Maxwell's equations

integral

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{encl}}}{\epsilon_0} \qquad \text{(Gauss's law)}$$

$$\oint \vec{B} \cdot d\vec{A} = 0 \qquad \text{(Gauss's law for magnetism)}$$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \left(i_C + \epsilon_0 \frac{d\Phi_E}{dt} \right)_{\text{encl}} \qquad \text{(Ampere's law)}$$

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt} \qquad \text{(Faraday's law)}$$

differential

$$\nabla \bullet \vec{E} = div(\vec{E}) = \frac{\rho_{encl}}{\varepsilon_0}$$

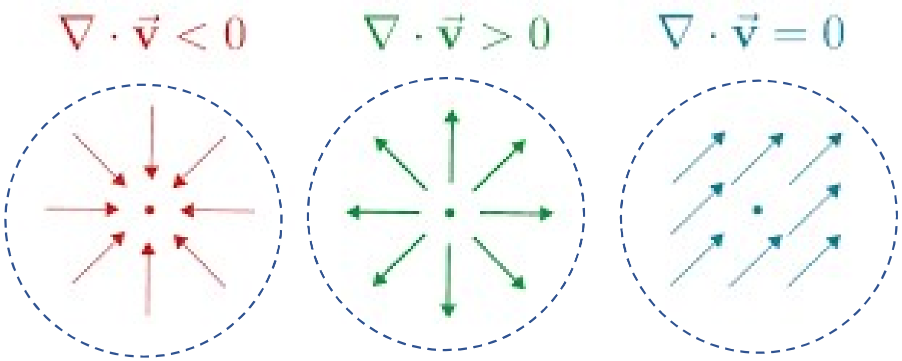
$$\nabla \bullet \vec{B} = 0$$

$$\nabla \times \vec{E} = curl(\vec{E}) = -\frac{d\vec{B}}{dt}$$

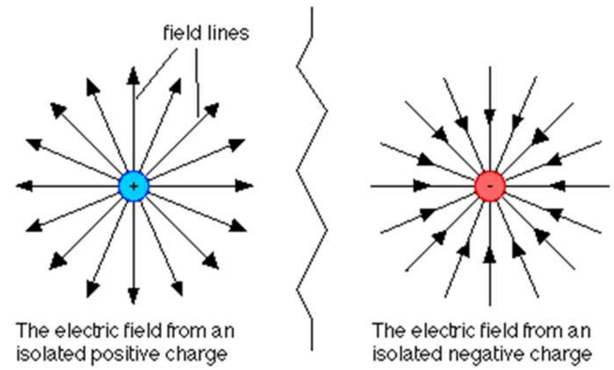
$$\nabla \times \vec{B} = curl(\vec{B}) = \mu_0 \left(\vec{j} + \varepsilon_0 \frac{d\vec{E}}{dt} \right)_{encl}$$

