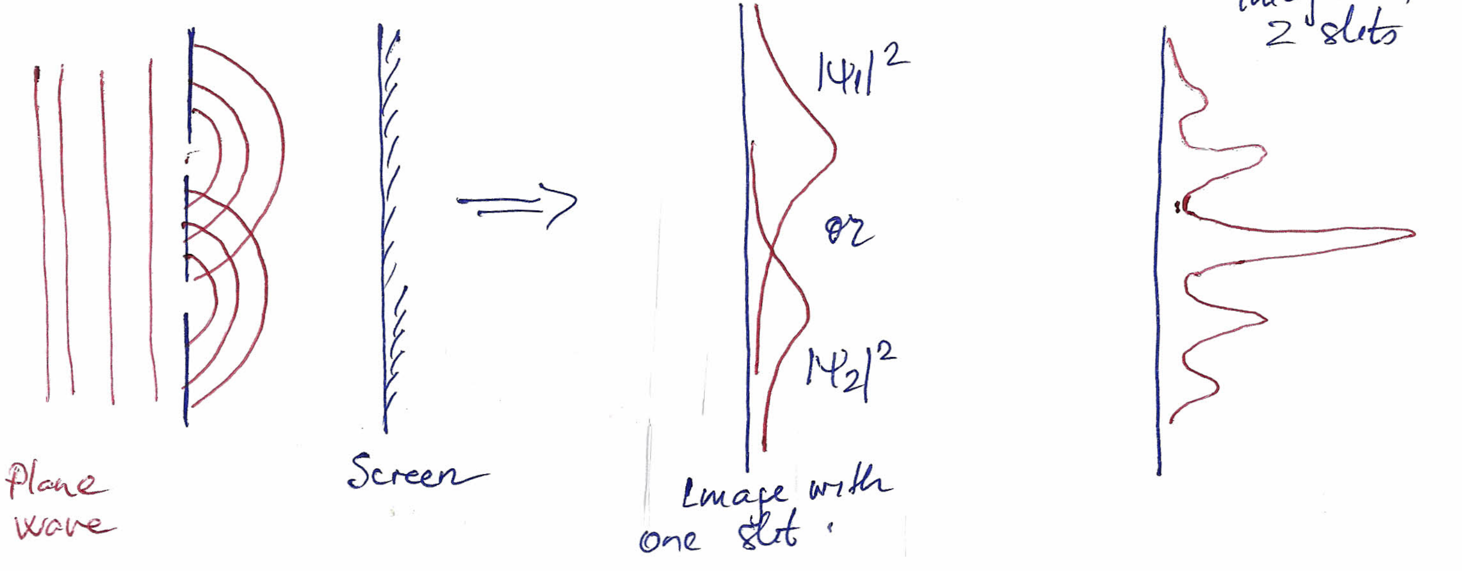
WAVE-PARTICLE DUALITY

① Photons : waves or particles ?

Young slits and failure of classical analysis

The concept of energy quanta introduced by Einstein to explain the photoelectric effect drove the scientists in a big puzzle. So far, following the theory of electromagnetic field light has been considered as a non-localized electromagnetic wave. This has been also supported by all the experiments of optical interference. But, how to understand the interference phenomena if one admits now that light is composed by particles named photons with energy $h\nu$?

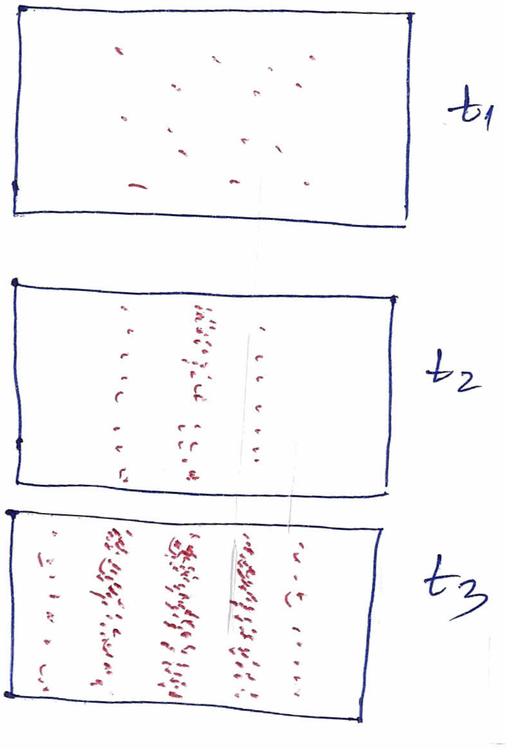
To get an answer to this question, recall the experiment of Young from optics. A monochromatic electromagnetic radiation is sent over a slide containing two pierced small slits (size analogous to the wavelength of radiation) and we analyze the image on a screen. The diffracted waves on each slit interfere constructivelly and destructively => interference fringes on the screen.



