COURSE no. 1

① Introduction

UNITS, PHYSICAL QUANTITIES, VECTORS

→ Physics is an experimental science.

Physicists observe phenomena of nature and tries to relate these phenomena ⇒ physical theories
physical laws, principles

Solving physical problem:

Identify the relevant concepts

↓

Set-up the problem

↓

Execute the solution

↓

Evaluate the answer

Model: - simplified version of a physical system which would be too complicated to be analyzed in all details.

ex: real ball in flight ⟶ point approximation.

Standards and units

→ experiments require measurements,

→ we use numbers to describe the result of experiments.

Physical quantity

= any number used to describe a physical phenomenon quantitatively

ex : weight, height

Reference standard

When we measure a quantity we allways compare it with some reference standard.

⇒ units

SI { time → [seconds]
length → [meters]
mass → [kg]

Multiple
Sub-multiple

ex : m
$mm = 10^{-3} m$
$\mu m = 10^{-6} m$
$km = 10^{3} m$
etc...

British system

Length : 1 inch = 2,54 cm length unit

1 pound = 4,448221615260 N (force unit)

$10^{26} m$ ← → 1m ← → $10^{-10} m$ — $10^{-14} m$

LIMIT OF OBSERVABLE UNIVERSE — HUMAN — ATOM RADIUS — NUCLEUS RADIUS

Uncertainty and accuracy

→ measurement allway have uncertainties (ex. instrumental)

⇒ accuracy : how close is the measured value with respect to true value

→ 5,44 ± 0,02 m ex.

Vectors and operations with vectors

Scalar quantities:

ex. time, temperature, mass, volume, density can be described completelly by a single number with a unit.

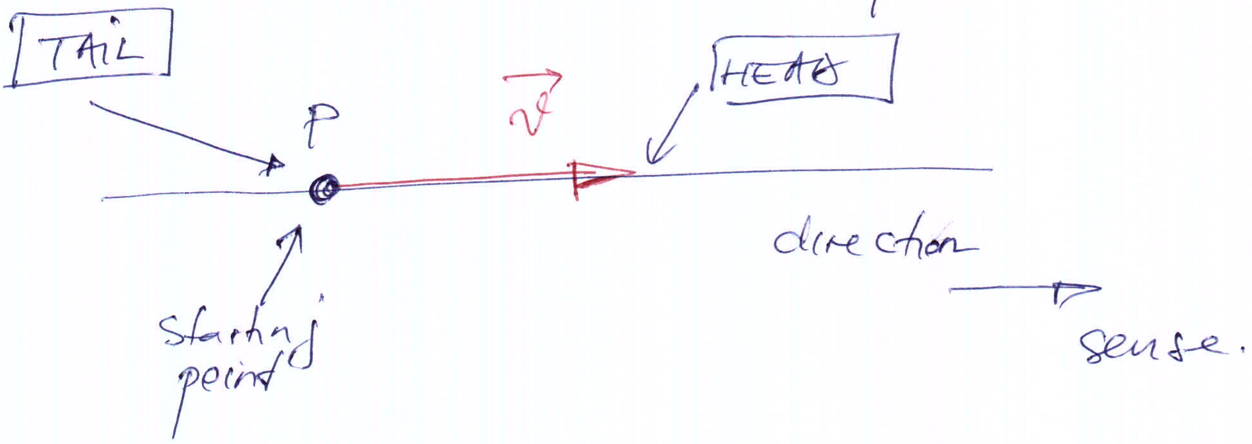
Vector quantities

$\vec{A}$

— described by

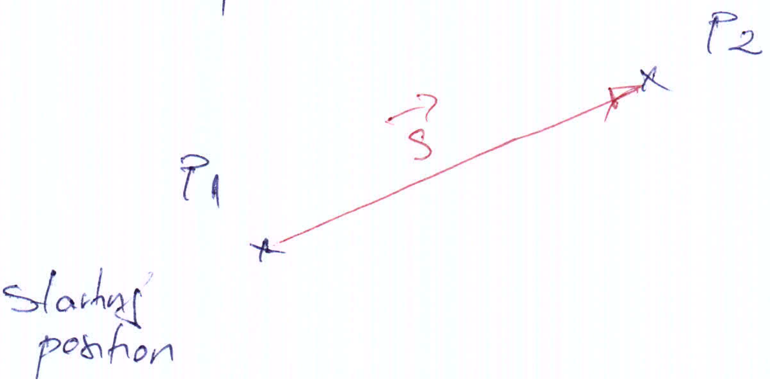
→ MAGNITUDE $\boxed{|\vec{A}| = A}$

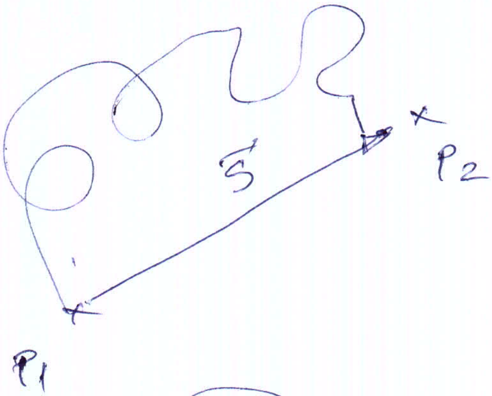
→ DIRECTION in space.



ex: displacement vector,
velocity
force

ex displacement vector → allways a stright line connecting starting and ending point.

