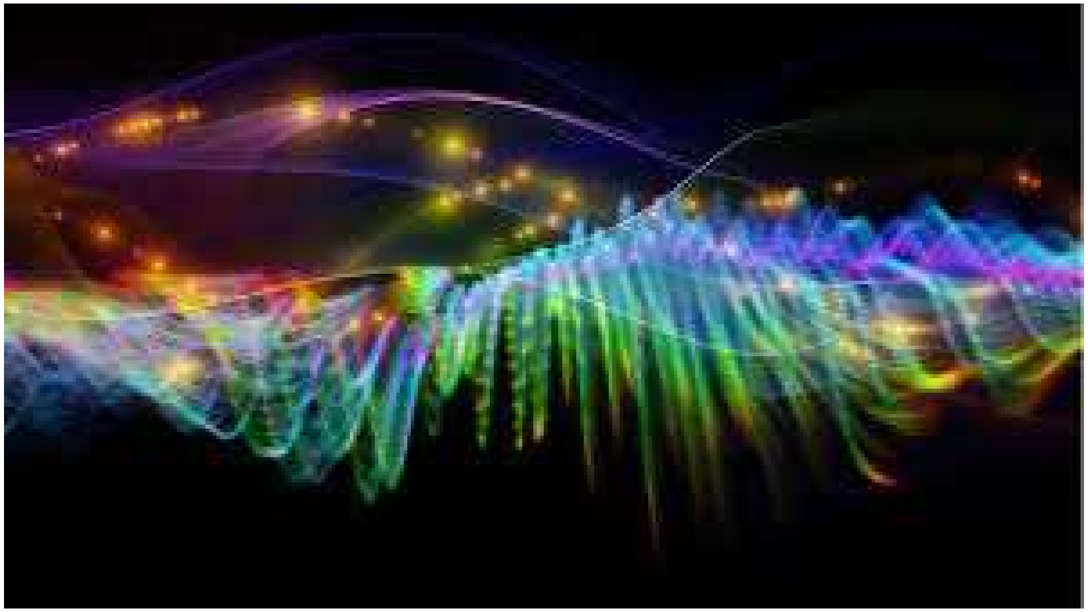
SOUND WAVES. ACOUSTICS.

Sound and hearing.



Introduction

Of all the mechanical waves that occur in nature, the most important in our everyday lives are **longitudinal waves in a medium**—usually air—called ***sound waves.***

The human ear is tremendously sensitive and can detect sound waves even of very low intensity.

We have described mechanical waves in terms of displacement;

However, a description of sound waves in terms of ***pressure fluctuations*** is often more appropriate, largely because the ear is primarily sensitive to changes in pressure.

We'll study the relationships among displacement, pressure fluctuation, and intensity and the connections between these quantities and human sound perception.

When a source of sound or a listener moves through the air, the listener may hear a frequency different from the one emitted by the source. This is the Doppler effect, which has important applications in medicine and technology.

LEARNING GOALS

By studying this chapter, you will learn:

• How to describe a sound wave in terms of either particle displacements

or pressure fluctuations.

• How to calculate the speed of sound waves in different materials.

• How to calculate the intensity of a sound wave.

• What determines the particular frequencies of sound produced by an organ or a flute.

• How resonance occurs in musical instruments.

• What happens when sound waves from different sources overlap.

• How to describe what happens when two sound waves of slightly different frequencies are combined.

• Why the pitch of a siren changes as it moves past you.

Acoustics: deals with the study of mechanical waves in gases, liquids, solids.

(I) Sound Waves

sound = longitudinal wave in a medium (gas, liquid, solid)

The simplest sound waves are **sinusoidal waves**, with definite:

- frequency f
- amplitude A
- wavelength λ

The **human ear** is sensitive to waves in the frequency range:

- **f** $\in$ **20 to 20,000 Hz**, called the **audible range**

- **f>20kHz → ULTRASONIC waves**
- **f< 20 Hz → INFRASONIC waves**

Some **other animals** have hearing limits with higher frequencies:

- **Dogs** can hear up to around 50,000 Hz
- **Cats** up to about 60,000 Hz
- **Mice** up to about 90,000 Hz
- **Bats**, which use ultrasound to move, up to 110 000 Hz
- some **whale** species hear up to about 150,000 Hz
- **Elephants** rarely exceed 12 000 Hz
- **Chickens** 2000 Hz.

Sound waves usually travel out in all directions from the source of sound, with an amplitude that depends on the direction and distance from the source (*see later*).

Idealized case: sound wave that propagates in the positive *x-direction only.*

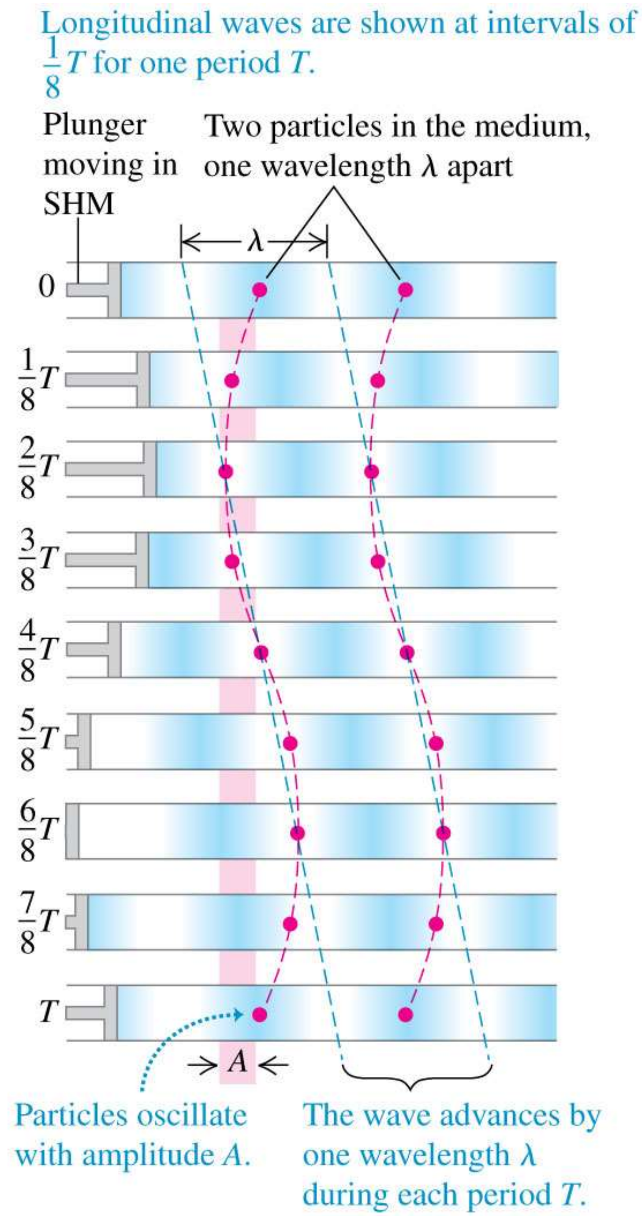
➡ Wave described by the wave function:

$$y(x,t) = A\cos(kx - \omega t)$$

gives the instantaneous displacement ***y*** *(measured along x direction) of a particle in the medium* ***at position x*** *and* ***at instant t***

In a Longitudinal wave it is along the propagation

The amplitude *A is the maximum* displacement of a particle in the medium from its equilibrium position
➔ *A=* **displacement amplitude.**



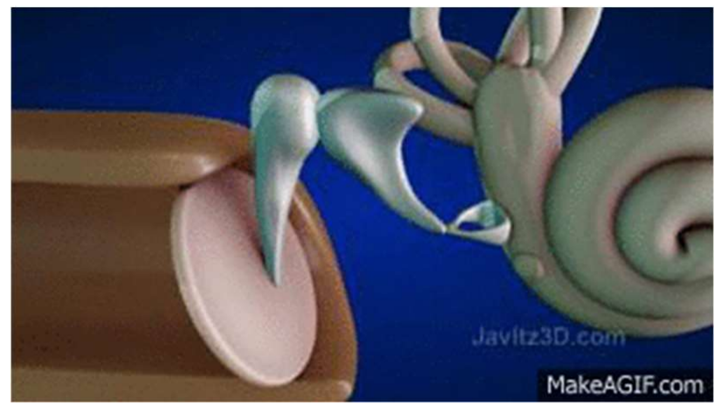
A sinusoidal longitudinal wave traveling to the right in a fluid.

Sound Waves As Pressure Fluctuations

Sound waves may also be described in terms of variations of *pressure at various* points.

In a sinusoidal sound wave in air, the pressure fluctuates above and below atmospheric pressure $\mathbf{p_a}$ in a sinusoidal variation with the same frequency as the motions of the air particles.

The human ear operates by sensing such pressure variations (put in motion the eardrum)



A sound wave entering the ear canal exerts a fluctuating pressure on one side of the eardrum; the air on the other side of the eardrum, vented to the outside by the Eustachian tube, is at atmospheric pressure. The pressure difference on the two sides of the eardrum sets it into motion.

Microphones and similar devices also usually sense pressure differences, not displacements, so it is very useful to develop a relationship between these two descriptions.

Let ***p(x,t)*** be the **instantaneous pressure fluctuation** in a sound wave at any point *x at time t.*

= the amount by which the pressure differs from the atmospheric pressure p_a

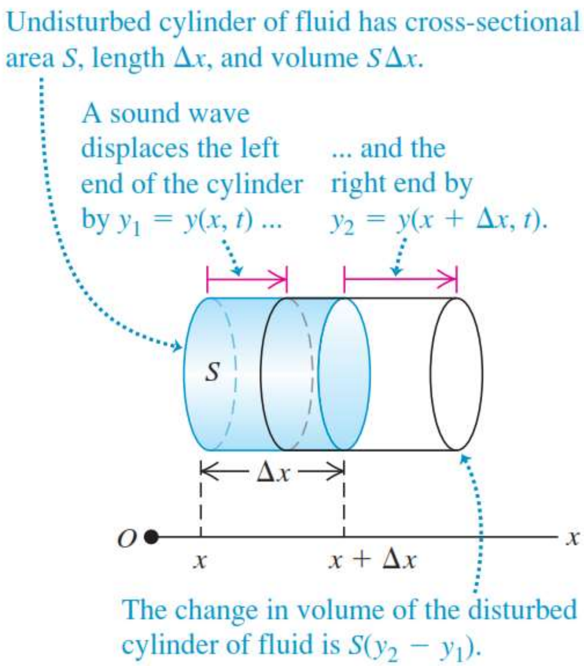
= *gauge pressure,* it can be either positive or negative

➔ The **absolute pressure** is $p_a+p(x,t)$

Connection between the pressure fluctuation p(x,t) and the displacement *y(x,t)*

sound wave propagating in the air

consider an imaginary cylinder of a wave medium (gas, liquid, or solid) with cross-sectional area S



When no sound wave is present, the cylinder has length Δx and volume $V=S\,\Delta x$

When a wave is present, at time t,

the left end is displaced by $y_1=y(x,t)$ and

the right end by $y_2= y(x+\Delta x,t)$.

the change in volume of the cylinder is:

$$\Delta V = S(y_2 - y_1) = S[y(x + \Delta x, t) - y(x, t)]$$

In the limit as $\Delta x \to 0$, the fractional change in volume dV/V (volume change divided by original volume) is

$$\frac{dV}{V} = \lim_{\Delta x \to 0} \frac{S[y(x + \Delta x, t) - y(x, t)]}{S\Delta x} = \frac{\partial y(x, t)}{\partial x}$$