Divergence

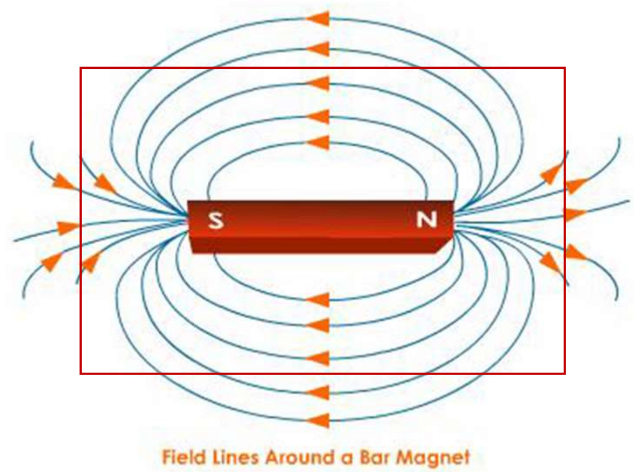
a measure of the escaping flow



Imaginary surface



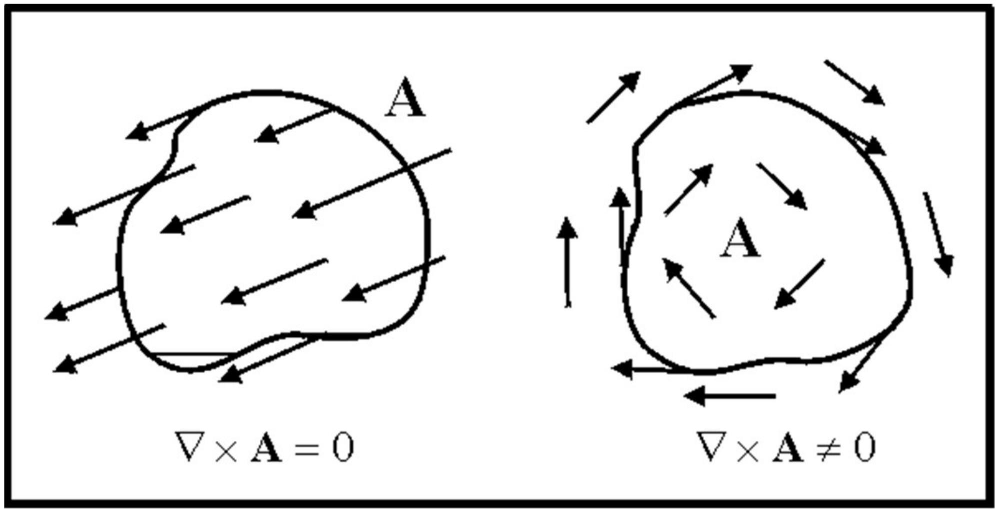
$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$$



$$\nabla \cdot \vec{B} = 0$$

Curl

A curl is a vector or a measure of the rotation of a 3-dimensional vector field in any point on that field.



$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

- the electric field curls because of a change in a nearby magnetic field meaning it flows in a circuit
- the negative sign indicates that the flow will happen in a direction that'll cause another magnetic field opposing to the first one(Lenz): the change causes a reaction that tries to oppose the initial change

A field that has a **curl** is a **non-conservative** field:

$$\varepsilon = \oint_{closed} \vec{E} \cdot \vec{dl} = -\frac{d\Phi_B}{dt}$$

Maxwell's equations

In vacuum

no charge: (ρ=0)

no currents: (**j**=0)

integral

$$\oint \vec{E}\cdot\overrightarrow{dA}=0$$

$$\oint \vec{E}\cdot\overrightarrow{dl}=-\frac{d\phi_B}{dt}$$

$$\oint \vec{B}\cdot\overrightarrow{dA}=0$$

$$\oint \vec{B}\cdot\overrightarrow{dl}=\mu_0\varepsilon_0\frac{d\phi_E}{dt}$$

differential

$$\nabla\bullet\vec{E}=div(\vec{E})=0$$

$$\nabla\bullet\vec{B}=0$$

$$\nabla\times\vec{E}=curl(\vec{E})=-\frac{d\vec{B}}{dt}$$

$$\nabla\times\vec{B}=curl(\vec{B})=\mu_0\varepsilon_0\frac{d\vec{E}}{dt}$$

A variable electric flux =source of magnetic field

A variable magnetic flux =source of electric field

➡ Time varying **E** produces **B** and vice versa

$$\vec{E}(t)\to\vec{B}(t)\to\vec{E}(t)\to\vec{B}(t)\ldots$$

= perturbation **(ELECTROMAGNETIC WAVE)** that propagates in vacuum with the velocity:

$$v=\frac{1}{\sqrt{\mu_0\varepsilon_0}}=\frac{1}{\sqrt{4\pi\times10^{-7}\,\frac{H}{m}\times8{,}854\times10^{-12}\,\frac{F}{m}}}=299\,792\,458\,\tfrac{m}{s}$$

That is the speed of light

=> Light is an electromagnetic wave

Electromagnetic Waves: Solutions to Maxwell's Equations

$$\vec{\nabla} = \frac{\partial}{\partial x}\vec{i} + \frac{\partial}{\partial y}\vec{j} + \frac{\partial}{\partial z}\vec{k}$$

$$\nabla \bullet \vec{E} = div(\vec{E}) = 0$$

$$\nabla \bullet \vec{B} = 0$$

$$\nabla \times \vec{E} = curl(\vec{E}) = -\frac{d\vec{B}}{dt}$$ Faraday

$$\nabla \times \vec{B} = curl(\vec{B}) = \mu_0 \varepsilon_0 \frac{d\vec{E}}{dt}$$ Ampere