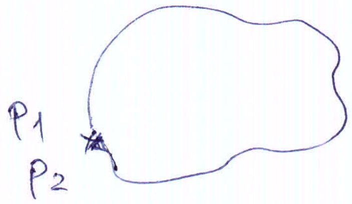




displacement does not depend on the path.

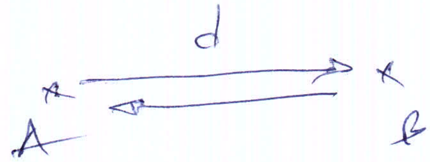
$\Downarrow$

total displacement on closed path = zero.



$\Downarrow$

displacement $\neq$ distance

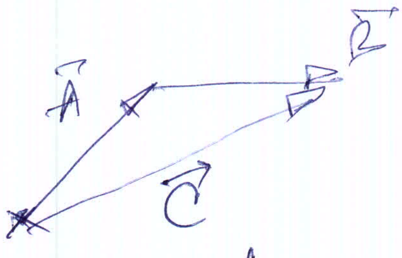


zero displacement, 2d distance (magnitude)

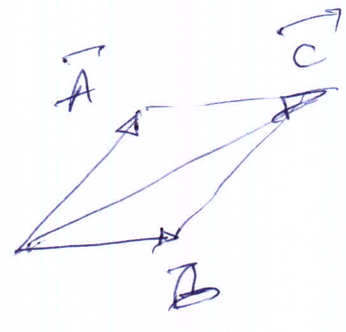
Operations with vectors

Vector addition

$$\vec{C} = \vec{A} + \vec{B}$$

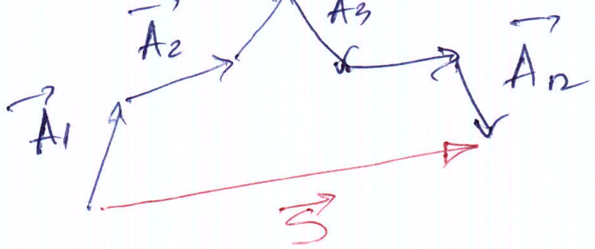


head to tail



paralelograme rule

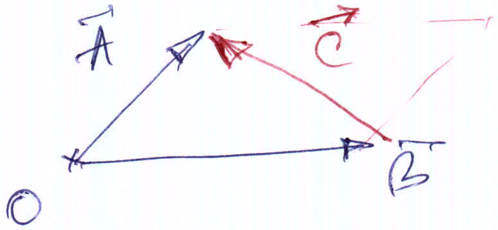
Polygone law $\rightarrow$ N vectors



$$\vec{S} = \vec{A_1} + \vec{A_2} + \dots + \vec{A_n}$$

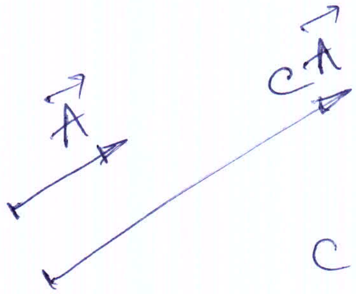
Vector substraction

$$\vec{C} = \vec{A} - \vec{B}$$



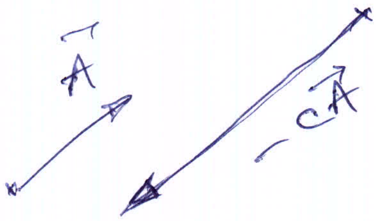
Multiplication

by positive scalar



$c > 0$

by negative scaler

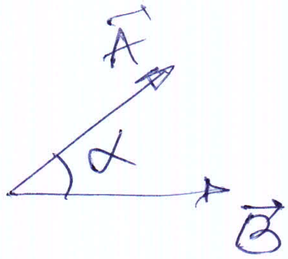


Scaler product

$\vec{A}, \vec{B}$

$$\boxed{C = \vec{A} \cdot \vec{B} = AB\cos\alpha}$$

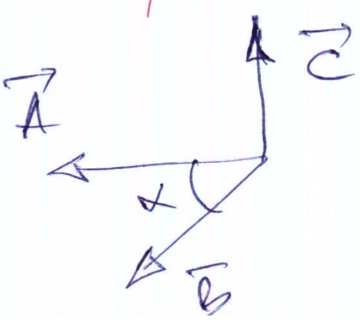
scalar quantity



Vector product

The magnitude C,

$$\boxed{\vec{C} = \vec{A} \times \vec{B} \text{ is } AB\sin\alpha}$$

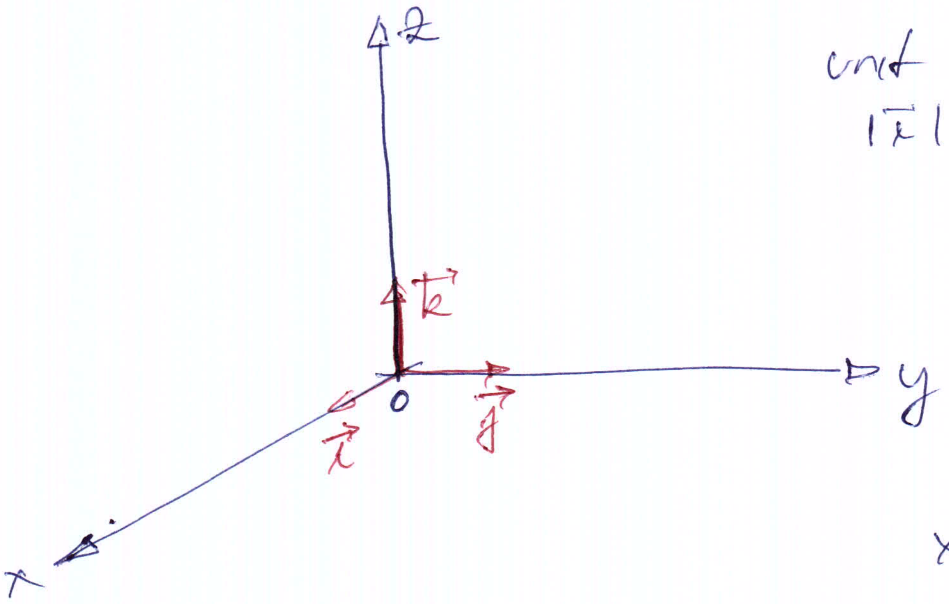


"right-hand" rule

(=) right screw

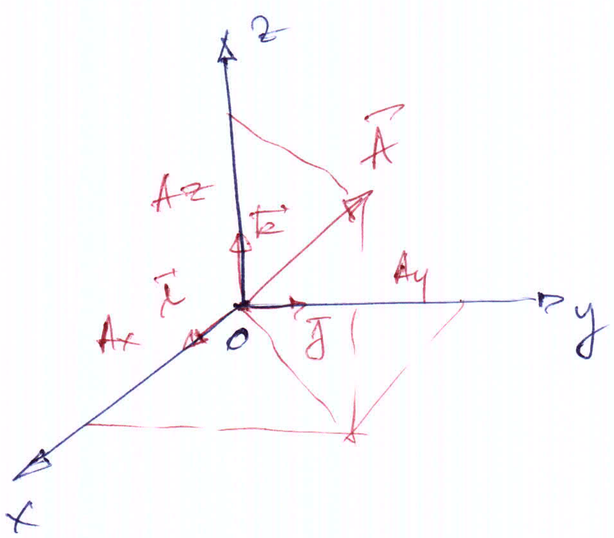