The fractional volume change is related to the pressure fluctuation by the **bulk modulus B**

$$B = -p(x, t)/(dV/V)$$

$$p(x,t) = -B\frac{dV}{V} = -B\frac{\partial y(x,t)}{\partial x}$$

But: $y(x,t) = A\cos(kx - \omega t)$

$$\Rightarrow \quad p(x, t) = BkA\sin(kx - \omega t)$$

The **pressure amplitude:**

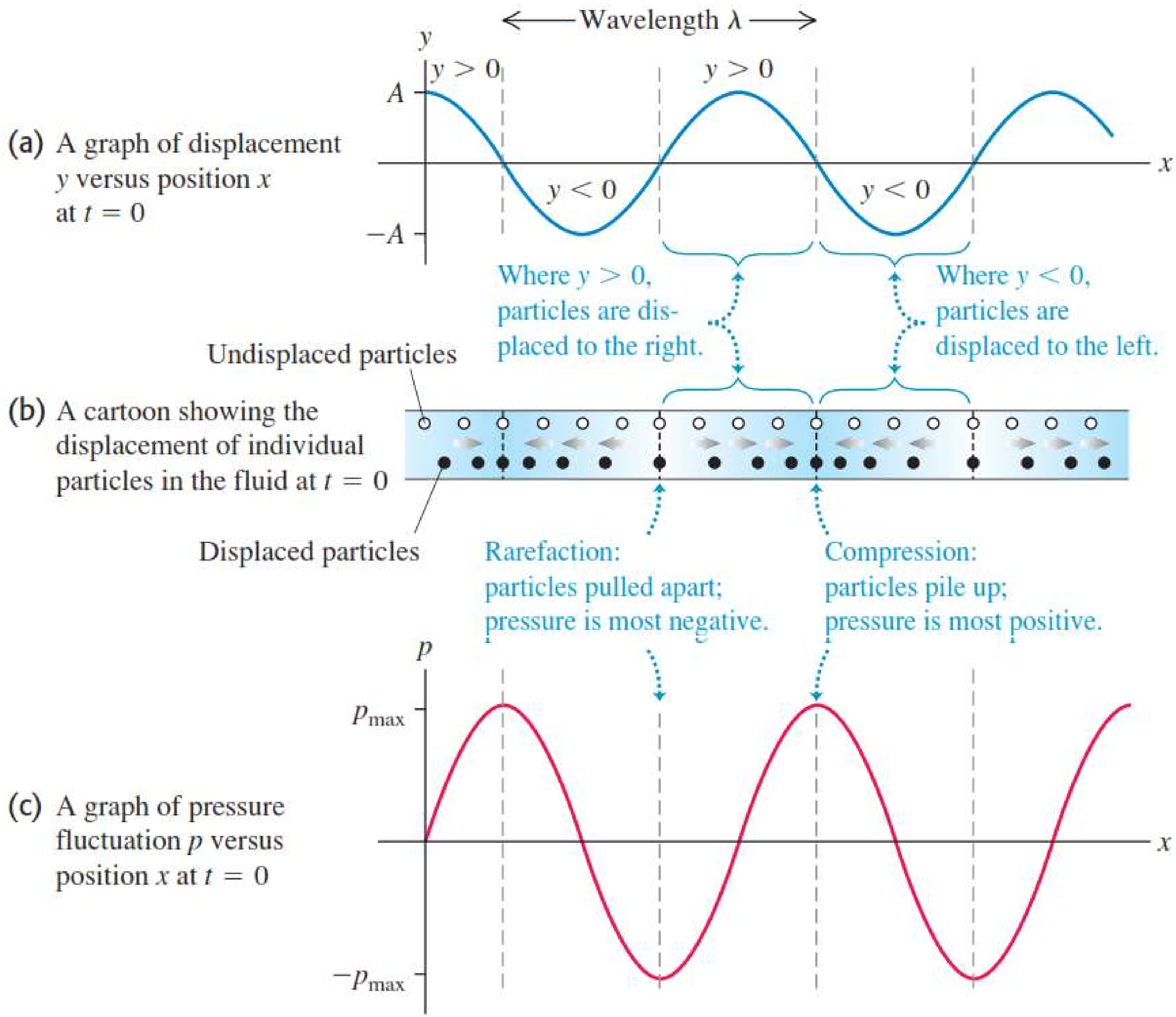
$$p_{\max} = BkA \qquad \text{(sinusoidal sound wave)}$$

$$k = \frac{2\pi}{\lambda} \quad \Rightarrow \quad p_{\max} = \frac{2\pi BA}{\lambda}$$

The pressure amplitude ~ to the: displacement amplitude *A,* bulk modulus B
depends on 1/wavelength.

Waves of shorter wavelength λ have larger pressure variation for given amplitude because the maxima and minima are squeezed closer together.

Three ways to describe a sound wave.



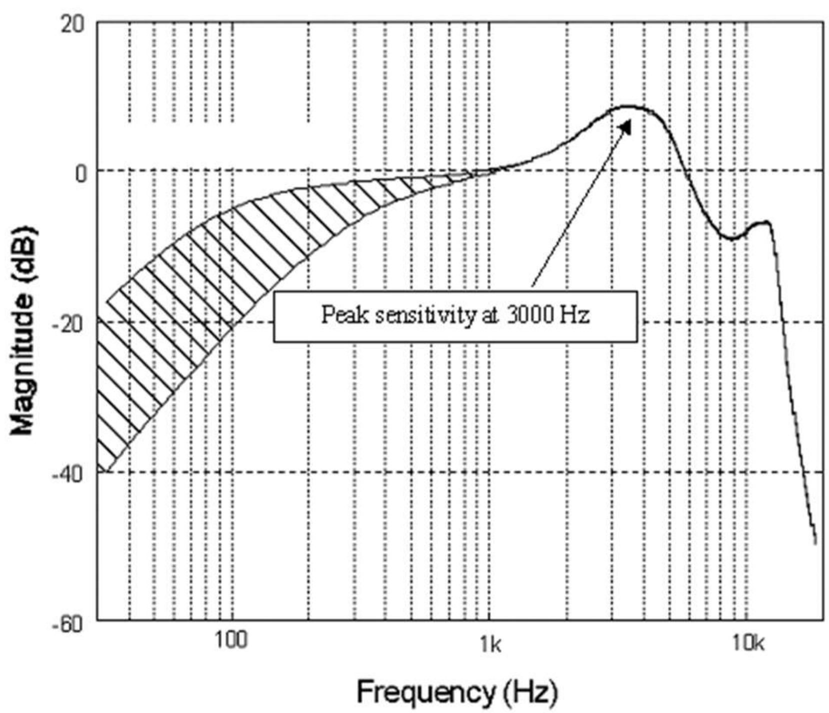
Perception of Sound Waves

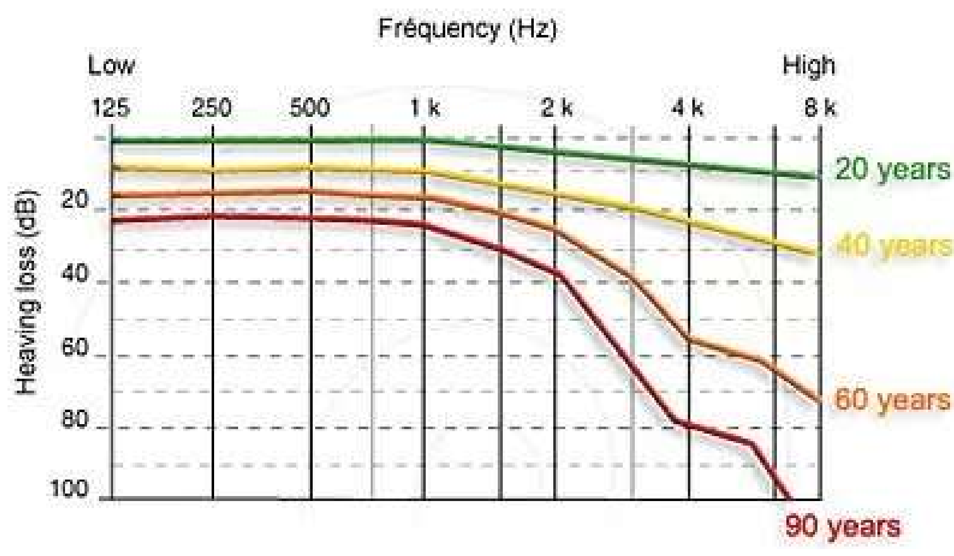
The physical characteristics of a sound wave are directly related to the perception of that sound by a listener. For a given frequency, the greater the pressure amplitude of a sinusoidal sound wave, the greater the perceived **loudness.**

The relationship between pressure amplitude and loudness is complex and it *varies from one person to another.*

The ear is not equally sensitive to all frequencies in the audible range.

Ear is most sensitive to sounds at the range of 1,000 to **5,000 Hz**, and particularly at **about 3,000 Hz**.





A loss of sensitivity at the high-frequency end usually happens naturally with age but can be further aggravated by excessive noise levels.

The pitch of a sound = the quality that lets us classify the sound as “high” or “low.”

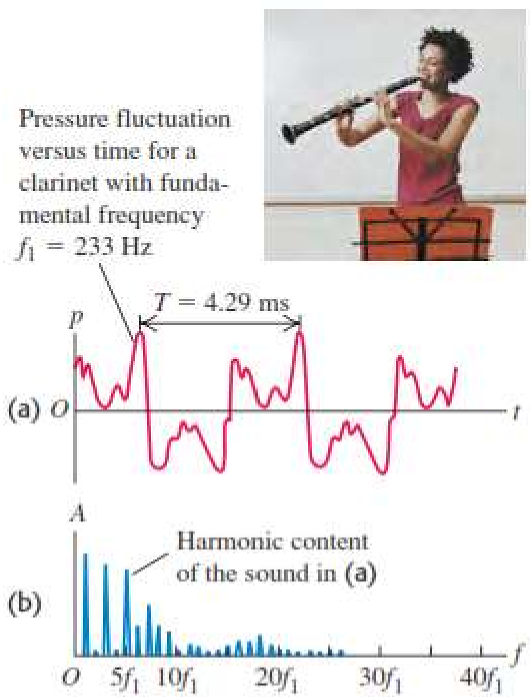
The higher the frequency of a sound (within the audible range), the higher the pitch that a listener will perceive.

Pressure amplitude also plays a role in determining pitch.

Sounds with **same f** but **larger p_{max}** are perceives as **louder** but slightly *lower in pitch*.

Spectral composition of sounds → Timber

Musical sounds have wave functions that are more complicated than a simple sine function.



Fourier analysis:
sound = **fundamental** frequency + many **harmonics**

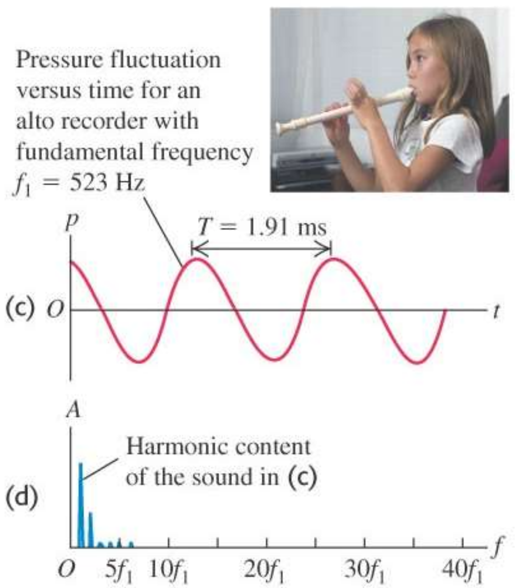
Two tones produced by different instruments might have the same fundamental frequency (and thus the same pitch) but sound different because of different harmonic content.

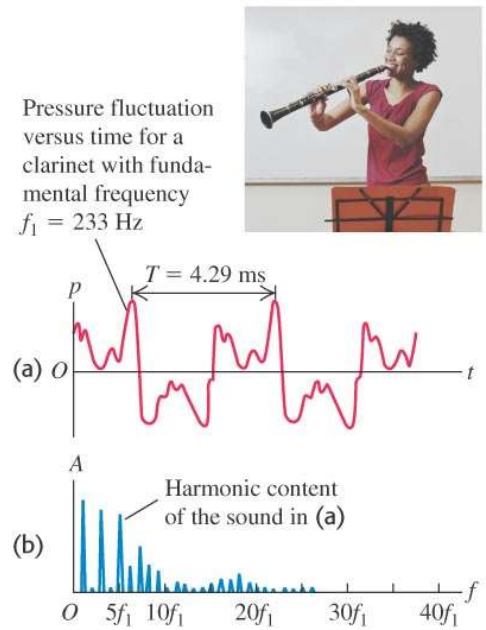
The difference in sound is called tone *color, quality, or timbre*

Same principle applies to the human voice, which is another example of a wind instrument; the vowels “a” and “e” sound different because of differences in harmonic content.