If one of the two slots is closed, we observe the diffraction image of one slot. When the two slots are open, we observe the interference figure (fringes) related to constructive interference of waves. (remember the principle of Fermat for mechanical waves).

The Young experiment demonstrates the wave nature of light.

Let's analyze now the problem considering that the light is made of particles (photons). The incident radiation is therefore constituted by a large number of particles that can pass through one slit or through the other. Each photon should have its own path and we expect on the screen to detect a lot of photons on maxima and few or no photon on minima. Is this correct? Let's perform the experiment, using a low flux of photons (so that one can count the impact of the photons on detector). Indeed, we can observe the impact of photons on the screen which confirms their particle aspect. Moreover, we observe an amazing fact: the interference pattern does not appear immediatelly but gradually with continous increasing the number of impacts.

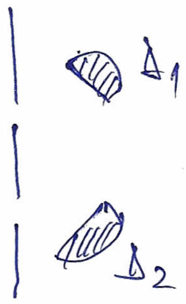


The impact patterns on the screen validates the particle aspect of photons, whereas the interference figure pattern confirms the wave nature of photons (light).

We underline also the aspect impossible to appear when considering only the wave aspect: the analyzed phenomenon has a probabilistic aspect. This experiment does not depend on the flux of photons. If we send the photons one by one, we will still obtain the interference patterns.

The influence of the measurement

The above aspects leads to a problem without solution in classical physics: when thinking in terms of wave, we can understand that the wave can pass through the two slits simultaneously. However, what happens when thinking in terms of particles? Can the photon pass through the two slits simultanously? To answer to this puzzling question we have to put detectors after the slits to see if the can detect simultaneously the photon. Experimentally, the detectors never detect simultanously!



This proves the particle nature of photons!

However, a strange phenomena occurs: if we detect exactly the slit where the photon passed through the interference figure disapears!

It means that by detecting (measuring) the photon, wanting to know exactly through which slit he passed, we loose the interference patten!. This idea is complettely astonishing, even Einstein did not believed on it and unsuccesfully imagined series of more sophisticated experiment to refute such a result!

Till now, all the other imagined experiments led to similar results: when we detect (with certitude) the photon and know through which slit he passed, the interference pattern disappears!!

We will see later that in quantum physics, the experiment is altering the state!

② Particles behaving as waves

Introduction: Elements of relativity

We aim to establish a correlation between what we know about matter (particles having a certain momentum and energy) and what we discovered about photons as waves (wavevector k and energy $E=h\nu=\hbar\omega$).

Within the classical Newton theory, this will be impossible: the energy of a photon as a free particle with zero mass would be zero ($E_c=\frac{1}{2}mv^2$). On the other hand, the classical mechanics is no longer valid for particles whose velocity approaches the speed of light.

We need to get through another theory which is the RELATIVITY theory, developed by Albert Einstein. This will bring corrections to the classical mechanics.

The theory of relativity is based on two main principles (postulates).

① The laws of physics are the same in every inertial frame of refference.

② The speed of light in vacuum is the same in all inertial frames of refferance and it is independent on the motion of the source.

The second postulat implicates that ~~it~~ it is impossible for an inertial observer to travel at c, the speed of light in vacuum => speed of light = ultimate speed limit.

An important aspect in relativity is that the time is not the same when changing the reference system.

$$\Delta t = \frac{\Delta t_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

time measured in a rest frame

time measured in a moving frame with v

The twin paradox: Consider two twin astronauts Eartha and Astrid. Eartha remains on Earth and Astrid takes off on a high-speed trip trough the galaxy. Because of time dilation, Eartha observes Astrid's hearts-beats and all life processes proceeding more slowly than on her own. Thus, to Eartha Astrid ages more slowly: when Astrid comes back to Earth she will younger (aged less) then Eartha.

Now, here is the paradox: all inertial frames are equivalent. Astrid can make same analysis and conclude that Eartha is younger.