$$\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$$

Analytical representation of vectors



unit vector:
$|\vec{i}| = |\vec{j}| = |\vec{k}| = 1.$

x, y, z coordinate system (CARTESIAN).



$$\boxed{\vec{A} = A_x \vec{i} + A_y \vec{j} + A_z \vec{k}}$$

(3D)

(2D) $\vec{A} = A_x \vec{i} + A_y \vec{j}$

Sum and difference

$$\vec{A} = A_x \vec{i} + A_y \vec{j} + A_z \vec{k}$$

$$\vec{B} = B_x \vec{i} + B_y \vec{j} + B_z \vec{k}$$

$$\vec{C} = \vec{A} + \vec{B} = (A_x + B_x)\vec{i} + (A_y + B_y)\vec{j} + (A_z + B_z)\vec{k}$$

$$\vec{C} = \vec{A} - \vec{B} = (A_x - B_x)\vec{i} + (A_y - B_y)\vec{j} + (A_z - B_z)\vec{k}.$$

Multiplication by scaler

$$c\vec{A} = cA_x\vec{i} + cA_y\vec{j} + cA_z\vec{k}$$

Scalar product

$$\vec{A}\cdot\vec{B} = (A_x\vec{i} + A_y\vec{j} + A_z\vec{k})\cdot(B_x\vec{i} + B_y\vec{j} + B_z\vec{k})$$

$$= A_xB_x\vec{i}\cdot\vec{i} + A_xB_y\vec{i}\cdot\vec{j} + A_xB_z\vec{i}\cdot\vec{k} +$$
$$A_yB_x\vec{j}\cdot\vec{i} + A_yB_y\vec{j}\cdot\vec{j} + A_yB_z\vec{j}\cdot\vec{k} +$$
$$A_zB_x\vec{k}\cdot\vec{i} + A_zB_y\vec{k}\cdot\vec{j} + A_zB_z\vec{k}\cdot\vec{k}$$

$$\vec{i}\cdot\vec{i} = \vec{j}\cdot\vec{j} = \vec{k}\cdot\vec{k} = 1\cdot 1\cdot\cos(0) = 1$$
$$\vec{i}\vec{j} = \vec{i}\vec{k} = \vec{j}\vec{k} = 1\cdot 1\cos 90 = 0$$