Math: $$\vec{\nabla} \times \left(\vec{\nabla} \times \vec{A}\right) = \vec{\nabla}\left(\vec{\nabla} \cdot \vec{A}\right) - \vec{\nabla}^2 \vec{A}$$

$$\vec{\nabla}^2 A = \frac{\partial^2 A}{\partial x^2} + \frac{\partial^2 A}{\partial y^2} + \frac{\partial^2 A}{\partial z^2}$$

$$\vec{\nabla} \times \left(\vec{\nabla} \times \vec{E}\right) = \vec{\nabla}\left(\vec{\nabla} \cdot \vec{E}\right) - \nabla^2 \vec{E} = -\vec{\nabla}^2 \vec{E}$$

=0 Gauss

Faraday

Ampere

$$\vec{\nabla} \times \left(-\frac{\partial \vec{B}}{\partial t}\right) = -\frac{\partial}{\partial t}\left(\vec{\nabla} \times \vec{B}\right) = -\mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} = \nabla^2 \vec{E}$$

Resembles to wave equation

$$\frac{1}{v^2} \frac{\partial^2 \vec{E}}{\partial t^2} = \nabla^2 \vec{E}$$

If $v = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$

Similarly:

$$\vec{\nabla} \times \left(\vec{\nabla} \times \vec{B}\right) = \vec{\nabla}\left(\vec{\nabla} \cdot \vec{B}\right) - \nabla^2 \vec{B} = -\vec{\nabla}^2 \vec{B}$$

=0 Gauss

$$\mu_0 \varepsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2} = \nabla^2 \vec{B}$$

$$\frac{1}{v^2} \frac{\partial^2 \vec{B}}{\partial t^2} = \nabla^2 \vec{B}$$

Generating Electromagnetic Radiation

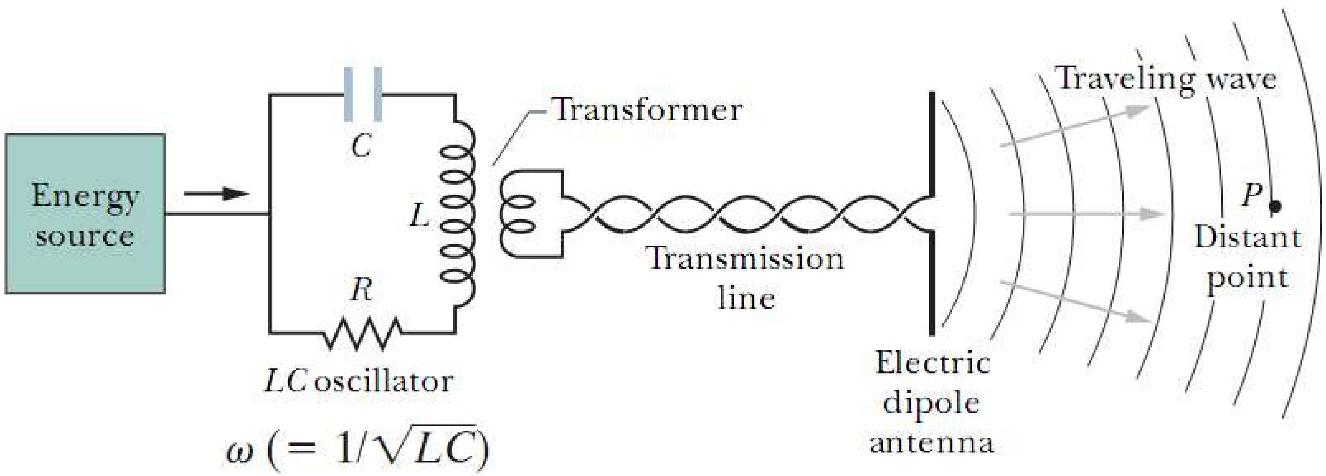
According to Maxwell's equations:

- a point charge at rest produces a static **E** field but no **B** field
- a point charge moving with a constant velocity produces both **E** and **B** and fields
- in order for a point charge to produce electromagnetic waves, the charge must *accelerate*