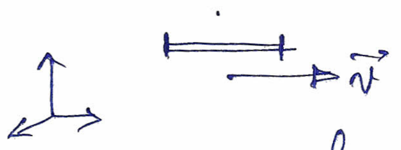
To resolve this paradox, we have to observe that the twins are not identical in all aspects. While Eartha remains in an approximatelly inertial frames all time, Astrid must accelerate with respect to that inertial frame when leaving on approaching Earth. This difference lead to a global difference in age and, finally, Astrid will be younger than Eartha.

Relativity of length:

$$\boxed{l = l_0\sqrt{1-\frac{v^2}{c^2}}}$$

length contraction

(moving parallel to length direction)



l_0 = length in fixed iRS

l = length in moving IRS

Ob: Length measured perpendicularly to the direction of motion remain uncorrected.

Quadri-vectors (space-time) Four-vector

(x, y, z, ct)

(time is translated in length by multiplying with c for homogeneity reasons.)

Considering the ox axis along the direction of the relative velocity between two IRS one can write the Lorentz transformations that connect the physical quantities from one IRS to the other

$$\begin{pmatrix} x' \\ y' \\ z' \\ ct' \end{pmatrix} = \begin{bmatrix} \gamma & 0 & 0 & -\gamma\beta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\gamma\beta & 0 & 0 & \gamma \end{bmatrix} \begin{pmatrix} x \\ y \\ z \\ ct \end{pmatrix}$$

with $\beta = \frac{v}{c}$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}}$$

ob: $\underbrace{x^2+y^2+z^2-c^2t^2 = x'^2+y'^2+z'^2-c'^2t'^2}_{\text{does not depend on iRS}}$ = the quadrivector norm.

Relativistic dynamics

Relativistic momentum

$\vec{p} = m\vec{v}$ with $m = \dfrac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$

m_0 = rest mass

Newton 2nd law

$$\vec{F} = \frac{d\vec{p}}{dt} = \frac{d}{dt}\,\frac{m_0\vec{v}}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$F = \frac{m_0}{\left(1-\frac{v^2}{c^2}\right)^{3/2}}\,a$$

($\vec{F}$ and $\vec{v}$ along the same line)

a = acceleration along the x axis

$$\Rightarrow \underline{a = \frac{F}{m_0}\left(1-\frac{v^2}{c^2}\right)^{3/2}}$$

⇒ constant force does not cause constant acceleration

Consequence when $v \to c \Rightarrow f \to 0$, $a \to 0$

⇒ impossible to accelerate a particle with $m_0 \neq 0$ at $v = c$.