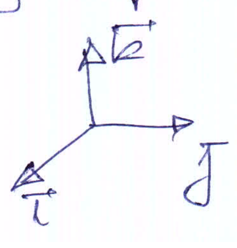
$$\Rightarrow \boxed{\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z}$$

Vector product

$$\vec{A}\times\vec{B} = (A_x\vec{i} + A_y\vec{j} + A_z\vec{k})\times(B_x\vec{i} + B_y\vec{j} + B_z\vec{k})$$

$$\vec{i}\times\vec{i} = \vec{j}\times\vec{j} = \vec{k}\times\vec{k} = 1\cdot 1\sin(0) = 0.$$

using def. $C = AB\sin\alpha$ and $\vec{i},\vec{j},\vec{k}$ orthogonal

$\Rightarrow$ 

$$\begin{cases} \vec{i}\times\vec{j} = -\vec{j}\times\vec{i} = \vec{k} \\ \vec{j}\times\vec{k} = -\vec{k}\times\vec{j} = \vec{i} \\ \vec{k}\times\vec{i} = -\vec{i}\times\vec{k} = \vec{j} \end{cases}$$

$\Rightarrow$

$$\vec{A} \times \vec{B} = (A_y B_z - A_z B_y)\vec{i} + (A_z B_x - A_x B_z)\vec{j} + (A_x B_y - A_y B_x)\vec{k} =$$

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

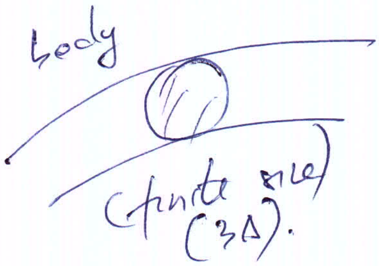
KINEMATICS

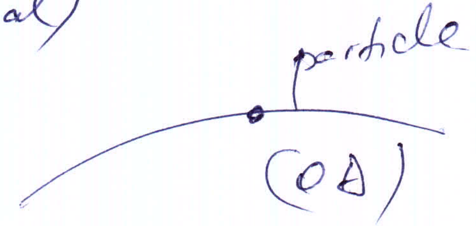
= branch of classical mechanics describing the motion of
→ points
→ bodies
→ system of bodies

without taking into account the cause of the motion (from Greek kinema = motion).

① Fundamental of kinematics

Particle (punct material)

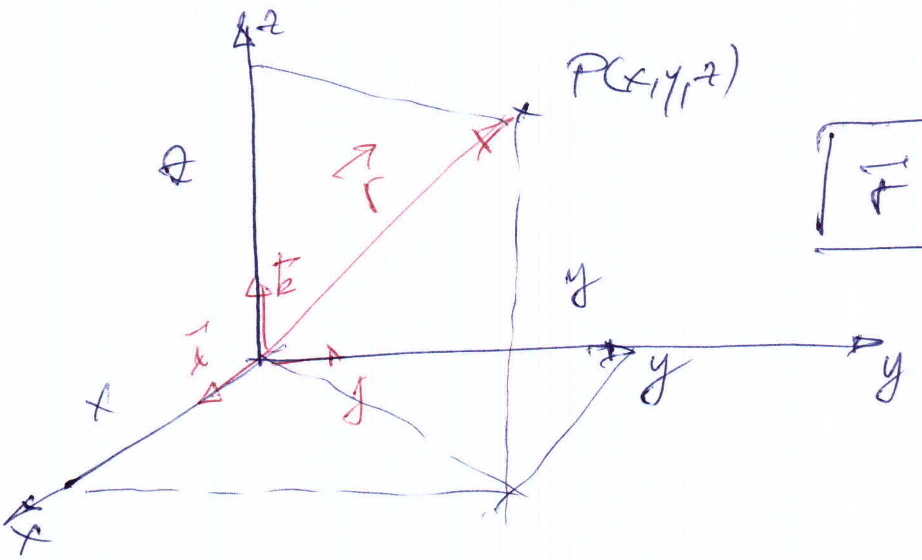




describing the movement of a body in space requires geometry notions.

Position vector

→ defines the particle position in space.