every accelerated charge radiates electromagnetic energy

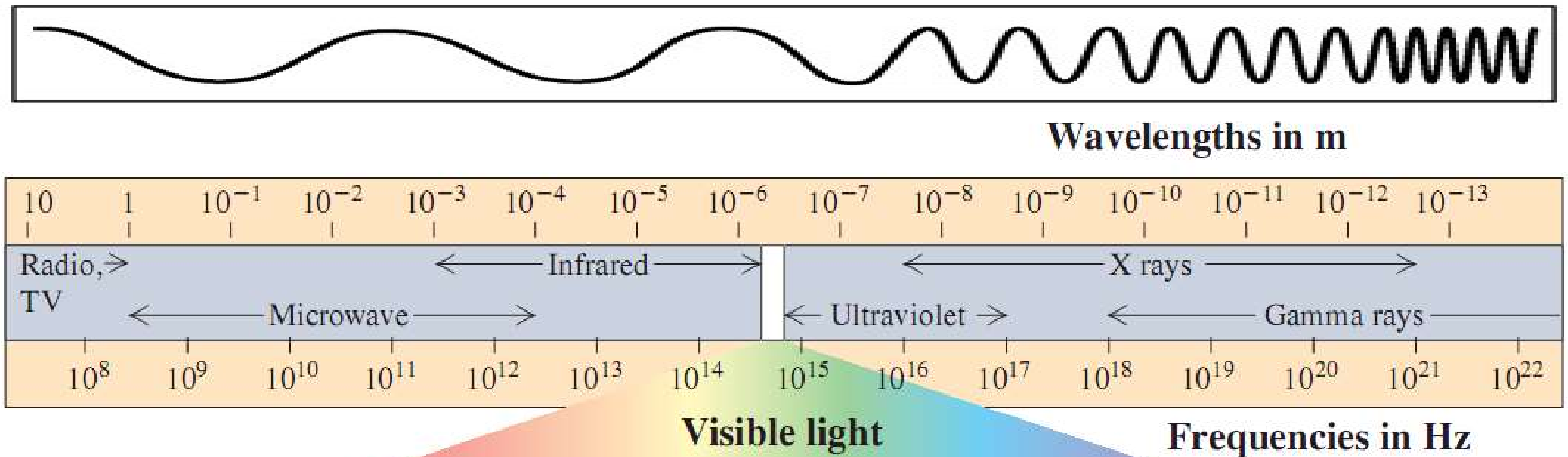
One way in which a point charge can be made to emit electromagnetic waves is by making it oscillate in simple harmonic motion

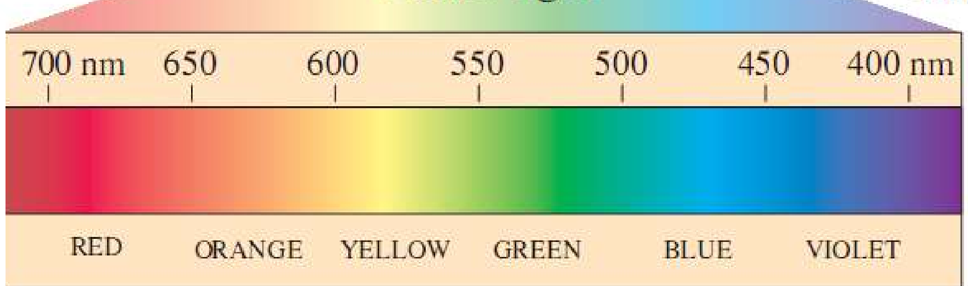
Emitter:



Receiver: **->** the antenna is also a conductor. The fields of wave emitted by the distant transmitter exert forces on the free charges within the receiver antenna producing an oscillating current that is detected and amplified by the receiver circuitry.
-> maximum power transfer at resonance