Relativistic energy

$$\boxed{E = mc^2}$$

$$\boxed{E^2 = p^2c^2 + m_0^2c^4}$$

total energy, momentum and rest energy

when $v \ll c$ ⇒

$$E(v \ll c) = m_0c^2 + \frac{1}{2}m_0v^2$$

rest energy

kinetic energy

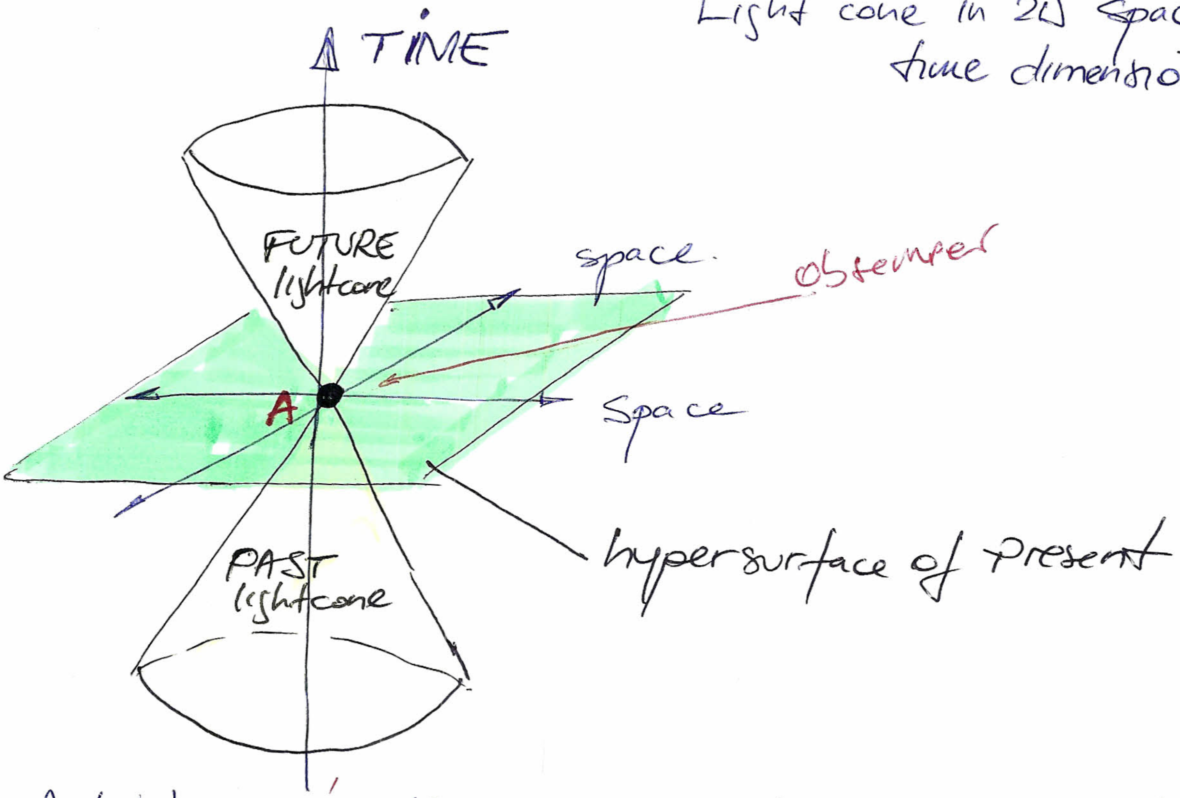
(obtained by developing in Taylor series)

Obs: The rest energy m_0c^2 (0,511 MeV for e^-
940 MeV for p, n) is disregarded in classical mechanics.

General theory of relativity

=> relativity theory extended to non-inertial reference frames (moving acceleratelly). The gravity affects the properties of the space (geometric properties). Within the general theory of relativity the effect of acceleration (gravity) is included in the metric of the space => curved metrics

Light cone in 2D space + time dimension



A light cone is the conic consisting of all events that can be connected to A via light rays. Events falling in the past light cone can reach or exert causal influence to A. Events in the future lightcone can be causally influenced by A. Events outside of A's lightcone cann'ot be causally linked to A, because in order to do so, information has to be transmitted faster than c. When we look to Universe we are actually seeing just our past light cone in all of spacetime!

The hypothesis of de BROGLIE

(pronounced "de Broy")

In 1924 a French scientist, Louis de Broglie, made a remarkable proposal about the nature of the matter. He did that during his Phd thesis and J. Perrin and P. Langevin, as members of the thesis defense jury, have been really puzzled about this challenging idea. De Broglie said: that the nature loves symmetry. If the light, considered as a wave has been finally demonstrated to have a dual behavior with photons as energy quanta, why not the electrons, considered as material elementary particles, would not behave as waves? Puzzled about this chalanging idea, Perrin and Langevin wrot to Einstein for advice and the geniust of Einstein admited that the hypothesis of de Broglie represents a major descovery and a major brick of a new science edifice represented by a new science branch that was quantum mechanics.

If a particle behave as wave, it should have a wavelenth and a frequency. De Broglie propose that:

$$\boxed{\lambda = \frac{h}{p} = \frac{h}{mv}}$$

$h = 6{,}625 \cdot 10^{-34}$ J·s
Planck's constant.

p = particle's momentum

If v is fraction of c

$$mv = \frac{m_0 v}{\sqrt{1 - \frac{v^2}{c^2}}}$$

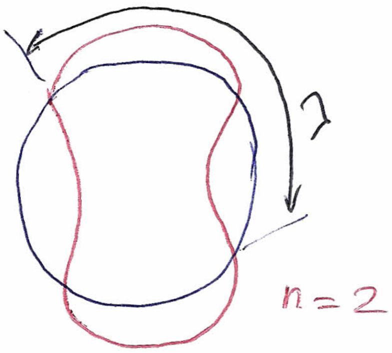
The frequency ν, according to de Broglie, is also related to the particle's energy E in the same way as for a photon:

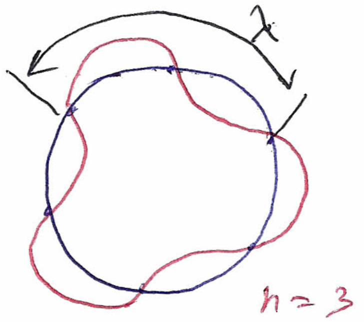
$$\boxed{E = h\nu = \hbar\omega}$$

Bohr's model justified

The approach of de Broglie allows justifying the hypothesis of Bohr concerning the quantification of the angular momentum. De Broglie proposes that the wavelength of an electron moving along a circular orbit around the nucleus has to fit the lenght of the orbit so that to form stationary waves:

$$\Rightarrow 2\pi r_n = n\lambda_n \qquad n = 1, 2, 3\ldots$$





The ideas come from the mechanisms of stationary electromagnetic waves in cavities

but $\lambda_n = \dfrac{h}{m v_n}$

$$\Rightarrow m v_n r_n = n\frac{h}{2\pi} = n\hbar \Rightarrow \boxed{L_n = n\hbar}$$

which is exactly the Bohr's quantification of orbital momentum.

This hypothesis is also in agreement with the particle behavior of the photon proposed by Einstein. If we write the equation:

$$E^2 = p^2c^2 + m_0^2c^4$$ for photon $m_0 = 0$ $\Rightarrow$

$$E = pc = \frac{h}{\lambda}c = h\nu$$

$\Rightarrow$ the hypothesis of de Broglie gives an unified framework of the wave-particle duality.

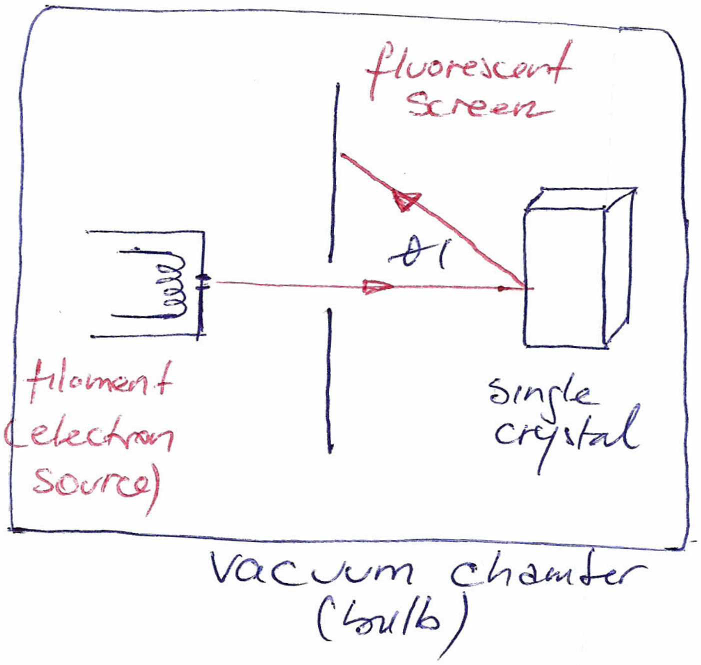
So, the particles can diffract?

To check if de Broglie was right one should do an experiment with Young slits but with particles. Unfortunatelly, a problem occurs; in order to have diffraction, the size of the slits has to be similar to λ. For light, this is technically easy to do because $\lambda \sim \mu m$. But what happens in case of electrons? They have small mass but large velocity. A thermally emitted electron from a tungsten filament heated at 2500 K has an energy: $\frac{1}{2}K_BT$ per degree of freedom:

$$\Rightarrow \frac{1}{2}mv^2 = \frac{3}{2}K_BT \quad \Rightarrow v = \sqrt{\frac{3K_BT}{m}}$$

$$\Rightarrow \lambda \triangleq \frac{h}{mv} = \frac{h}{\sqrt{3mK_BT}} \simeq \underline{\underline{2nm}} \ !!$$

On early 20th century wass impossible to build a Young experiment with so small slits. However people knew that the distance between the atoms in the matter was about 0,2-0,4 nm. Then, the beam of particles should be sent against an order structure of atoms (e.g. a single crystal) to be able to produce diffraction = reflexion + interference. Each atom plays the role of a slit in the Young's experiment. This experiment has been done in 1927 by Davisson and Germer (Bell Laboratories).