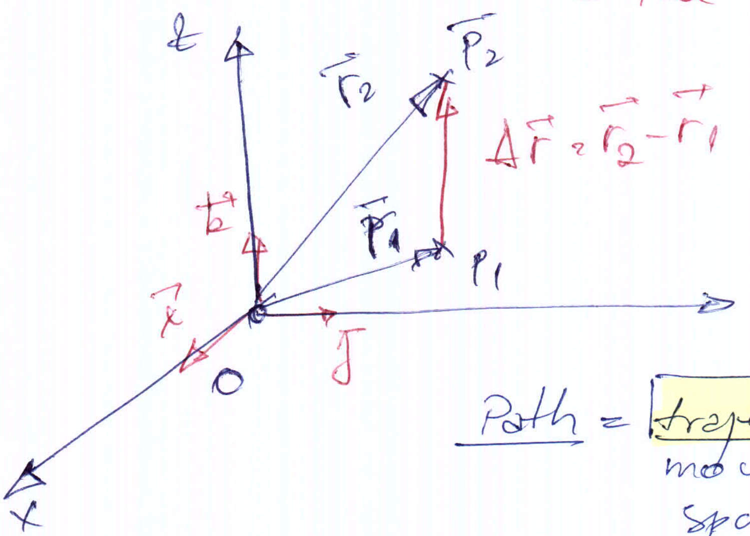
$$\boxed{\vec r = x\vec i + y\vec j + z\vec k}$$

• Displacement vector

During the time interval $\Delta t = t_2 - t_1$, the point moves from $P_1(\vec{r_1})$ to $P_2(\vec{r_2})$ $\Rightarrow$
the change in position is:

$$\boxed{\Delta\vec{r} = \vec{r_2} - \vec{r_1} = (x_2 - x_1)\vec{i} + (y_2 - y_1)\vec{j} + (z_2 - z_1)\vec{k}}$$

the displacement vector



Path = trajectory = path that a moving object follows through space as a function of time.

• The average velocity

$$\boxed{\vec{v}_{av} = \frac{\vec{r_2} - \vec{r_1}}{t_2 - t_1} = \frac{\Delta\vec{r}}{\Delta t}}$$

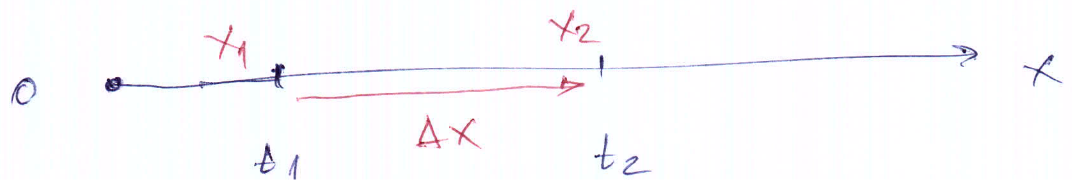
rate of change in particle position

vector quantity

$$[v]_{SI} = \frac{[\Delta r]}{[\Delta t]} = \frac{m}{s}$$

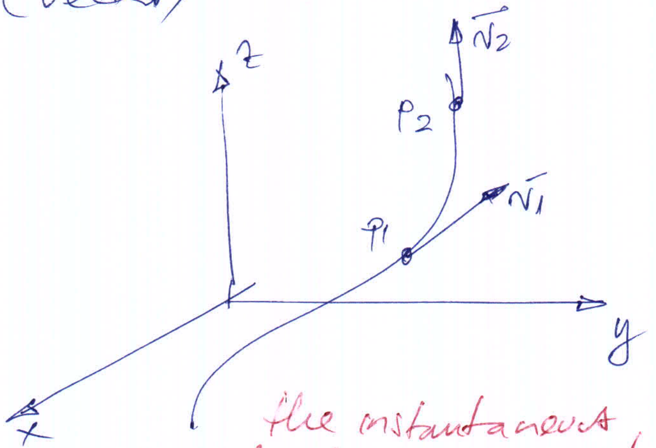
1D motion

$$v_{av-x} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}$$



Instantaneous velocity (vector)

$$\boxed{\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}}$$



the instantaneous velocity is tangent to the path in each point

magnitude of velocity (vector) is the speed (scalar)

$$v = |\vec{v}| = \sqrt{v_x^2 + v_y^2 + v_z^2}$$

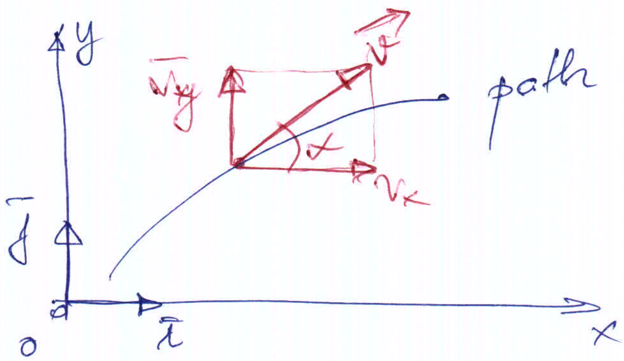
$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{d}{dt}(x\vec{i} + y\vec{j} + z\vec{k}) = \frac{dx}{dt}\vec{i} + \frac{dy}{dt}\vec{j} + \frac{dz}{dt}\vec{k}$$