Noise

Unlike the tones made by musical instruments or the vowels in human speech, noise is a combination of all frequencies, not just frequencies that are integer multiples of a fundamental frequency.

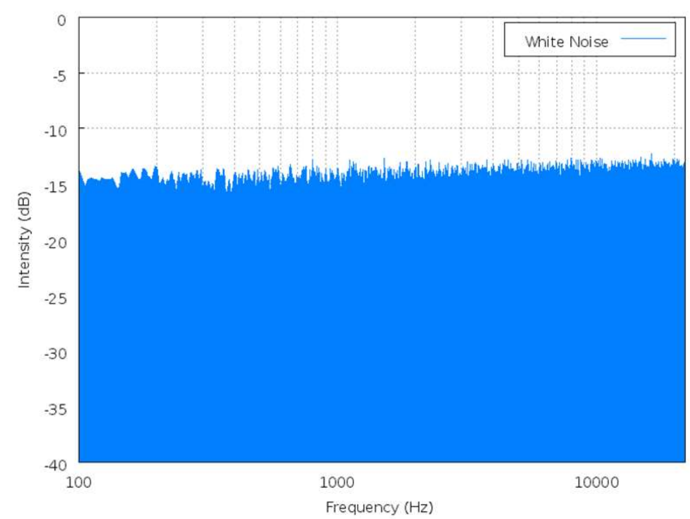




Noise

White noise

An extreme case is “**white noise,”** which contains equal amounts of all frequencies across the audible range). Examples include the sound of the wind and the hissing sound you make in saying the consonant “s.”



(II) SPEED OF SOUND WAVES

$$v = \sqrt{\frac{\text{Restoring force returning the system to equilibrium}}{\text{Inertia resisting the return to equilibrium}}}$$

- The speed of sound depends on the characteristics of the medium (see *Table*)
- The speed of sound:

$$v=\sqrt{\frac{B}{\rho}} \quad \text{(fluid)}$$

$$v=\sqrt{\frac{Y}{\rho}} \quad \text{(solid rod)}$$

$$v=\sqrt{\frac{\gamma RT}{M}} \quad \text{(ideal gas)}$$

$$R = 8.314472(15)\ \text{J/mol}\cdot\text{K}$$

γ=ratio of heat capacities (1.4)

Material	Speed of Sound (m/s)
Gases	
Air (20°C)	344
Helium (20°C)	999
Hydrogen (20°C)	1330
Liquids	
Liquid helium (4 K)	211
Mercury (20°C)	1451
Water (0°C)	1402
Water (20°C)	1482
Water (100°C)	1543
Solids	
Aluminum	6420
Lead	1960
Steel	5941

In air: T=20C =293K
M=28,8 10^{-3} kg/mole (mean molecular mass air =N2/O2)

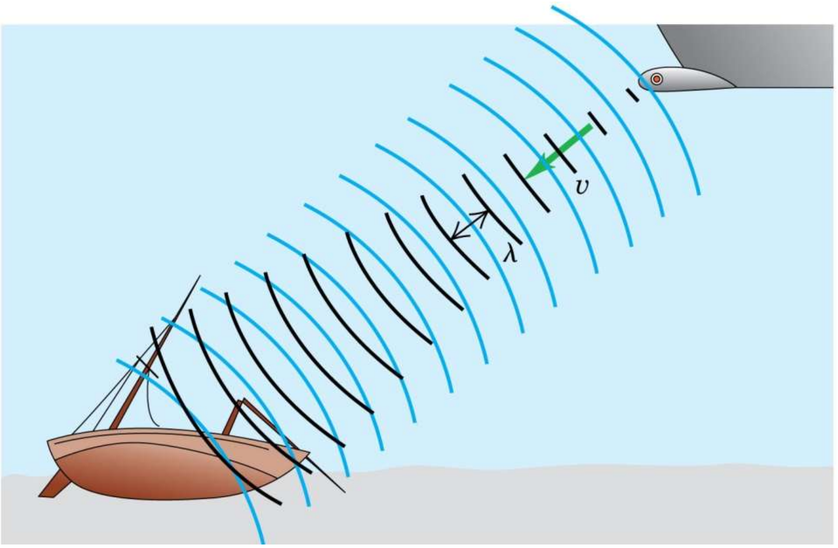
$$v = \sqrt{\frac{\gamma RT}{M}} = \sqrt{\frac{(1.40)(8.314\ \text{J/mol}\cdot\text{K})(293\ \text{K})}{28.8 \times 10^{-3}\ \text{kg/mol}}} = 344\ \text{m/s}$$

Using this value of v in $\lambda = v/f$, we find that at 20°C the frequency $f = 20$ Hz corresponds to $\lambda = 17$ m and $f = 20{,}000$ Hz to $\lambda = 1.7$ cm.

Applications

1. Sonar waves

A sonar system uses underwater sound waves to detect and locate submerged objects.



Uses a wavelength of a f=262-Hz wave

$$\lambda = \frac{v}{f} = \frac{1480\ \text{m/s}}{262\ \text{s}^{-1}} = 5.65\ \text{m}$$

Dolphins emit high-frequency sound waves (typically 100,000 Hz) and use the echoes for guidance and for hunting. The corresponding wavelength in water is 1.48 cm. With this high-frequency “sonar” system they can sense objects that are roughly as small as the wavelength (but not much smaller).

The bat does not see as human but uses echo-location.

It doubles the emitted frequency (40 →80kHz) when attacking with respect to flight to double the precision by a factor 2 when catching a fly.

2. Echography

Ultrasound are used in medicine based on sonar concept ➔ echography: we often use **2 MHz** ultrasound. With the reflections of this sound on the different organs of the body, we can recreate an image showing the inside of the body, including babies …

$\lambda = v / f$ $\quad$ $v_{water} = 1840 m/s$ $\quad$ *The precision become equal to micron.*

- For the exploration of the abdomen and pelvis we use probes whose frequency varies between 2 and 5 MHz (λ=0,3mm).
- For the explorations of the superficial organs, probes whose frequency varies from 5 to 20 MHz are used.
- Some explorations may require probes whose frequency reaches 50 MHz (ocular biomicroscopy).
- Ultrasound is more sensitive than x rays in distinguishing various kinds of tissues and does not have the radiation hazards associated with x rays.

(III) SOUND INTENSITY

Propagating waves transfer energy from one region of the medium to another.

We define **the intensity, I , of the wave** to be *the average energy* transferred *per unit time* per *unit area* perpendicular to the direction of propagation.

$$I \equiv \frac{\Delta E}{A\Delta t} \quad \Leftrightarrow \quad I = \frac{P_{av}}{A} \qquad \mathrm{W/m^2}$$

the *instantaneous power could be written:* $P = \vec{F}\cdot\vec{v}$

Sinusoidal wave:

$$y(x,t) = A\cos(kx - \omega t)$$

$$p(x,t) = BkA\sin(kx - \omega t)$$

$$v_y(x,t) = \frac{\partial y(x,t)}{\partial t} = \omega A \sin(kx - \omega t)$$

$$\Rightarrow \quad p(x,t)v_y(x,t) = [BkA\sin(kx - \omega t)][\omega A\sin(kx - \omega t)]$$

$$= B\omega k A^2 \sin^2(kx - \omega t)$$

The intensity is, by definition, the time average value of $p(x,t)v_y(x,t)$

$$\left\langle \sin^2(kx - \omega t)\right\rangle_T = \frac{1}{2}$$

$$\Rightarrow \quad I = \tfrac{1}{2}B\omega k A^2$$

By using the relationships $\omega = vk$ and $v^2 = B/\rho$

$$\Rightarrow \quad I = \tfrac{1}{2}\sqrt{\rho B}\,\omega^2 A^2$$

in a stereo system, a low-frequency woofer has to vibrate with much larger amplitude than a high-frequency tweeter to produce the same sound intensity.

Intensity and Pressure Amplitude

It is usually more useful to express *I* in terms of the pressure amplitude $p_{pmax}=BkA$ ⇨ $A = \frac{p_{max}}{Bk}$

⇨ $$I = \frac{1}{2}\sqrt{\rho B}\omega^2 A^2 = \frac{1}{2}\sqrt{\rho B}v^2 k^2 \frac{p_{max}{}^2}{B^2 k^2} = \frac{1}{2}\sqrt{\rho B}\frac{B}{\rho}\frac{p_{max}{}^2}{B^2}$$

⇨ $$I = \frac{p_{max}{}^2}{2\rho v} = \frac{p_{max}{}^2}{2\sqrt{\rho B}}$$

The total average power carried across a surface by a sound wave equals the:
product of the intensity at the surface **and the surface area**,
if the intensity over the surface is uniform.

- The average total sound power emitted by a person speaking in an ordinary conversational tone is about 10^{-5} W, while a loud shout corresponds to about $3\text{x}10^{-2}$ W.

- If all the residents of New York City were to talk at the same time, the total sound power would be about 100 W, equivalent to the electric power requirement of a medium-sized light bulb.

Geometrical attenuation of amplitude

If the sound source emits waves in all directions equally, the intensity decreases with increasing distance r from the source according to the **inverse-square law**: The intensity is proportional to $1/r^2$.

If the sound goes predominantly in one direction, the inverse-square law does not apply and the intensity decreases with distance more slowly than $1/r^2$.

By cupping your hands like this, you direct the sound waves emerging from your mouth so that they don't propagate to the sides. Hence the intensity decreases with distance more slowly than the inverse-square law would predict, and you can be heard at greater distances.

The inverse-square relationship also does not apply indoors because sound energy can reach a listener by reflection from the walls and ceiling. Indeed, part of the architect's job in designing an auditorium is to tailor these reflections so that the intensity is as nearly uniform as possible over the entire auditorium.

The Decibel Scale

Because the ear is sensitive over a broad range of intensities, a logarithmic intensity scale is usually used.

The *sound intensity level* β of a sound: $$\beta = (10\ \text{dB}) \log \frac{I}{I_0} \qquad \text{(definition of sound intensity level)}$$

- $I_0 = 10^{-12}\ W/m^2$ reference intensity, approximately the threshold of human hearing at 1000 Hz.
- "log" means the logarithm to base 10
- dB=decibel=1/10 bel (Alexander Graham Bell –inventor of telephone).

$$\begin{cases} I = 10^{-12} W/m^2 \Rightarrow \beta = 0dB \\ I = 1W/m^2 \Rightarrow \beta = 120dB \end{cases}$$

Table 16.2 Sound Intensity Levels from Various Sources (Representative Values)

Source or Description of Sound	Sound Intensity Level, β (dB)	Intensity, I (W/m^2)
Military jet aircraft 30 m away	140	10^2
Threshold of pain	120	1
Riveter	95	3.2×10^{-3}
Elevated train	90	10^{-3}
Busy street traffic	70	10^{-5}
Ordinary conversation	65	3.2×10^{-6}
Quiet automobile	50	10^{-7}
Quiet radio in home	40	10^{-8}
Average whisper	20	10^{-10}
Rustle of leaves	10	10^{-11}
Threshold of hearing at 1000 Hz	0	10^{-12}

(IV) STANDING SOUND WAVES AND NORMAL MODES

When longitudinal (sound) waves propagate in a fluid in a pipe with finite length, the waves are reflected from the ends in the same way that transverse waves on a string are reflected at its ends.

The superposition of the waves traveling in opposite directions again forms a standing wave.

Just as for transverse standing waves on a string, standing sound waves (normal modes) in a pipe can be used be used to create sound waves in the surrounding air.

operating principle of the human voice as well as many musical instruments, including woodwinds, brasses, and pipe organs

Transverse waves on a string, including standing waves, are usually *described only in terms of the displacement of the string*.