The electromagnetic spectrum





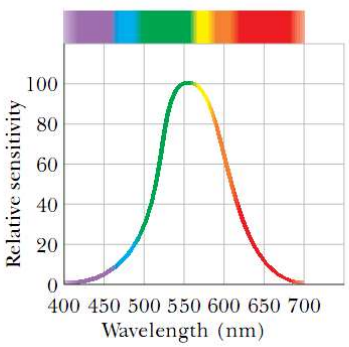
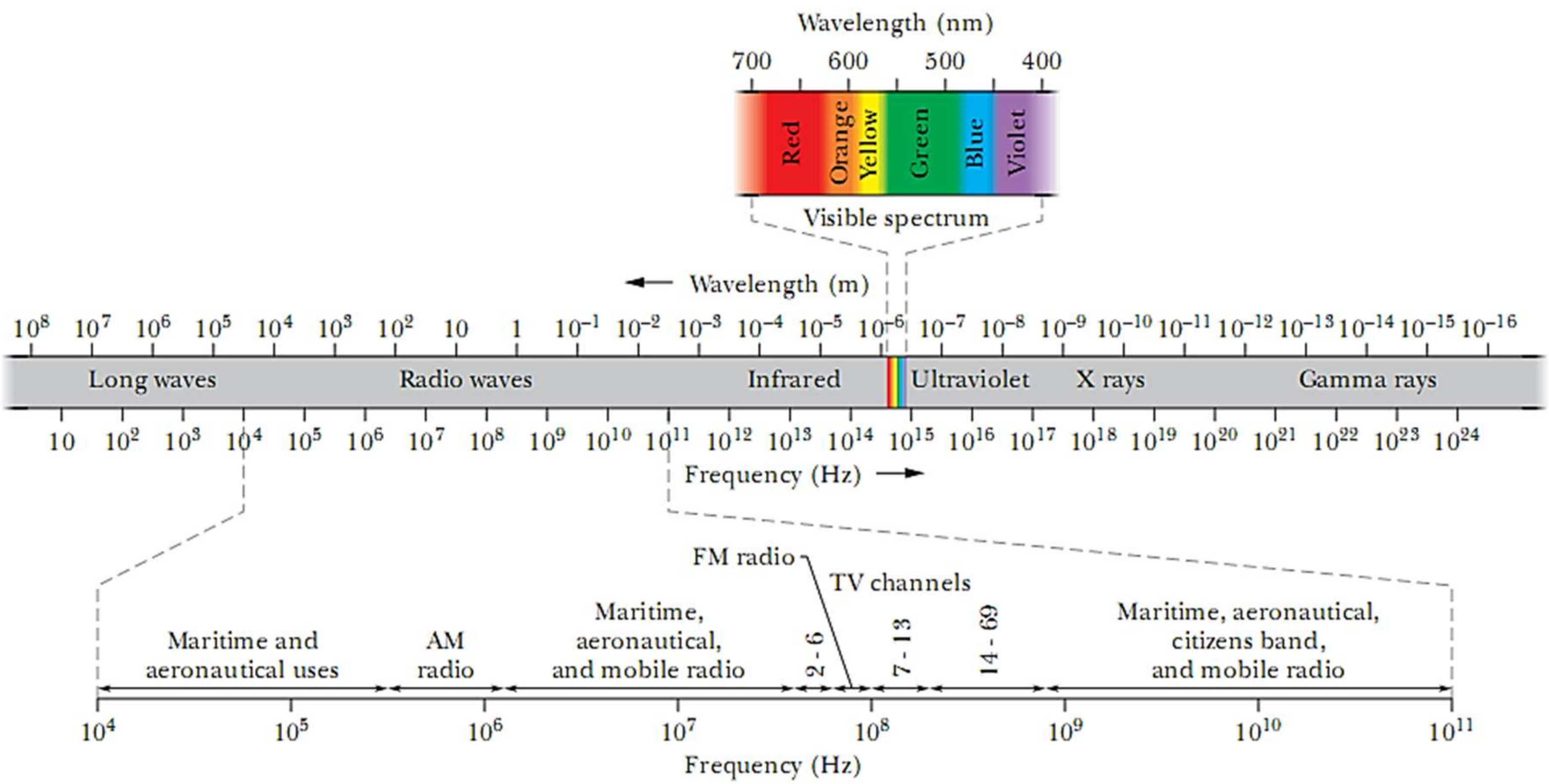


Fig. 33-2 The relative sensitivity of the average human eye to electromagnetic waves at different wavelengths. This portion of the electromagnetic spectrum to which the eye is sensitive is called *visible light*.

Table 32.1 Wavelengths of Visible Light

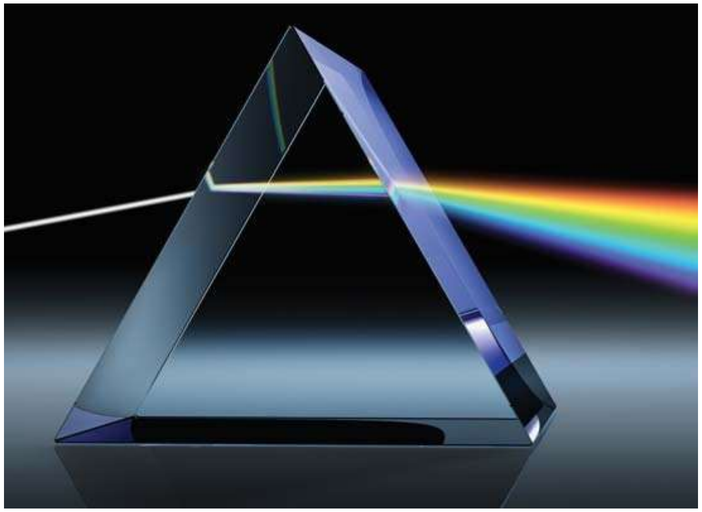
380–450 nm	Violet
450–495 nm	Blue
495–570 nm	Green
570–590 nm	Yellow
590–620 nm	Orange
620–750 nm	Red