$$= v_x\vec{i} + v_y\vec{j} + v_z\vec{k}$$

$\Rightarrow$ movement decomposition

total movement = movement along ox + movement along oy + movement along oz

Particular case (2D)



path in xy plane

$\vec{v}(v_x, v_y)$

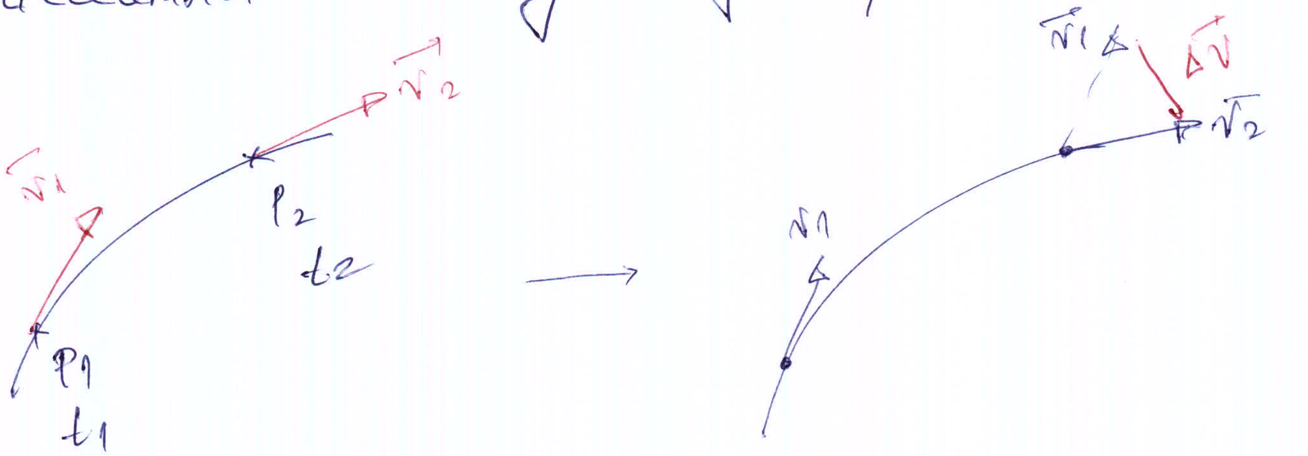
movement along ox + movement along oy

$$\begin{cases} v = \sqrt{v_x^2 + v_y^2} \\ \tan\alpha = \dfrac{v_y}{v_x} \end{cases}$$

$v_x = v\cos\alpha$
$v_y = v\sin\alpha$

The acceleration vector

acceleration = rate of change in particle velocity.



Average acceleration (vector)

$$\boxed{\vec{a}_{av} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1} = \frac{\Delta\vec{v}}{\Delta t}}$$

$\rightarrow$ has the direction of $\Delta\vec{v}$

$$[a]_{SI} = \frac{m/s}{s} = m/s^2$$

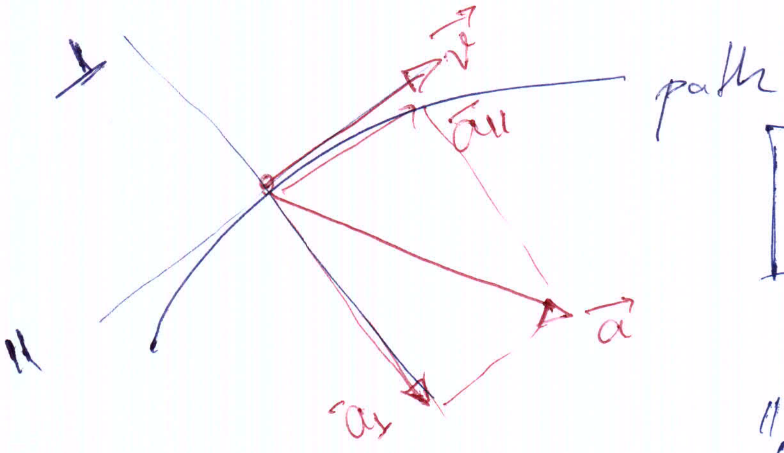
Instantaneous acceleration (vector)

$$\boxed{\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta\vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}}$$

again

$\vec{a}_x = \vec{a}_x + \vec{a}_y + \vec{a}_z$

2D case:



$$\boxed{\vec{a} = \vec{a}_\perp + \vec{a}_\parallel}$$

(chose 2 particular directions
$\parallel$ to $\vec{v}$ and
$\perp$ to $\vec{v}$

1D motion

ex : movement along ox, with constant acceleration

$a_{av} = a_x$

$$a_x = \frac{v_{2x} - v_{1x}}{t_2 - t_1} \quad ; \; t_1 = 0 \; , \; t_2 = t$$

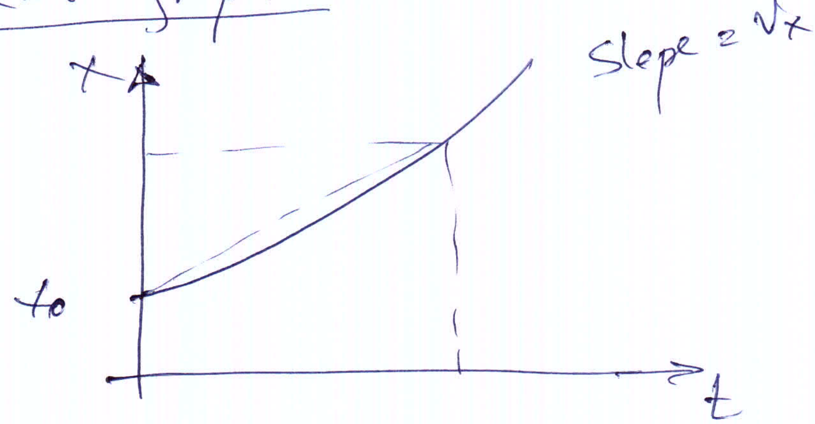
$$\Rightarrow a_x = \frac{v_x - v_{0x}}{t - 0} \Rightarrow \boxed{v_x = v_{0x} + a_x t}$$

$$v_{av} = \frac{x - x_0}{t - 0} = \frac{x - x_0}{t} \Rightarrow \boxed{x = x_0 + v_0 t}$$

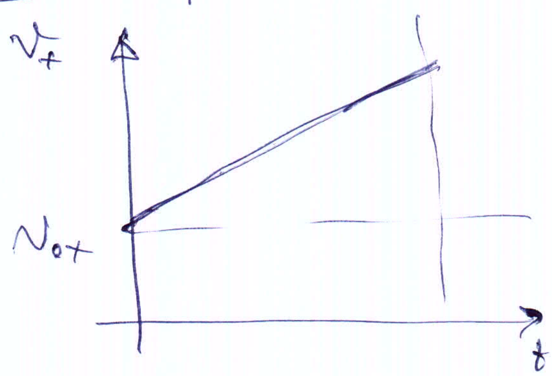
$$\Rightarrow \boxed{x = x_0 + v_{0x} \cdot t + 1/2 \, a_x t^2}$$

parabola equation

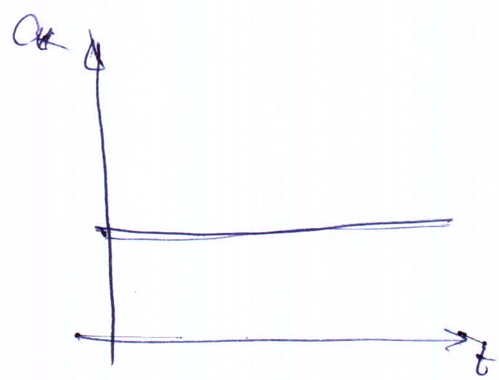
x-t graph



v-t graph



a-t graph



Galileo equation

From
$$\begin{cases} x - x_0 = v_0 t + 1/2\, a_x t^2 \\ v_x = v_{0x} + a_x t \end{cases}$$

$$\Rightarrow \boxed{v_x^2 = v_{0x}^2 + 2a_x (x - x_0)}$$ Galileo equation

$\Rightarrow$ EQUATIONS of MOTION in case of constant a_x acceleration

$$\begin{cases} v_x = v_{0x} + a_x t \\ x = x_0 + v_{0x} t + 1/2\, a_x t^2 \\ v_x^2 = v_{0x}^2 + 2a_x (x - x_0) \end{cases}$$

using this equation we can solve any problem involving straight line motion with constant acceleration.

Velocity and position by integration

We analyzed the situation when $a_x = ct.$

If $a_x = a_x(t)$ the equations of movement presented before are not anymore valid.

One can write:

$$a_x = \frac{dv_x}{dt} \Rightarrow dv_x = a_x\, dt$$

within the interval $t_1, t_2 \longleftrightarrow v_{x1}, v_{x2}$

one can integrate $\Rightarrow$

$$\boxed{v_{x2} - v_{x1} = \int_{v_{x1}}^{v_{x2}} dv_x = \int_{t_1}^{t_2} a_x(t)\, dt}$$

We can carry on the same for v_x

$$v_x = \frac{dx}{dt} \Rightarrow dx = v_x\, dt$$

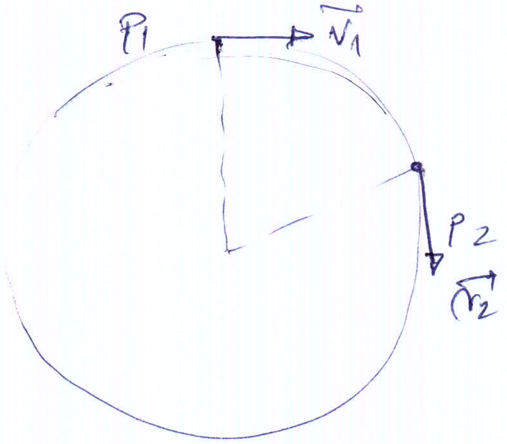
$$\Rightarrow \boxed{x_2 - x_1 = \int_{x_1}^{x_2} dx = \int_{t_1}^{t_2} v_x(t)\, dt.}$$

If $t_1 = 0$, $t_2 = t$

$$\Rightarrow \boxed{\begin{aligned} v_x &= v_{0x} + \int_0^t a_x(t)\, dt \\ x &= x_0 + \int_0^t v_x(t)\, dt. \end{aligned}}$$

general case of eq. of movement with $a_x = a_x(t)$.