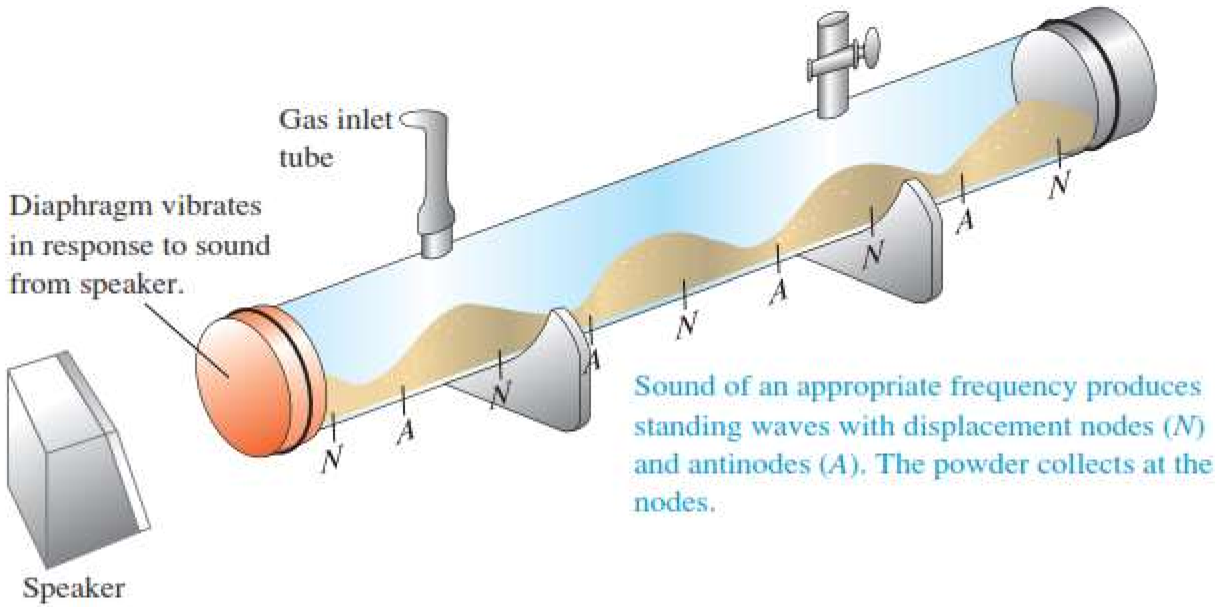
Sound waves in a fluid may be *described either in terms of the displacement* of the fluid or *in terms of the pressure variation* in the fluid.

displacement node ⇔ **pressure antinode**

displacement antinode ⇔ **pressure node**

$y(x,t) = A\cos(kx - \omega t)$ $\quad$ $p(x,t) = BkA\sin(kx - \omega t)$ $\quad$ Vary in antiphase

We can demonstrate standing sound waves in a column of gas using an apparatus called a **Kundt's tube**



The blue shading represents the density of the gas at an instant when the gas pressure at the displacement nodes is a maximum or a minimum.

As we vary the frequency of the sound, we pass through frequencies at which the amplitude of the standing waves becomes large enough for the powder to be swept along the tube at those points where the gas is in motion. The powder therefore collects at the displacement nodes (where the gas is not moving).

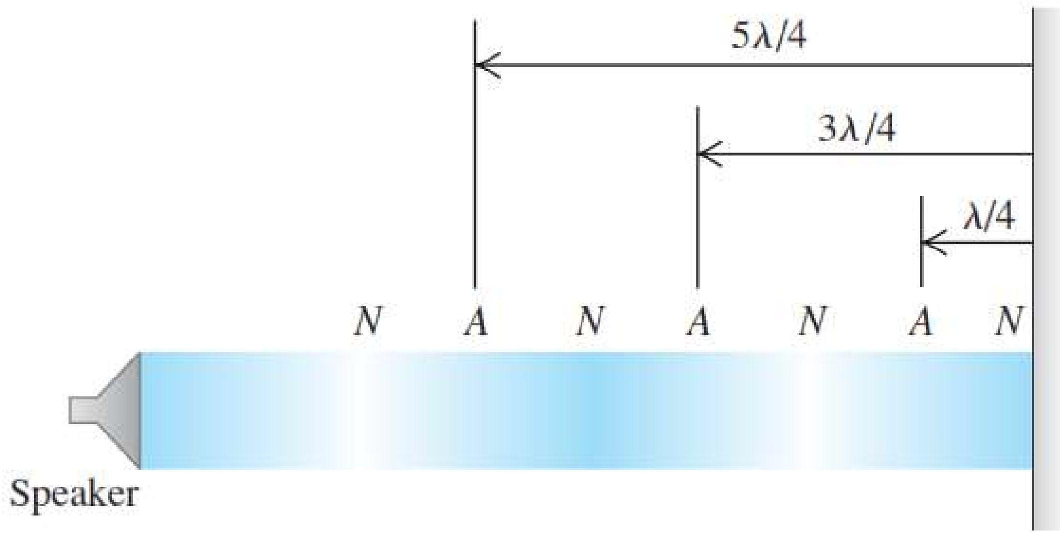
Adjacent nodes are separated by a distance equal to $\lambda/2$ and we can measure this distance. Given the wavelength, we can use this experiment to determine the wave speed: We read the frequency from the oscillator dial, and we can then calculate the speed of the waves from the relationship $v = \lambda f$.

The sound of silence

A directional loudspeaker directs a sound wave of wavelength λ at a wall. At what distances from the wall could you stand and hear no sound at all?

The ear detects pressure variations in the air
$\Rightarrow$ to hear no sound if the ear is at a pressure node, which is a displacement antinode.

the wall is at a displacement node; the distance from any node to an adjacent antinode is $\lambda/4$ and the distance from one antinode to the next is $\lambda/2$.



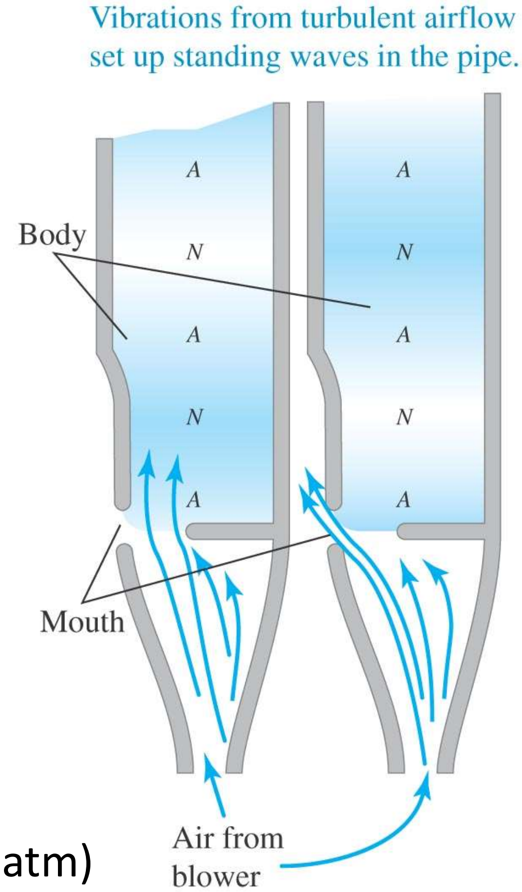
the displacement antinodes (pressure nodes), at which no sound will be heard, are at d= $\lambda/4$, $\lambda/4+ \lambda/2= 3\lambda/4$,…

=> "*Acoustical screening*" in noisy environments: place the working area in pressure nodes

Organ Pipes and Wind Instruments

The most important application of standing sound waves is the production of musical tones by wind instruments.

- Organ pipes of different sizes produce tones with different frequencies (bottom figure).
- The figure at the right shows displacement nodes in two cross-sections of an organ pipe at two instants that are one-half period apart. The blue shading shows pressure variation.



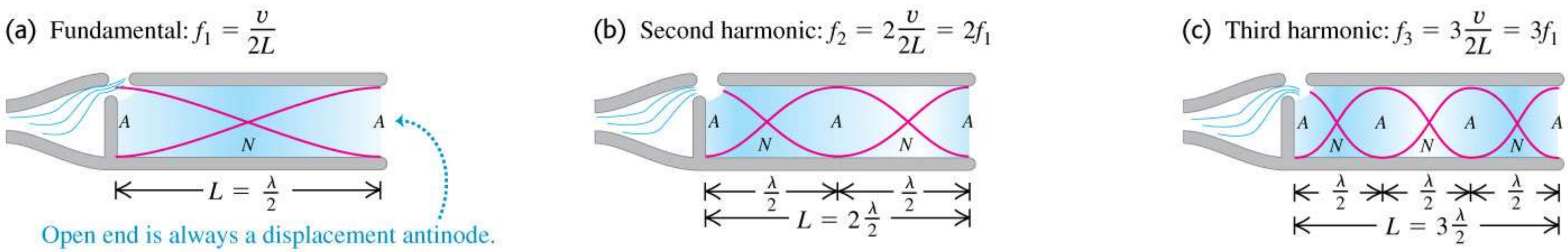
$p \sim 10^3$ Pa (10^{-2} atm)

The column of air in the pipe is set into vibration, and there is a series of possible normal modes, just as with the stretched string. The mouth always acts as an open end; thus it is a pressure node and a displacement antinode. The other end of the pipe (at the top in Fig.) may be either open or closed.

Harmonics in an open pipe

- An *open pipe* is open at both ends.
- For an open pipe $\lambda_n = 2L/n$

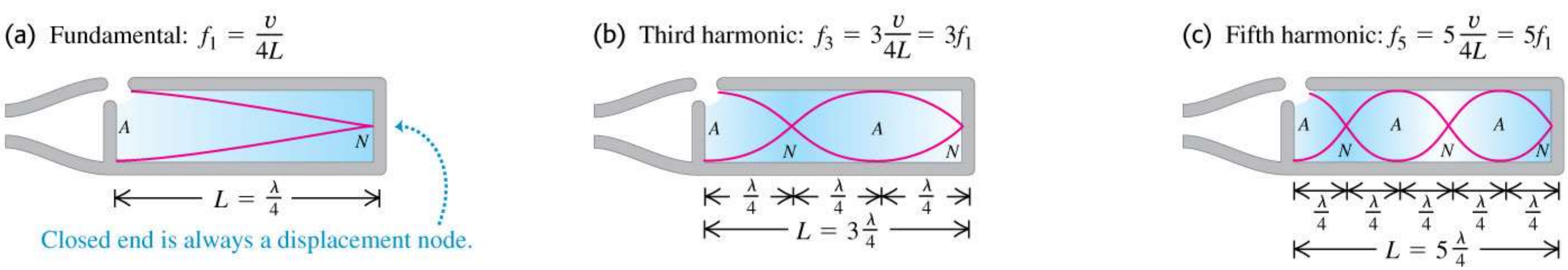
$$f_n = \frac{nv}{2L} \qquad (n = 1, 2, 3, \ldots)$$



Harmonics in an closed pipe

- A *closed pipe* is open at one end and closed at the other end.
- For a closed pipe $\lambda_n = 4L/n$

$$f_n = \frac{nv}{4L} \qquad (n = 1, 3, 5, \ldots)$$



Frequency ~ speed of sound *v* in the air column inside the instrument. *v* =f(temperature); it increases when T increases => the pitch rises with increasing T. An organ that has some of its pipes at one temperature and others at a different temperature is bound to sound out of tune.