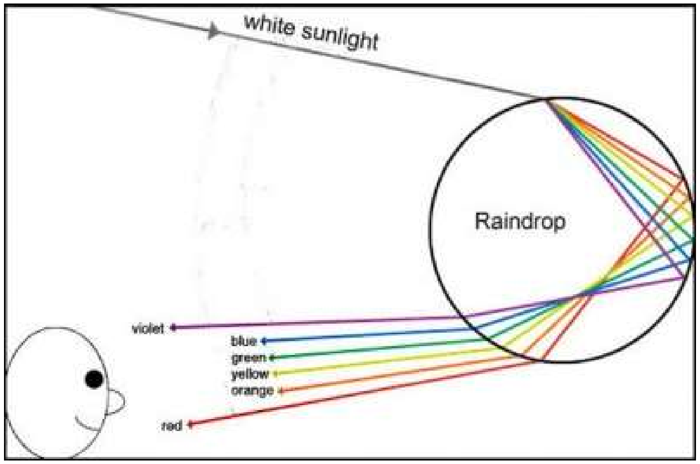


Wavelength (nm)
700
600
500
400
Red
Orange
Yellow
Green
Blue
Violet
Visible spectrum
Wavelength (m)
10^8 10^7 10^6 10^5 10^4 10^3 10^2 10 1 10^-1 10^-2 10^-3 10^-4 10^-5 10^-6 10^-7 10^-8 10^-9 10^-10 10^-11 10^-12 10^-13 10^-14 10^-15 10^-16
Long waves
Radio waves
Infrared
Ultraviolet
X rays
Gamma rays
10 10^2 10^3 10^4 10^5 10^6 10^7 10^8 10^9 10^10 10^11 10^12 10^13 10^14 10^15 10^16 10^17 10^18 10^19 10^20 10^21 10^22 10^23 10^24
Frequency (Hz)
FM radio
TV channels
Maritime and aeronautical uses
AM radio
Maritime, aeronautical, and mobile radio
2 - 6
7 - 13
14 - 69
Maritime, aeronautical, citizens band, and mobile radio
10^4 10^5 10^6 10^7 10^8 10^9 10^10 10^11
Frequency (Hz)