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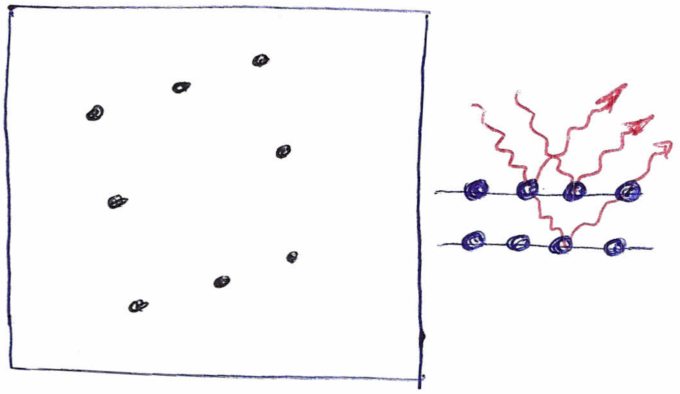


Obtained image (for a cubic monocrystal)

Obs: the fluorescent screen can be (alternatively) a detector which can be moved to detect scattered electrons at any angle θ

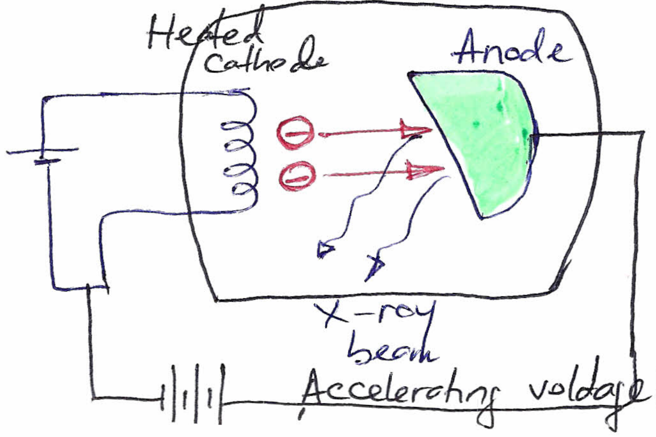
-> the above experiment is also the principle of modern LEED (low energy electron diffraction)

The Davisson-Germer experiment demonstrates the wave behaviour of electrons.

Later, diffraction experiments have been done with other particles with non-zero mass:

- neutrons produced in nuclear reactors
- ultra-cold atoms

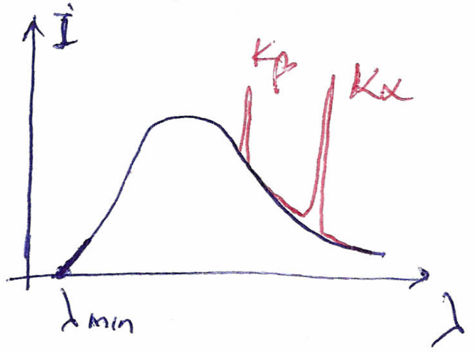
This diffraction experiments on crystals can be done with radiation with small λ (< nm). This is the case of X Rays descovered by Röntgen, produced in a



Röntgen tube:

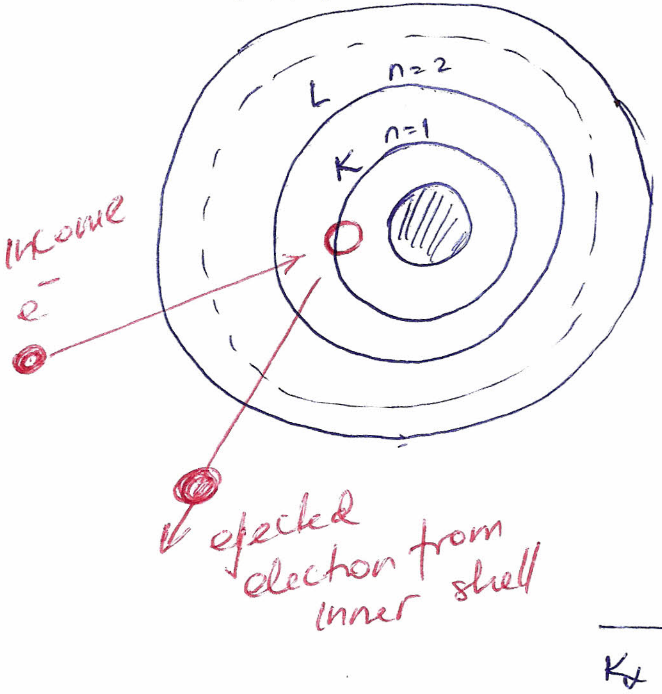
Electrons are emitted thermoionically from the heated cathode and accelerated towards the anode; when they strike it they lead to X ray production.