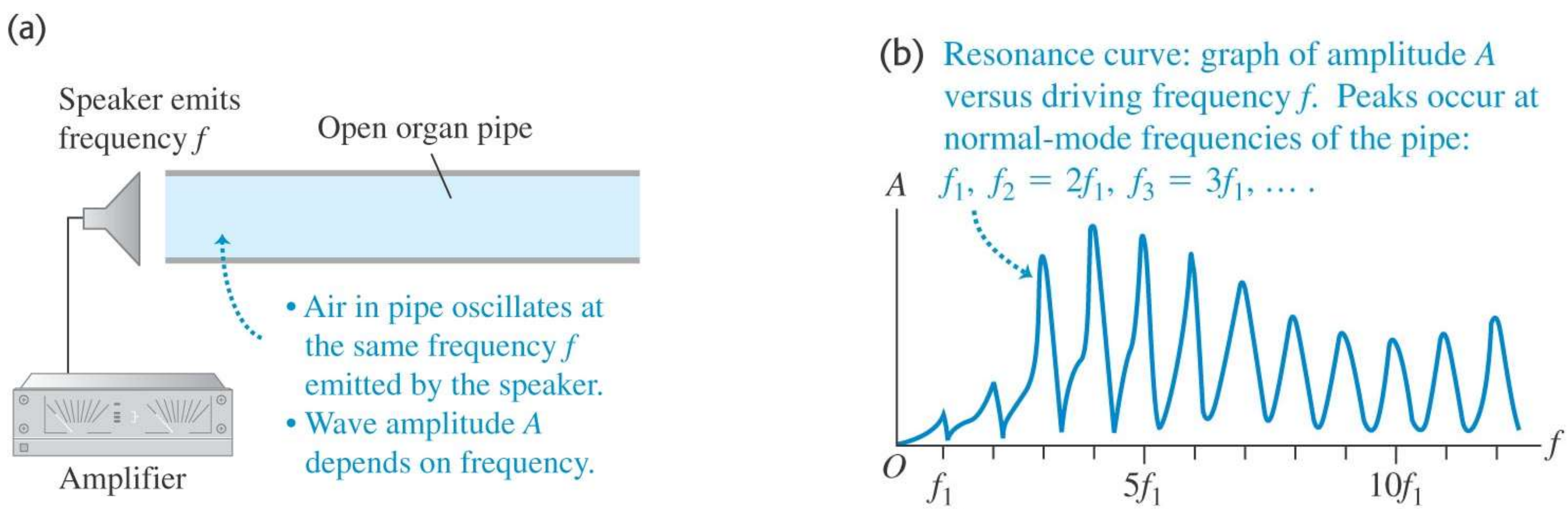
(V) RESONANCE AND SOUND

Many mechanical systems have normal modes of oscillation. In each mode, every particle of the system oscillates with SHM at the same frequency as the mode.

Air columns and stretched strings have an infinite series of normal modes, but the basic concept is closely related to the SHM, which has only a single normal mode (that is, only one frequency at which it oscillates after being disturbed).

Suppose we apply a periodically varying force to a system that can oscillate. The system is then forced to oscillate with a frequency equal to the frequency of the applied force (called the *driving frequency). => forced oscillation.*

In the forced oscillation regime the resonance occurs when the frequency of the force is close to one of the normal-mode frequencies.



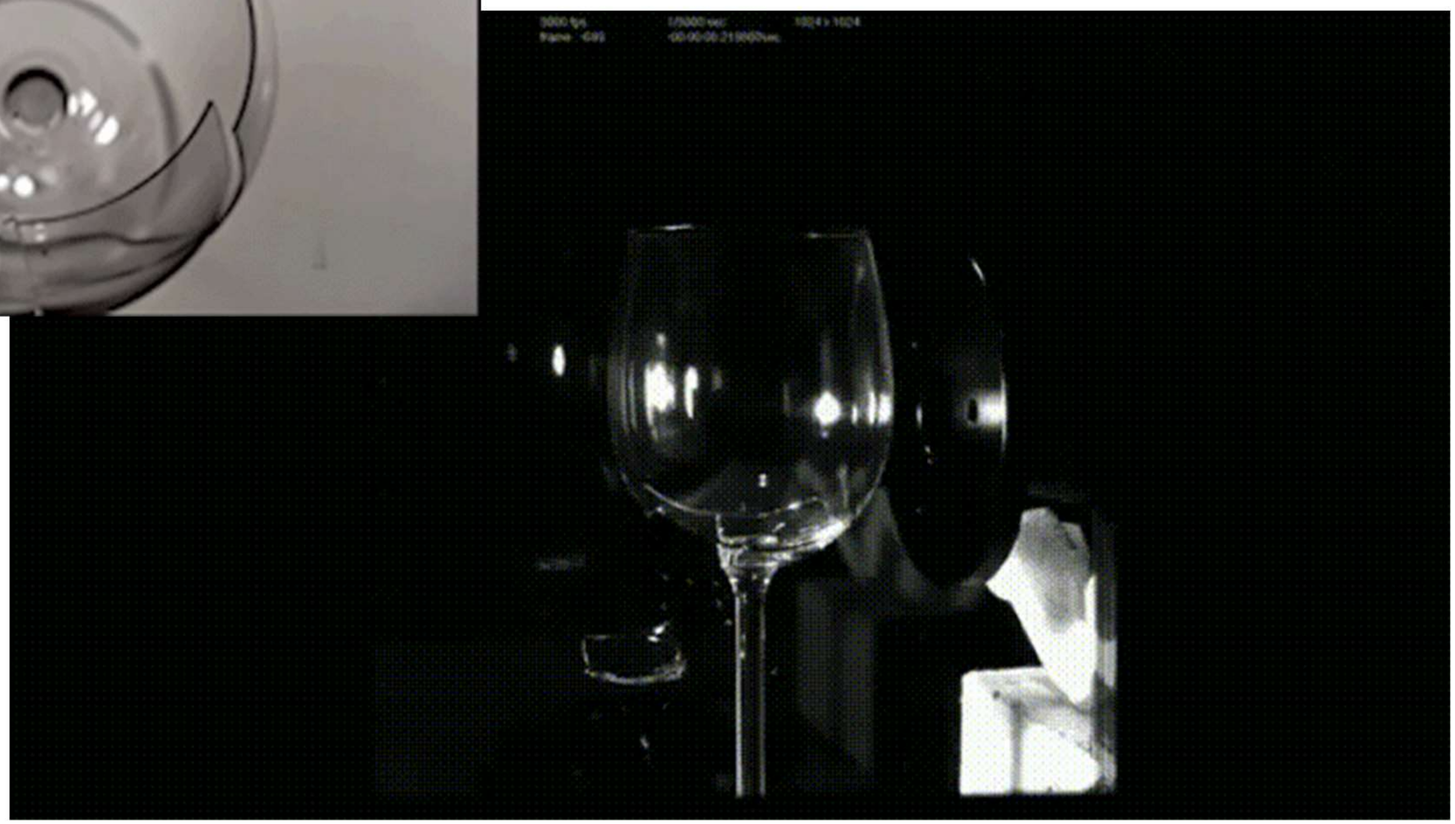
If the frequency of the force is precisely equal to a normal-mode frequency, the system is in resonance, and the amplitude of the forced oscillation is maximum. If there were no friction or other energy-dissipating mechanism, a driving force at a normal-mode frequency would continue to add energy to the system, and the amplitude would increase indefinitely. In such an idealized case the peaks in the resonance curve would be infinitely high. But in any real system there is always some dissipation of energy, or damping, the amplitude of oscillation in resonance may be large, but it cannot be infinite.

The "sound of the ocean" you hear when you put your ear next to a large seashell is due to resonance. The noise of the outside air moving past the seashell is a mixture of sound waves of almost all audible frequencies, which forces the air inside the seashell to oscillate. The seashell behaves like an organ pipe, with a set of normal-mode frequencies; hence the inside air oscillates most strongly at those frequencies, producing the seashell's characteristic sound.

Resonance and the Sensitivity of the Ear

The auditory canal of the human ear is an air-filled pipe open at one end and closed at the other (eardrum) end. The canal is about 2.5 cm = 0.025 m long, so it has a resonance at its fundamental frequency f_1*=v/4L=(344 m s)/[4(0.025m)]= 3440 Hz. The resonance means that a sound at this frequency produces a strong oscillation of the eardrum. That's why your ear is most sensitive to sounds near 3440 Hz.*

A more spectacular example is a singer breaking a wine glass with her amplified voice. A good-quality wine glass has normal-mode frequencies that you can hear by tapping it. If the singer emits a loud note with a frequency corresponding exactly to one of these normal-mode frequencies, large-amplitude oscillations can build up and break the glass.



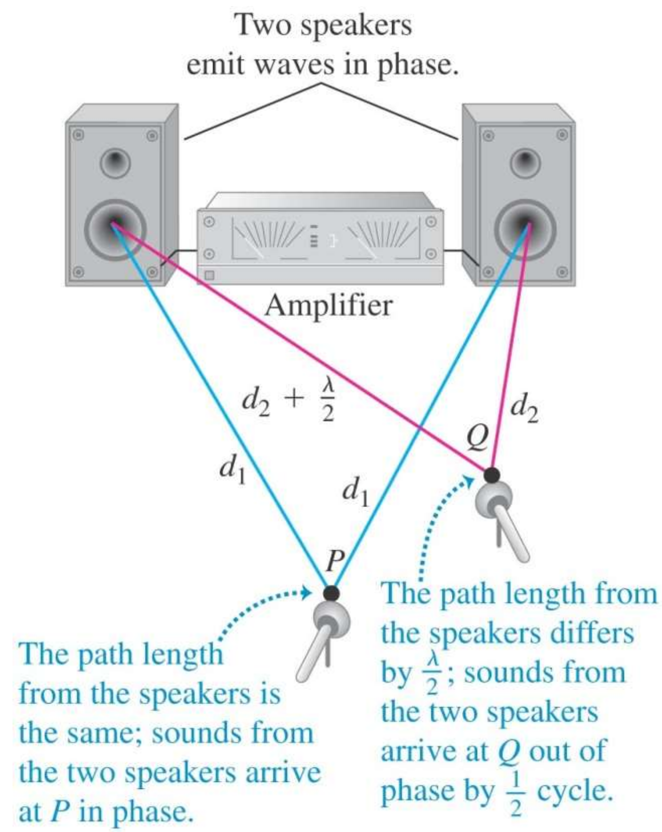
(VI) INTERFERENCE OF WAVES

When two or more waves overlap in the same region of space => **interference.**

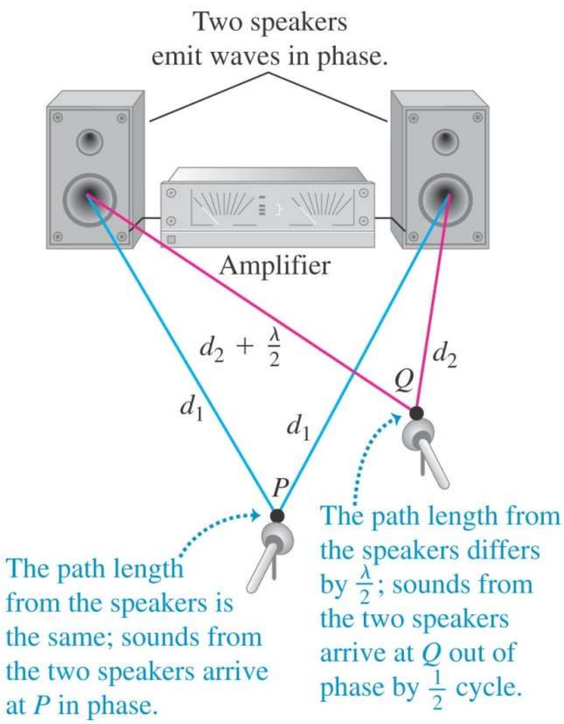
standing waves are a simple example of an interference effect: Two waves traveling in opposite directions in a medium combine to produce a standing wave pattern with nodes and antinodes that do not move.

