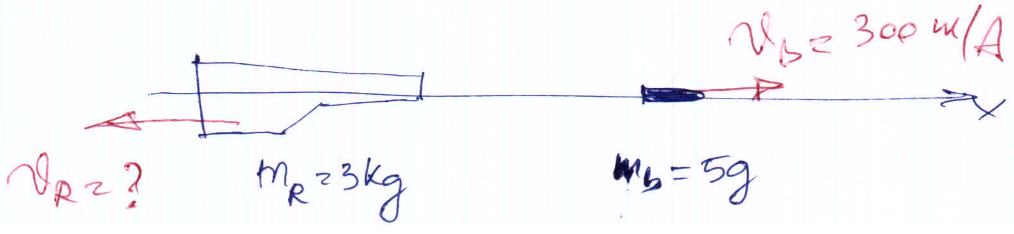


Example Recoil of a rifle

Before :



After :



Initially: $P_x = 0$ rifle and bullet in rest

Finally: $P_x = P_{bx} + P_{Rx} = m_b V_{bx} + m_R V_{Rx}$

momentum conservation:

$$0 = P_x = m_b v_{bx} + m_R V_{Rx} \quad ;$$

$$\boxed{v_{Rx} = -\frac{m_b v_{bx}}{m_R}}$$

negative sign means recoil in opposite direction with respect to the bullet.

$$v_{Rx} = -\frac{5 \cdot 10^{-3}\, 300}{3} = -0{,}5\, m/s$$

$$\sum \vec{F} \cdot \Delta t = \Delta \vec{p} \quad ; \quad \sum \vec{F} = \frac{\Delta p}{\Delta t} = \frac{m \Delta v}{\Delta t}$$

short (

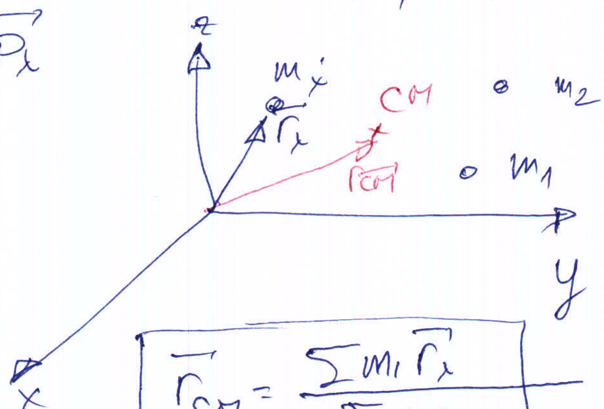
$$\bar{F} = \frac{3kg \cdot 0{,}5\, m/s}{10^{-3} s} = 1{,}5 \cdot 10^{3}\, N$$

$(1 s)$ $(1{,}5\, N)$

Center of mass

many particles: $m_i, \vec{v}_i$

$$\begin{cases} \vec{P} = \sum m_i \vec{v}_i = \sum \vec{P}_i \\ \vec{P} = M \vec{v}_{CM} \end{cases}$$



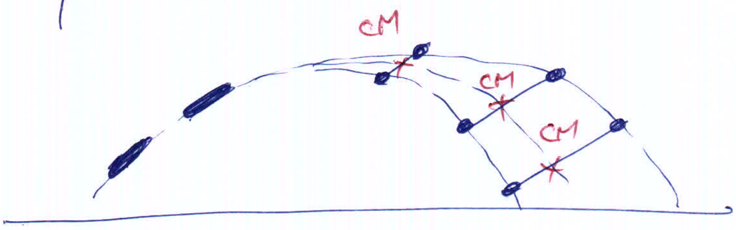
$$\boxed{\bar{r}_{CM} = \frac{\sum m_i \bar{r}_i}{\sum m_i}}$$

$$\boxed{\sum \vec{F}_{ext} = M\, \vec{a}_{cm}}$$

body or collection of particles

when a body or collection of particles is acted by external forces, the center of mass moves as though the whole mass would be concentrated at this point and that were acted by a net force equal to the sum of forces on the system.

Ex: Projectile exploding in two pieces during its parabolic motion



the two particles follow individual trajectories but the center of mass continues the initial trajectory.