⇒ continuous spectrum of X Rays produced when the anode (anti-cathode) is strucked by accelerated electrons. The radiation is emitted due to the braking of electrons (cf. Maxwell's theory).

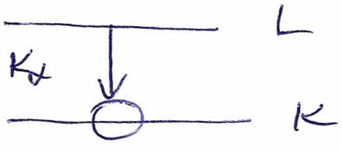


$$eV_{AC} = hf_{max} = \frac{hc}{\lambda_{min}}$$

Over the continuous braking radiation spectrum there are characteristic pics at certain λ (Kα, Kβ..) related to emission by electronic transition towards inner shells:



An incoming energetical electron can eject an inner shell electron of the anode's atom: The empty place will be subsequently filled by an electron from an outer shell with emission of corresponding characteristic frequency:



$$\boxed{\frac{hc}{\lambda_{K_\alpha}} = E_L - E_K}$$

Other examples of experiments with particles behaving as waves

① De Broglie waves and macroscopic world

The principle reason we do not see the wave effect of massic particles at human scale is the small value of the Plank constant h. As a result the corresponding λ is small and wave effects neglectible.

ex.: wave length of a falling grain of sand

$m = 5 \cdot 10^{-10}$ Kg falling with $v \simeq 0{,}4$ m/s, and diameter $d = 0{,}07$ mm $\simeq 7 \cdot 10^{-5}$ m

$$\lambda = \frac{h}{mv} = \underline{3 \cdot 10^{-24} \, m}$$

for smaller than the size of an atom ($\simeq 10^{-10}$ m)

A more massive object will have even smaller λ.
$\Rightarrow$ negligible effects of wave behavior

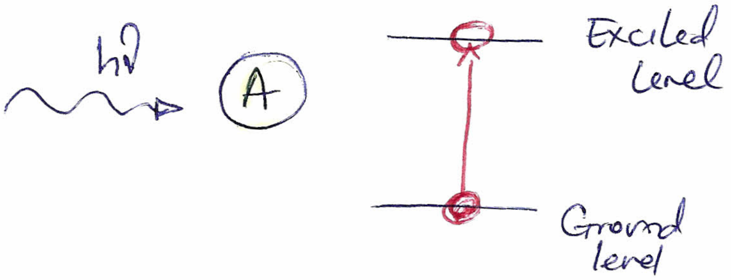
② The electron microscope

The resolution of an optical microscope is limited by λ. Here, $\lambda = h/p$ can be done extremelly small by accelerating e^- at large $v \Rightarrow$ large p.
$\Rightarrow$ magnifications thousand of times larger than the ones of optical microscopes $\Rightarrow$ acces to the NANO WORLD.

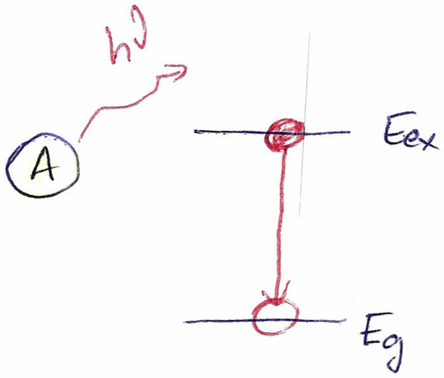
③ From LASERS with photons to LASERs with electrons

LASER = light Amplification by Stimulated Emission of Radiation

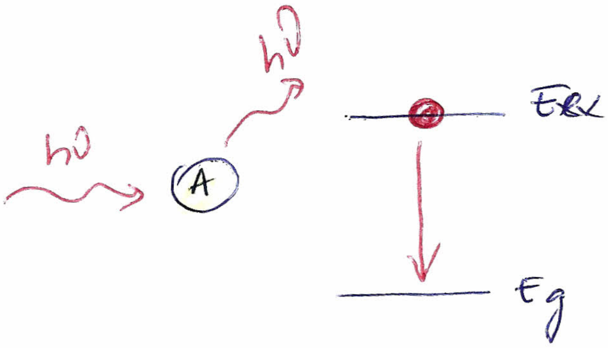
Three processes in which atoms interact with light