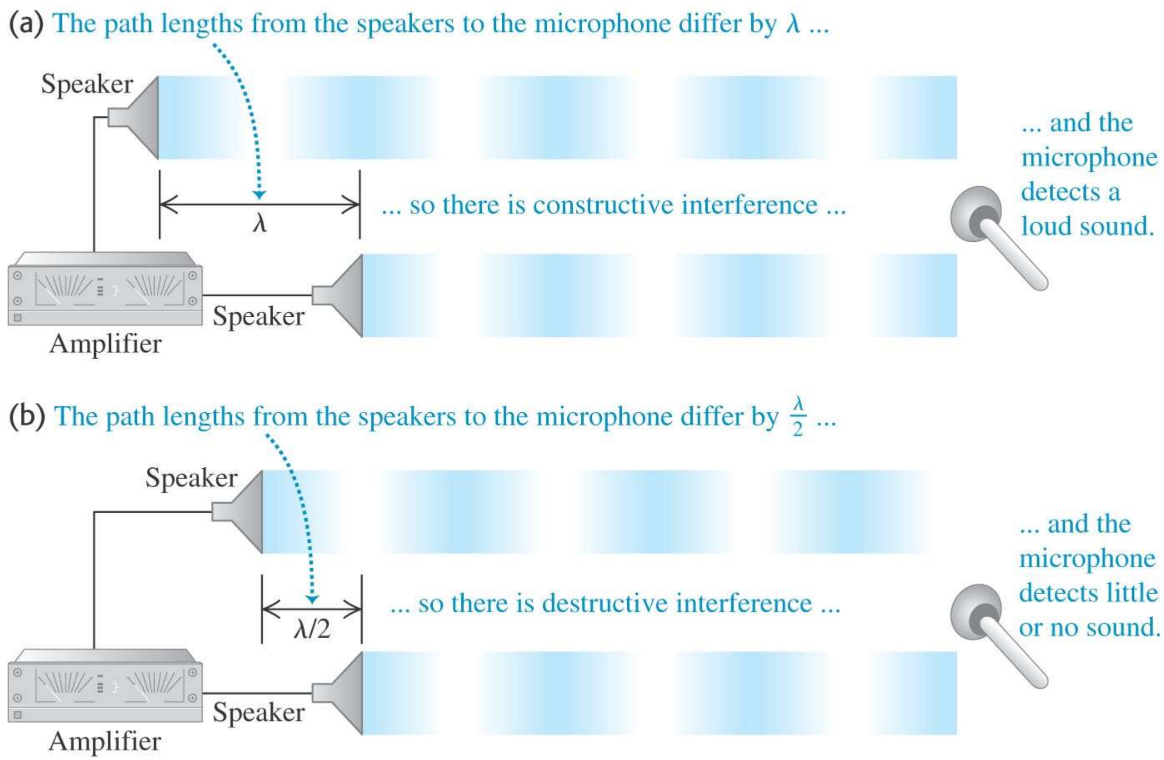
another type of interference that involves waves that spread out in space. Two speakers, driven in phase by the same amplifier, emit identical sinusoidal sound waves with the same constant frequency.

We place a microphone at point P, equidistant from the speakers. Wave crests emitted from the two speakers at the same time travel equal distances and arrive at point P at the same time; hence the waves arrive in phase, and there is constructive interference. The total wave amplitude at P is twice the amplitude from each individual wave, and we can measure this combined amplitude with the microphone.



Now let's move the microphone to point *Q, where the distances from the two* speakers to the microphone differ by a half-wavelength. Then the two waves arrive a half-cycle out of step, or *out of phase; a positive crest from one speaker* arrives at the same time as a negative crest from the other. Destructive interference takes place, and the amplitude measured by the microphone is much *smaller* than when only one speaker is present. If the amplitudes from the two speakers are equal, the two waves cancel each other out completely at point *Q,* and the total amplitude there is zero.





The difference in the lengths of the paths traveled by the sound determines whether the sound from two sources interferes constructively or destructively

(VII) BEATS

- We talked about interference effects that occur when two different waves with the same frequency overlap in the same region of space.
- Now let's look at what happens when we have two waves with equal amplitude but slightly different frequencies. This occurs, for example, when two tuning forks with slightly different frequencies are sounded together, or when two organ pipes that are supposed to have exactly the same frequency are slightly "out of tune."

$$y_a(x,t) = A\sin(k_a x + \omega_a t) \xrightarrow{x=0} A\sin(2\pi f_a x)$$

$$y_b(x,t) = A\sin(k_b x - \omega_b t) \xrightarrow{x=0} -A\sin(2\pi f_b x)$$

$$\sin a - \sin b = 2\sin\tfrac{1}{2}(a-b)\cos\tfrac{1}{2}(a+b)$$

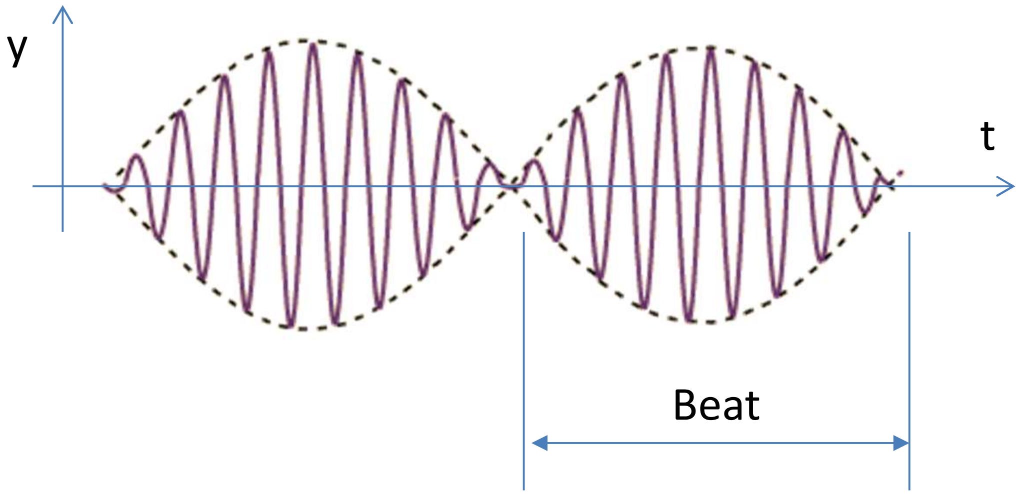
$$y_a(t) + y_b(t) = \left[2A\sin\tfrac{1}{2}(2\pi)(f_a - f_b)t\right]\cos\tfrac{1}{2}(2\pi)(f_a + f_b)t$$

The amplitude factor (the quantity in brackets) varies slowly with frequency $\frac{1}{2}(f_a - f_b)$. The cosine factor varies with a frequency equal to the *average* frequency $\frac{1}{2}(f_a + f_b)$. The *square* of the amplitude factor, which is proportional to the intensity that the ear hears, goes through two maxima and two minima per cycle. So the beat frequency f_{beat} that is heard is twice the quantity $\frac{1}{2}(f_a - f_b)$, or just $f_a - f_b$.

$$y_a(t) + y_b(t) = \left[2A \sin \tfrac{1}{2}(2\pi)(f_a - f_b)t\right] \cos \tfrac{1}{2}(2\pi)(f_a + f_b)t$$

Slow variation amplitude with frequency $f=(f_a-f_b)/2$

Rapid variation amplitude with frequency $f=(_{fa}+f_b)/2$



The sound intensity ~A^2 and
the frequency of $\sin^2[2\pi(f_a-f_b)/2]$ is $2\,(f_a-f_b)/2= f_a-f_b$

$$f_{\text{beat}} = f_a - f_b \qquad \text{(beat frequency)}$$

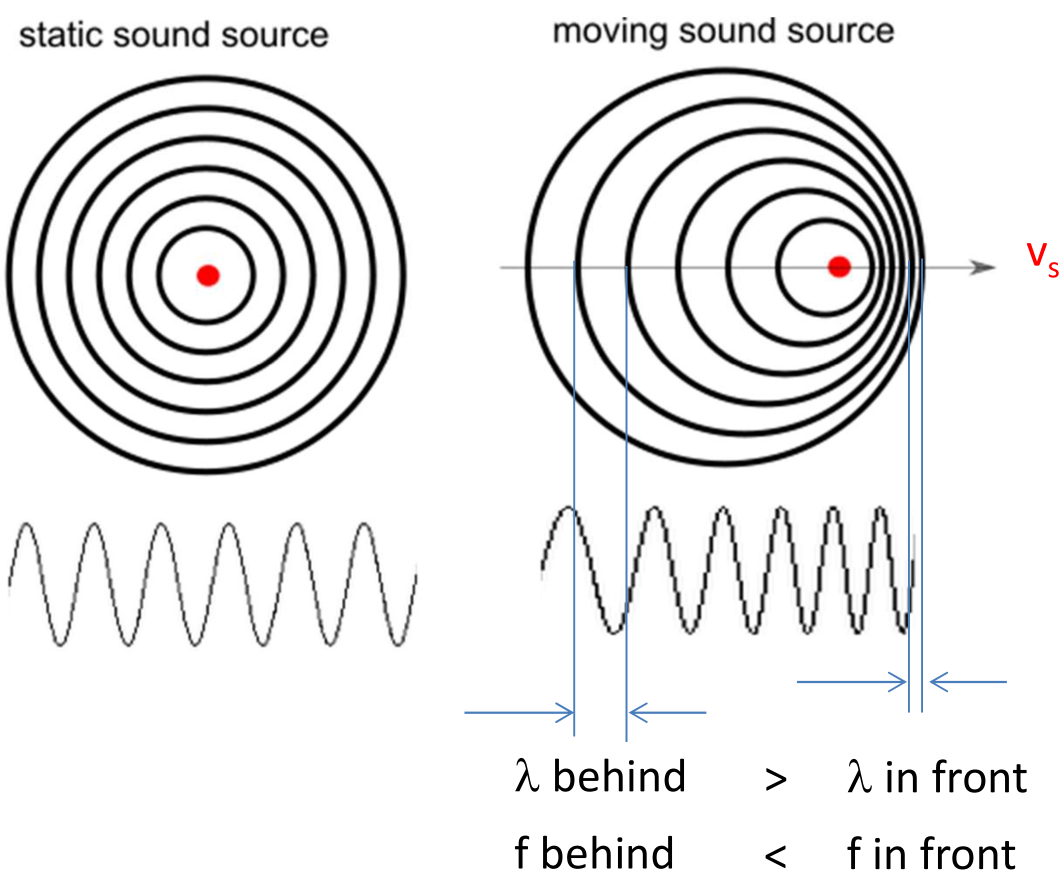
Beats between two tones can be heard up to a beat frequency of about 6 or 7 Hz. Above => the sensation merges into one of *consonance or dissonance.*

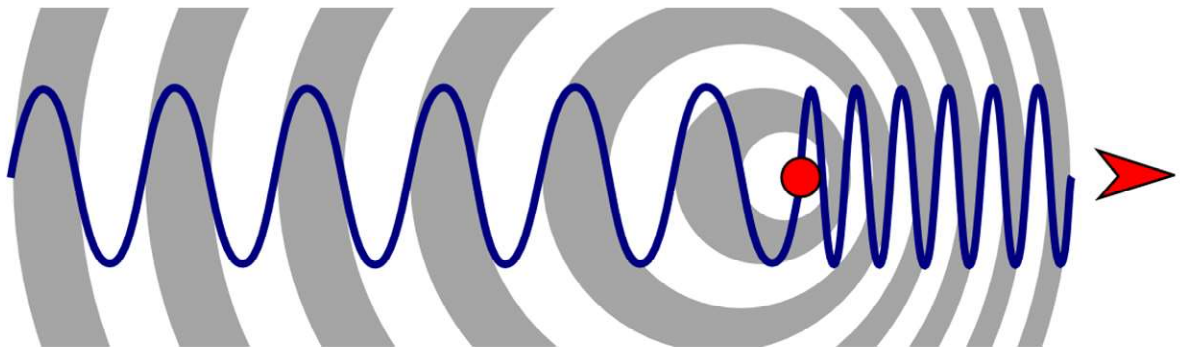
The engines on multiengine propeller aircraft have to be synchronized so that the propeller sounds don’t cause annoying beats, which are heard as loud throbbing sounds.

(VIII) THE DOPPLER EFFECT

You've probably noticed that when a car approaches you with its horn sounding, the pitch seems to drop as the car passes. This phenomenon, first described by the 19th-century Austrian scientist *Christian Doppler*, is called the **Doppler effect**.

When a source of sound and a listener are in motion relative to each other, the frequency of the sound heard by the listener is not the same as the source frequency.