Circular motion

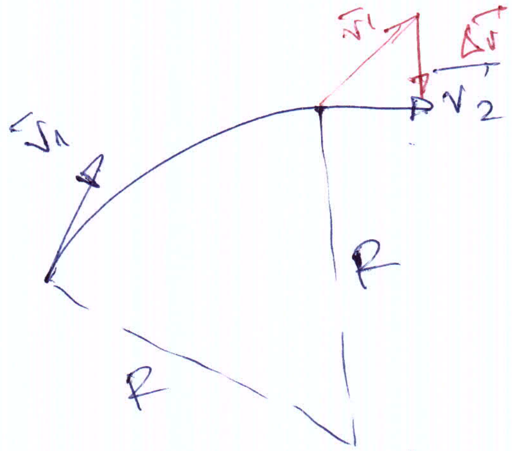


→ uniform circular motion $|\vec{v}_1| = |\vec{v}_2| = v$ constant speed

→ non-uniform circular motion $|\vec{v}_1| \neq |\vec{v}_2|$

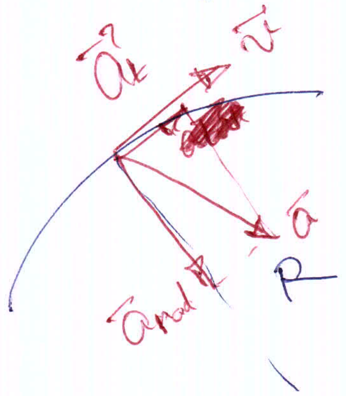
Uniform circular motion

$\Delta\vec{v} = \vec{v}_2 - \vec{v}_1 \neq 0$ even if $|\vec{v}_1| = |\vec{v}_2|$ because the vector direction changes on a curved path.



$\vec{a} \to$ direction of $\Delta\vec{v}$

$\vec{a} = \vec{a}_{rad} + \vec{a}_{tan}$



→ uniform circular motion:

$\vec{a}_{tan} = 0$

→ non uniform circ motion:

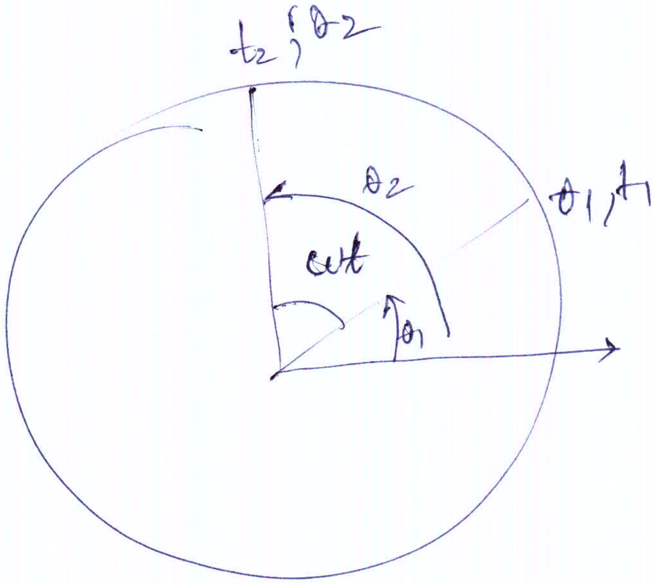
$\vec{a}_{tan} \neq 0$

$a_{tan} > 0 \Rightarrow |v_2| > |v_1| \Rightarrow$ particle accelerate

$a_{tan} < 0 \Rightarrow |\vec{v}_2| < |\vec{v}_1| \Rightarrow$ decelerate

$$\boxed{a_{rad} = \frac{v^2}{R}}$$

oriented towards the center of the circle
$\Rightarrow$ centripetal acceleration



$\omega = \frac{d\theta}{dt}$ angular velocity

$[\omega] = rad/s$

$$\boxed{v = \omega R}$$

$v = \frac{2\pi R}{T}$ ← period

$\Rightarrow$ $\boxed{a_{rad} = \frac{4\pi^2 R}{T^2} = \omega^2 R}$

$\omega = 2\pi / T$

$T = \frac{2\pi}{\omega}$

$\nu = 1/T$ $\Rightarrow$

frequency

Problems: (kinematics)

1. → Free falling body (1D)
   → Projectile motion (2D)