Absorbtion



Spontaneous emission



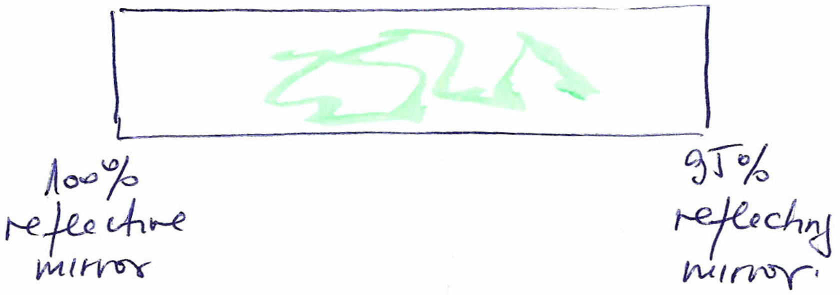
Stimulated emission

=> light amplification

Step towards LASER realization

1) Enhancing stimulated emission => population inversion (Nobl: A. Kastler) / optical pumping.

2) Active medium in rezonant cavity (Fabry-Perrot)
=> monochromatic radiation (reduces the with of wavelength dispersion mainly caused by Doppler effect)



③ The Uncertainty principle

In classical mechanics we think of a particle as a point. We can describe its location and state of motion precisely at any instant of time t with 3 spatial coordinates (x,y,z) and three components of velocity (v_x, v_y, v_z).

But because matter has also wave aspect and when we look at a small enough scale - comparable to the de Broglie wavelength of the particle - we cannot use anymore the Newtonian description

Niels Bohr in 1928 enounced a PRINCIPLE OF COMPLEMENTARITY that claims that wave and particle description are complimentary. We never use both simultanously to describe a phenomenon as a single part of an occurance. Wave and particle models cannot apply simultaneously to describe a single element of an experiment. This principle applies when analyzing the diffraction experiment with particles. We cannot predict exactly where in the pattern (a wave phenomenon) any individual electron (a particle) will land. We cannot ask which slit an individual electron passed through. When trying to look at it, the interference pattern dissapears!

For quantum particles (or photons) there are fondamental uncertainities inseparably related.

=> Heisenberg uncertainty principles

position ⟷ momentum energy ⟷ time

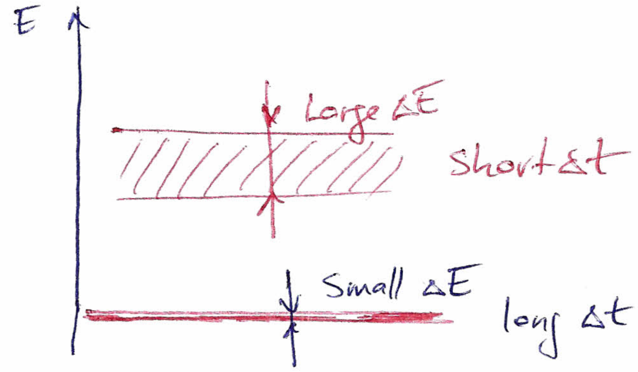
$$\left\{ \begin{array}{l} \left. \begin{array}{l} \Delta x \, \Delta p_x \geq \hbar/2 \\ \Delta y \, \Delta p_y \geq \hbar/2 \\ \Delta z \, \Delta p_z \geq \hbar/2 \end{array} \right] \\ \left. \Delta t \, \Delta E \geq \hbar/2 \right] \end{array} \right.$$

Heinsenberg uncertainity principle for position and momentum

Heisenberg uncertainity principle for time interval and energy.

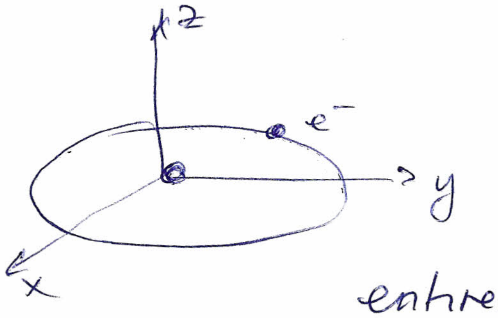
Obs : The uncertainity of a quantity is described in terms of the statistical concept of standard deviation which is a measure of the spread of dispersion of a set of numbers around an average value.

Uncertainty principle and limits of the Bohr's model



The longer the lifetime Δt of a state, the smaller its spread in energy

The Heisenberg uncertainity principle does not allow the electron to move precisely in a certain plane (e.g. xy plane) with $z=0$ with zero uncertainities in Δz and δp_z.



To get a correct insight of electrons moving inside atom or elswere we need a description entirelly based on electron's wave properties.

$\Longrightarrow$ goal of QUANTUM MECHANICS (ondulatory approach)