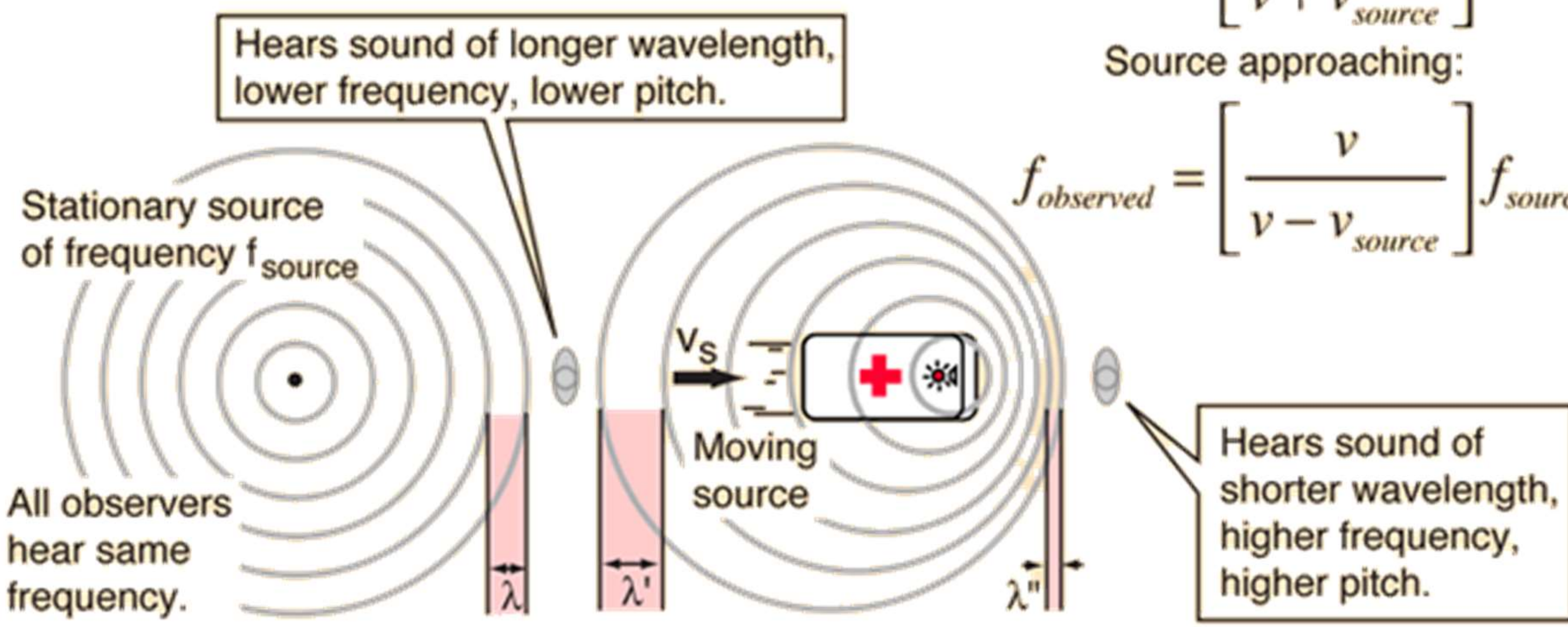


Source receding:

$$f_{observed} = \left[\frac{v}{v + v_{source}} \right] f_{source}$$

Source approaching:

$$f_{observed} = \left[\frac{v}{v - v_{source}} \right] f_{source}$$



Doppler Effect for Electromagnetic Waves

In this case there is no medium that we can use as a reference to measure velocities, and all that matters is the relative velocity of source and receiver.

To derive the expression for the Doppler frequency shift for light, we have to use the special theory of relativity.

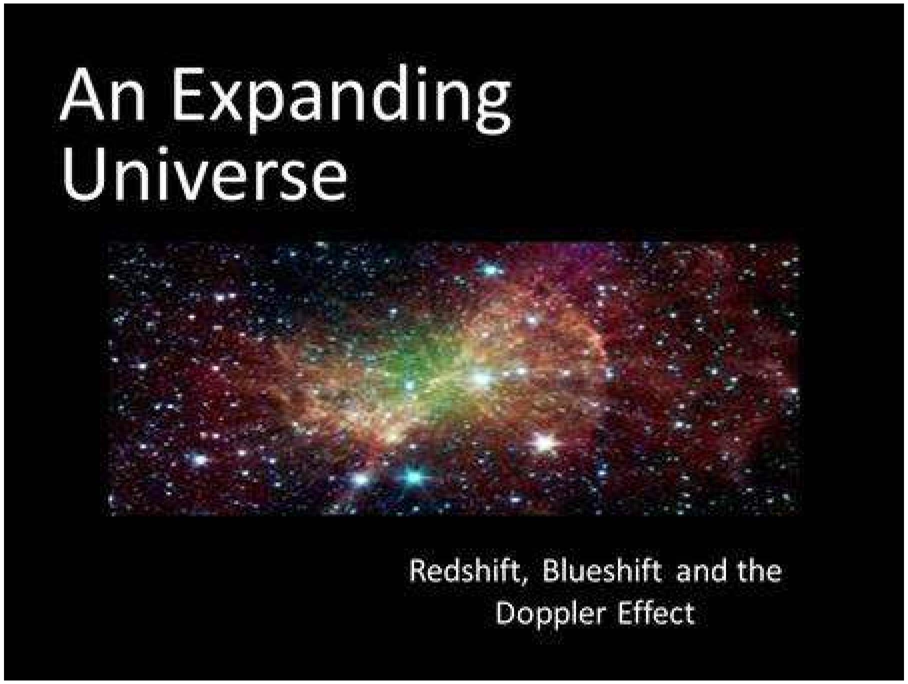
The wave speed is the speed of light denoted by *c, and it is the same for both source and receiver. In the frame* of reference in which the receiver is at rest, the source is moving away from the receiver with velocity v.

$$f_R = \sqrt{\frac{c - v}{c + v}} f_S$$

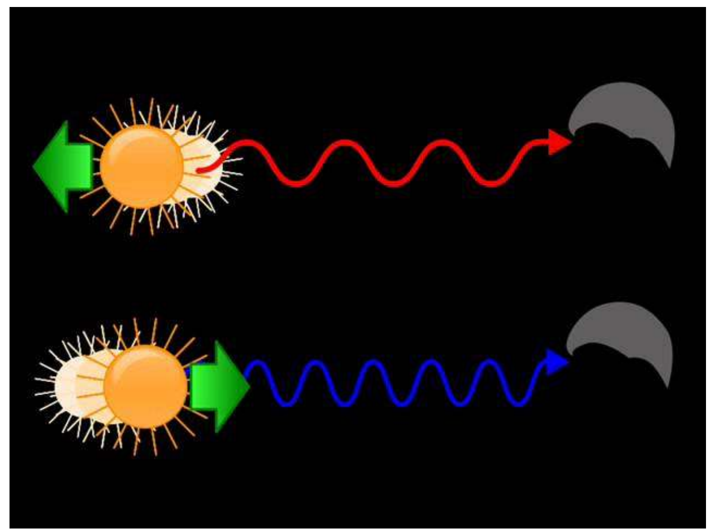
When v is positive, the source is moving directly *away* from the receiver and f_R is always *less* than f_S; when v is negative, the source is moving directly *toward* the receiver and f_R is *greater* than f_S. The qualitative effect is the same as for sound, but the quantitative relationship is different.

moving away from receiver => f_R smaller than f_s Red-shift
moving towards receiver=> f_R greater than f_s => Blue-shift

Doppler Effect and expansion of Universe

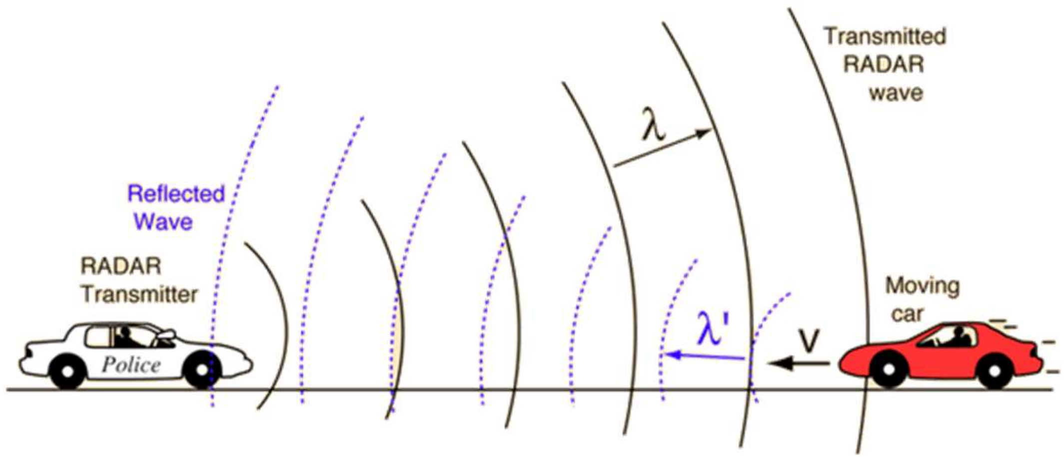


Edwin Hubble used the Doppler effect to determine that the universe is expanding. Hubble found that the light from distant galaxies was shifted toward lower frequencies, to the red end of the spectrum. This is known as a red Doppler shift, or a red-shift.



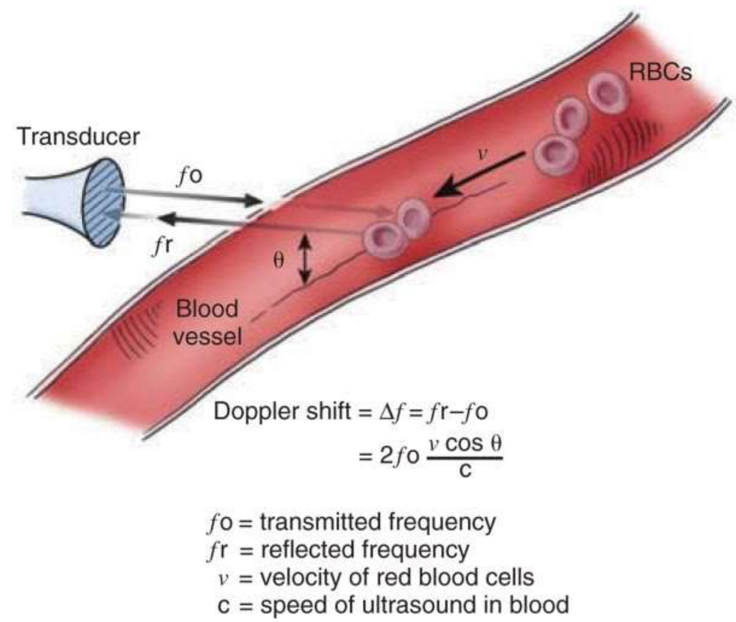
Doppler effect can be used to determine velocity of stars, galaxies…

Doppler radar device mounted on the side window of a police car to check other cars' speeds. The electromagnetic wave emitted by the device is reflected from a moving car, which acts as a moving source, and the wave reflected back to the device is Doppler shifted in frequency. The transmitted and reflected signals are combined to produce beats, and the speed can be computed from the frequency of the beats.



Similar techniques ("Doppler radar") are used to measure:

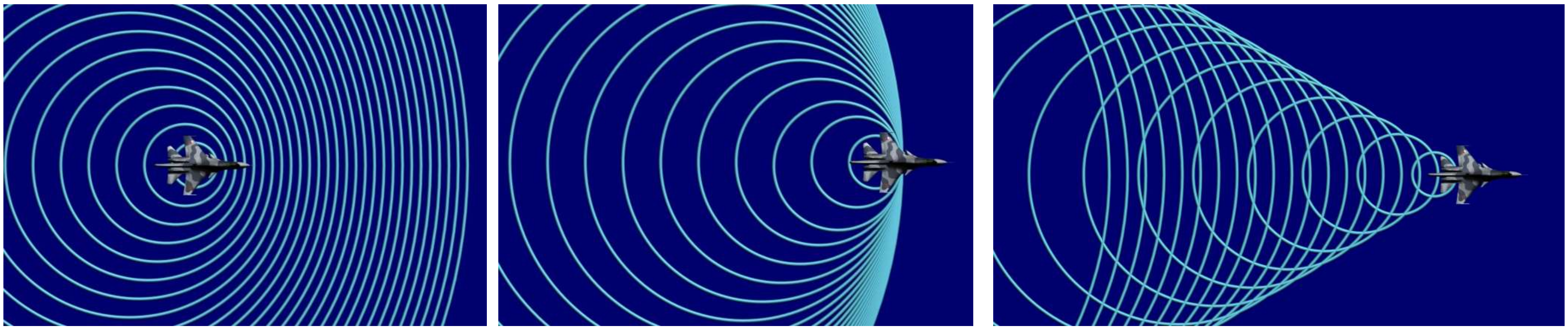
- wind velocities in the atmosphere
- **echo- cardiograph** (measure blood velocity: reflexing of sound waves of an echo-graph on red-blood cells.
  - We measure the speed of the blood through valves, in the vessels we can detect laminar regimes, turbulences/puffs…



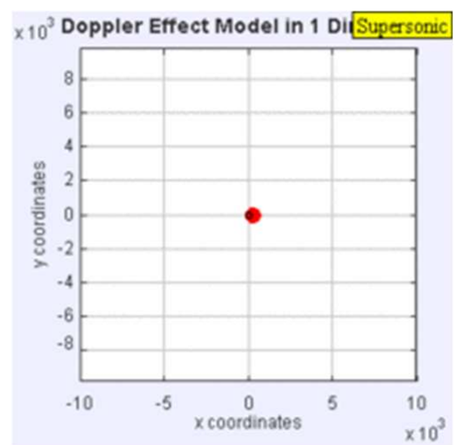
(IX) SHOCK WAVES

From Doppler regime to shock waves

What happens when the speed of wave source approaches the wave velocity in the medium (sound source velocity approaches sound velocity in air ?)