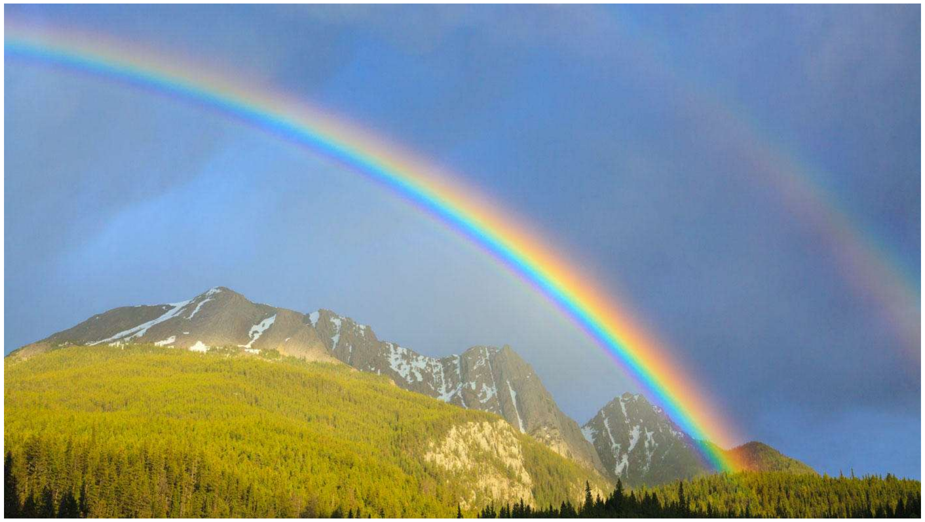
? White light

Mixture of all colors

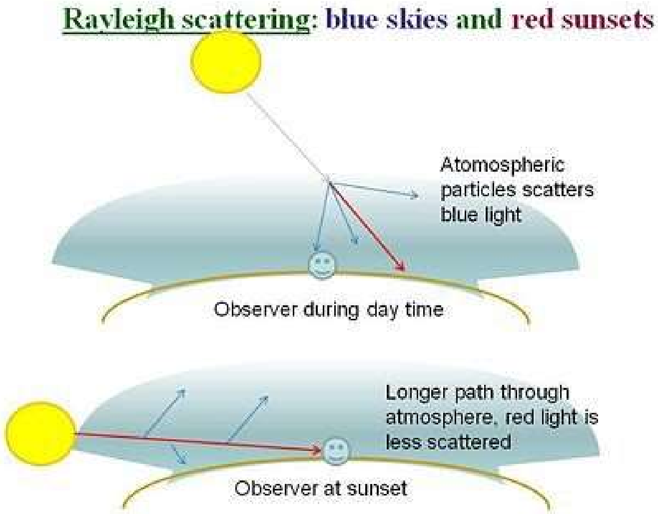


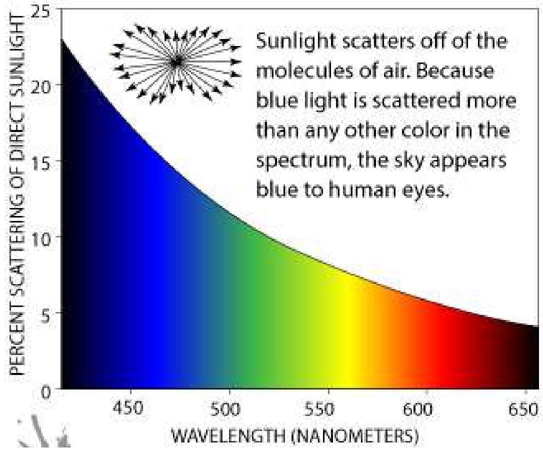


The rainbow



Rayleigh scattering

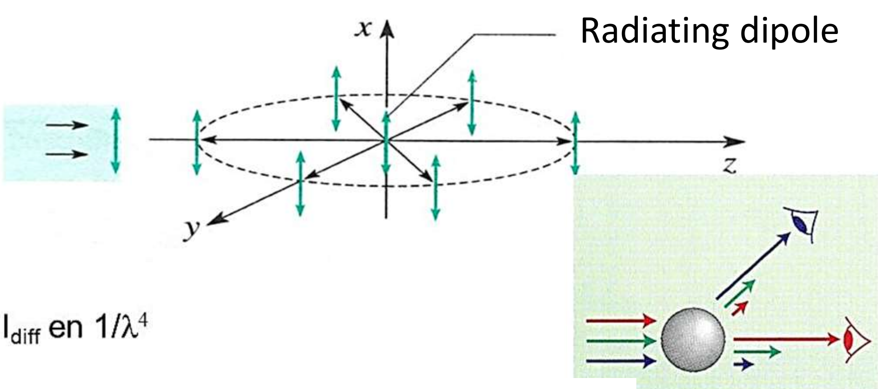


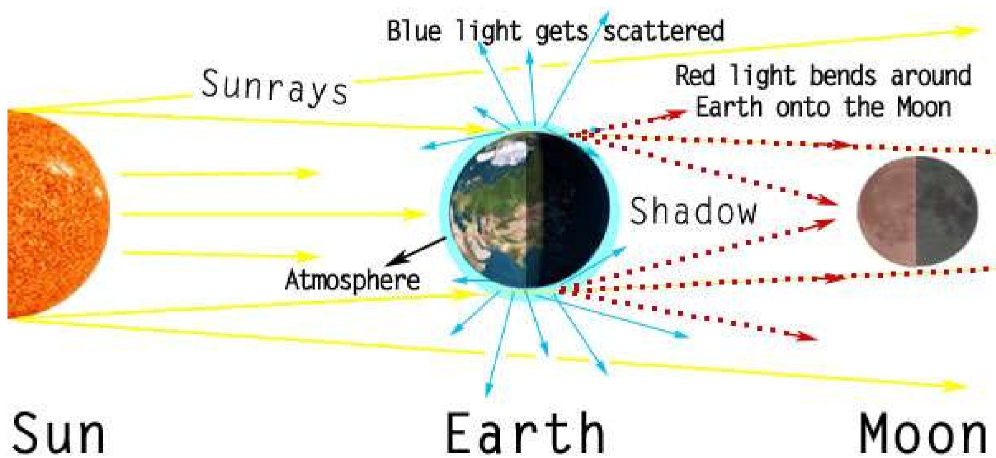


Red moon

- $kr << 1$

Radiating dipole





- I_{diff} en $1/\lambda^4$
- Scattered light linearly polarized

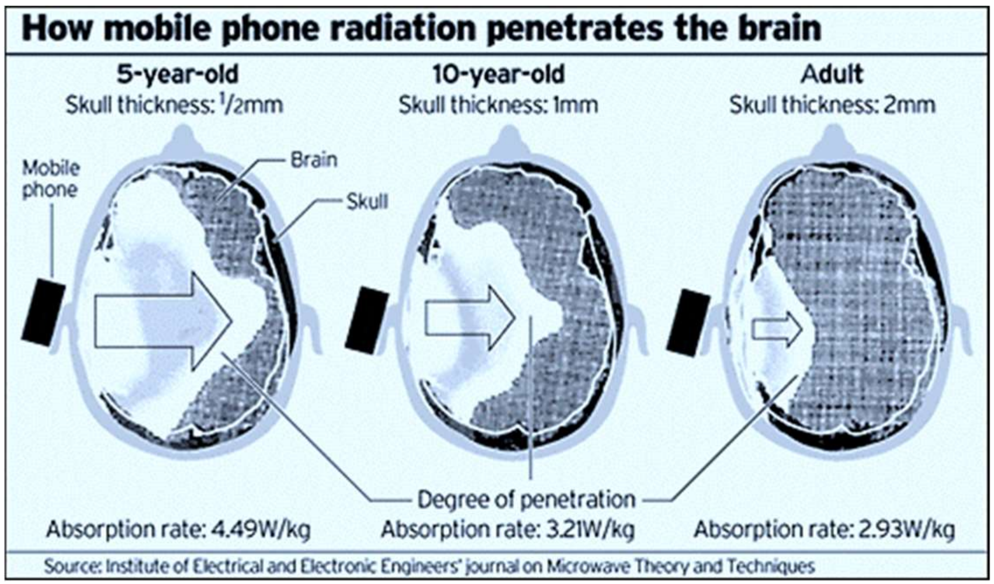
Mobile phone

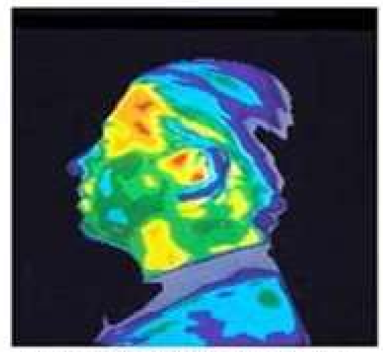
Cell phones use Electromagnetic radiation in the Microwave range around 2.5 GHz range

also WLAN channels: 802.11 workgroup (2.4 GHz, 3.6 GHz, 4.9 GHz, 5 GHz, and 5.9 GHz)

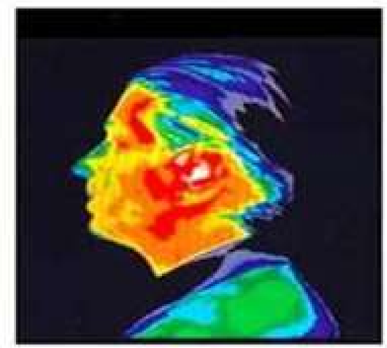
Consumer microwave ovens usually use 2.45GHz

However the intensity of a Wi-Fi signal is around is 100,000 times less than a microwave oven +1/r^2 attenuation law





Thermographic Image of the head with no exposure to harmful cell phone radiation.



Thermographic Image of the head after a 15-minute phone call. Yellow and red areas indicate thermal (heating) effects that can cause negative health effects.

Thermal/non-thermal effects: heating of brain tissues, resonant affecting brain waves responsible on mood and alertness, brain waves such as alpha beta ,delta waves will be affected when exposed to pulsed radiations.