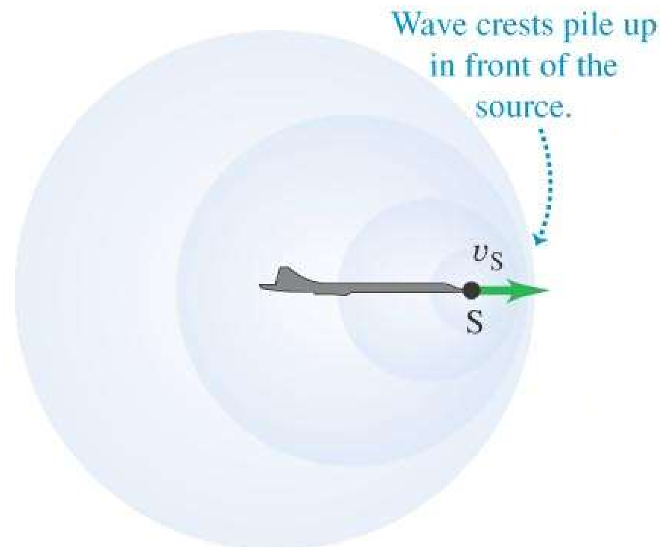
$v_{source} < v_{wave}$ $\qquad$ $v_{source} = v_{wave}$ $\qquad$ $v_{source} > v_{wave}$



(IX) SHOCK WAVES

You may have experienced "sonic booms" caused by an airplane flying overhead faster than the speed of sound.

(a) Sound source S (airplane) moving at nearly the speed of sound



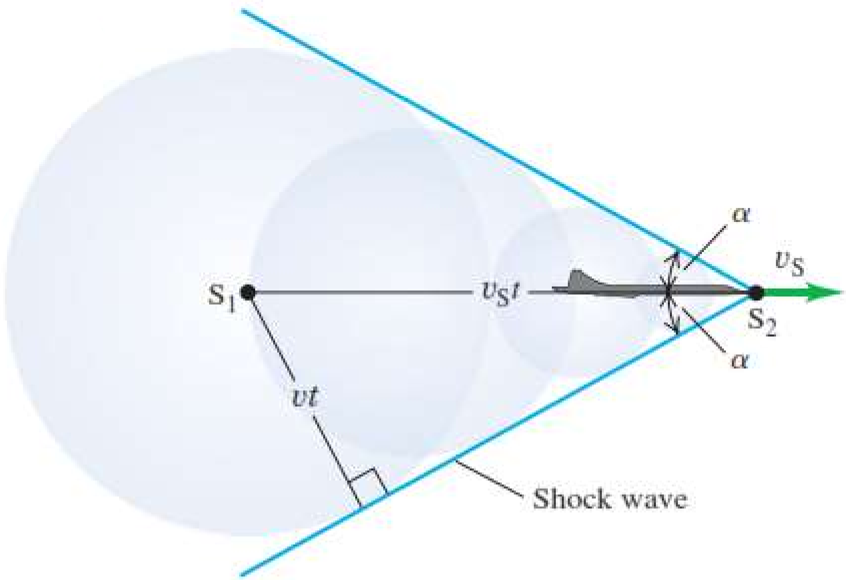
Let denote v_s the speed of the airplane relative to the air, so that it is always positive. The motion of the airplane through the air produces sound; if is less than the speed of sound the waves in front of the airplane are crowded together with a smaller wavelength (Doppler).

$$\lambda_{\text{in front}} = \frac{v - v_S}{f_S}$$

As the speed of the airplane v_s approaches the speed of sound v the wavelength approaches zero and the wave crests pile up on each other. The airplane must exert a large force to compress the air in front of it; by Newton's third law, the air exerts an equally large force back on the airplane. Hence there is a large increase in aerodynamic drag (air resistance) as the airplane approaches the speed of sound, a phenomenon known as the "sound barrier."

When v_s is greater in magnitude than v the source of sound is **supersonic, and** eqs. for the Doppler effect no longer describe the sound wave in front of the source.

(b) Sound source moving faster than the speed of sound



As the airplane moves, it displaces the surrounding air and produces sound. A series of wave crests is emitted from the nose of the airplane; each spreads out in a circle centered at the position of the airplane when it emitted the crest.

$$\sin\alpha = \frac{vt}{v_S t} = \frac{v}{v_S}$$

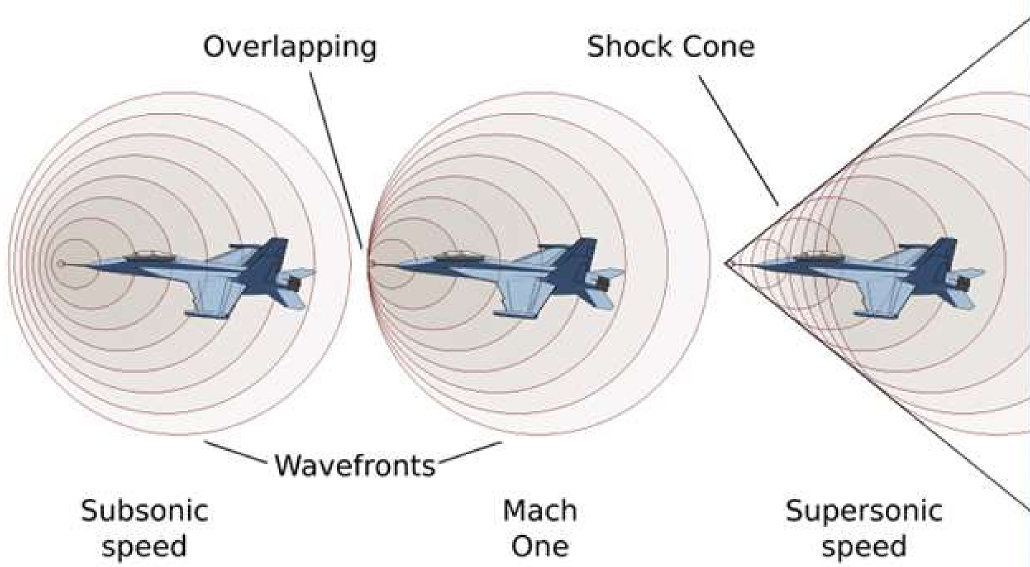
After a time *t the* crest emitted from point S1 has spread to a circle with radius v_t and the airplane
has moved a greater distance $v_s t$ to position S2.
The circular crests interfere constructively at points along the blue line that makes an angle α with the direction of the airplane velocity,
=> very-large-amplitude wave crest
called a **shock wave.**

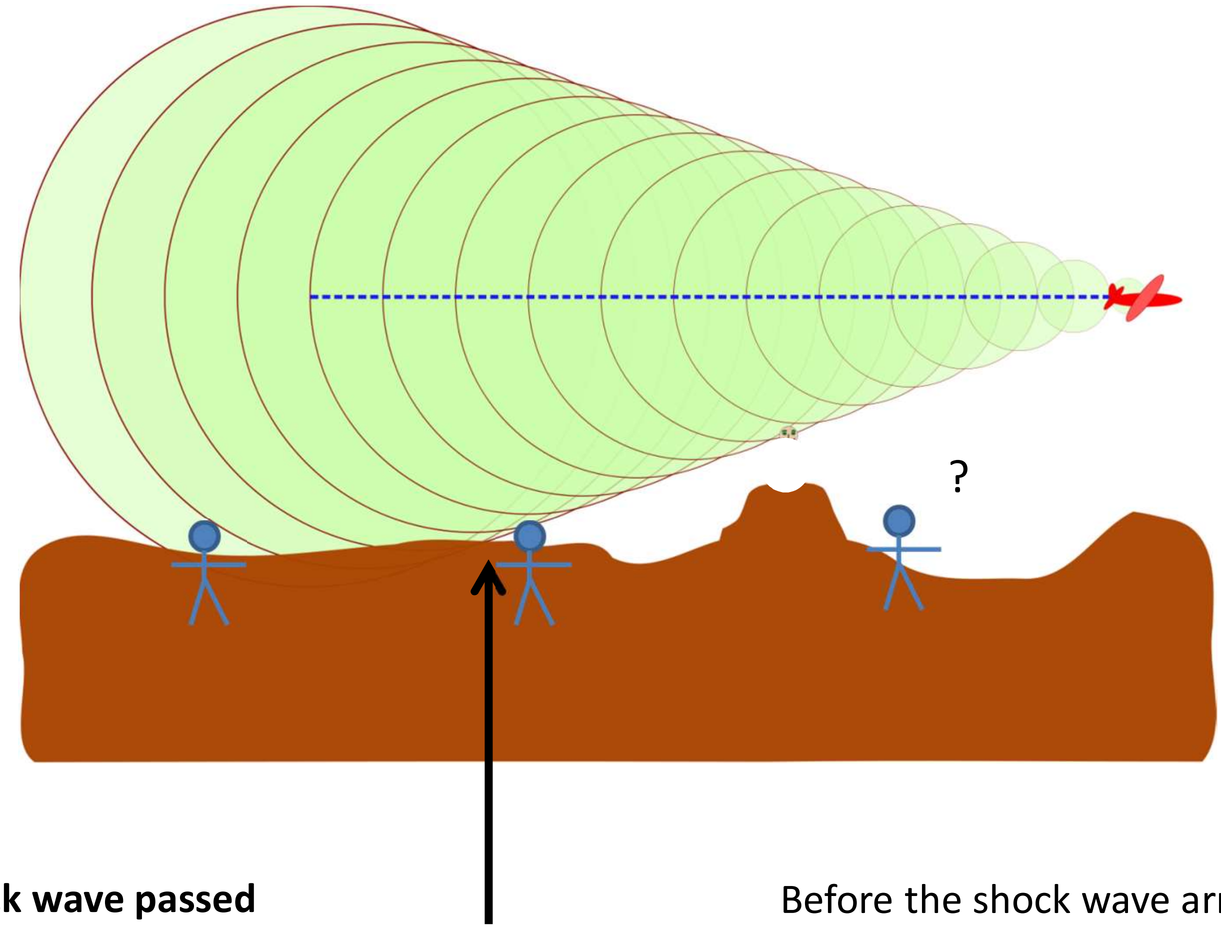
The ratio **$v_s/v = 1/\sin\alpha$** is called the **Mach number.**
>1 for supersonic velocities

The first person to break the sound barrier was Capt. Chuck Yeager of the U.S. Air Force, flying the Bell X-1 at Mach 1.06 on October 14, 1947.

Sonic boom

Shock waves are actually three-dimensional; a shock wave forms a cone around the direction of motion of the source. If the source (possibly a supersonic jet airplane or a rifle bullet) moves with constant velocity, the angle α is constant, and the shock-wave cone moves along with the source. It's the arrival of this shock wave that causes the **sonic boom** you hear after a supersonic airplane has passed by. The larger the airplane, the stronger the sonic boom; the shock wave produced at ground level by the (now retired) Concorde supersonic airliner flying at 12,000 m (40,000 ft) caused a sudden jump in air pressure of about 20 Pa. In front of the shock-wave cone, there is no sound. Inside the cone a stationary listener hears the Doppler-shifted sound of the airplane moving away.





After the shock wave passed
Hearing Doppler shifted sound

Before the shock wave arrived
No sound =>Nothing hearing

Boom