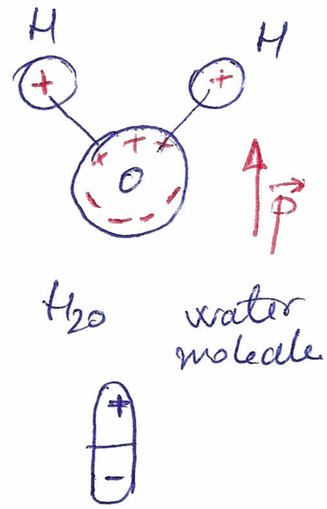
Electric dipoles

Electric dipole = pair of point charges with equal magnitude and opposite sign, separated by a distance d.



→ important concept because many physical systems from molecules to TV antennas can be described as electric dipoles

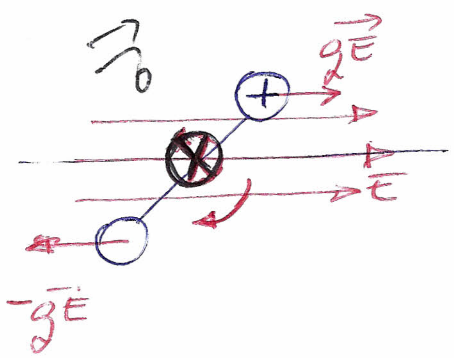
The electric dipole moment $\vec{p}$ is directed from negative to the positive end of the molecule

Force and torque on an electric dipole

↳ placed in an uniform external field $\vec{E}$

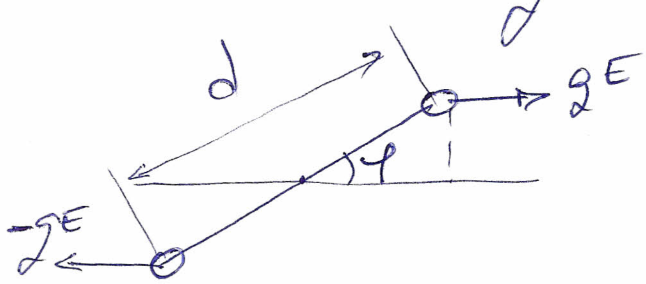
⇒ forces $\vec{F}_+$ and $\vec{F}_-$ on the two charges have same magnitude qE but opposite in direction

⇒ net zero force but non-zero torque ⇒ rotation



The torque: $\tau = \tau_1 + \tau_2 =$

$$= 2 \cdot qE \frac{d}{2} \sin\varphi$$



$$\boxed{\tau = qE\, d \sin\varphi}$$

$\boxed{p = qd}$ ⇒ electric dipole moment $\quad [p] = C\,m$

$\Rightarrow \tau = pE\sin\varphi$

In the vector form $\boxed{\vec{\tau} = \vec{p} \times \vec{E}}$

<u>Obs</u> : One can use the right-hand rule to find the direction of $\vec{\tau}$ (see. fig before)

Potential Energy of an Electric dipole

When a dipole changes the direction with $d\varphi$ in an electric field to do the forque τ

$dW = \tau\, d\varphi$

Because the forque τ is in the direction of decreasind φ

$\Rightarrow \tau = -pE\sin\varphi$

$\Rightarrow dW = -pE\sin\varphi\, d\varphi$

In a finite displacement from φ_1 to φ_2; the total work is:

$$W = \int_{\varphi_1}^{\varphi_2} (-pE\sin\varphi)\, d\varphi = pE\cos\varphi_2 - pE\cos\varphi_1$$

but from mechanics we know that

$W = -\Delta U = U_1 - U_2$; U = potential energy

and if $pE\cos\varphi = \vec{p}\cdot\vec{E}$ $\Rightarrow$

$$\boxed{U = -\vec{p}\cdot\vec{E}}$$

represent the potential energy for a dipole in an electric field.

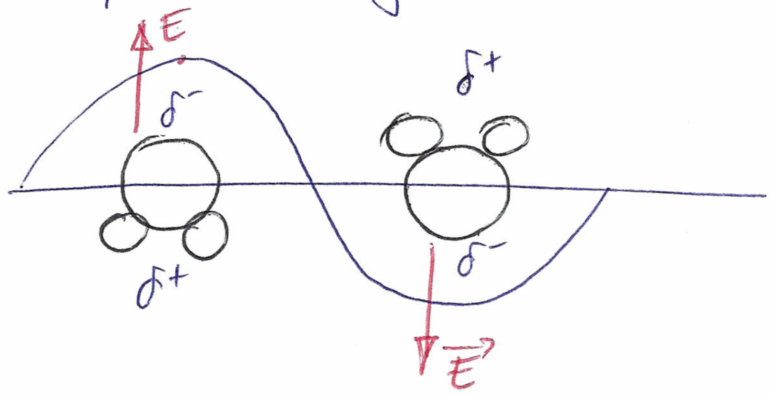
Obs: The potential energy has a minimum (most negative) for $\varphi = 0$ and $\vec{p} \parallel \vec{E}$. This represents the stable equilibrium position.

- At $\varphi = \pi/2$ when $\vec{p} \perp \vec{E}$; $U = 0$

- When brought out of equilibrium, the dipole will oscillate around the equilibrium position.

Heating food with microwaves

Microwaves produced by a magnetron source are directed and absorbed by the water contained in food. The molecule of water behaves as a dipole. In the presence of the electric field component of the microwave a force will be applied to both dipoles of the molecule which cause its rotation! Because the electric field is continuously oscillating the water molecule continuously rotate, at a frequency imposed by the ~~ext~~ normal field of 2,45 GHz. As the water molecules rotate, they bump into other molecules surrounding them and transfer some of their kinetic energy => food heating.

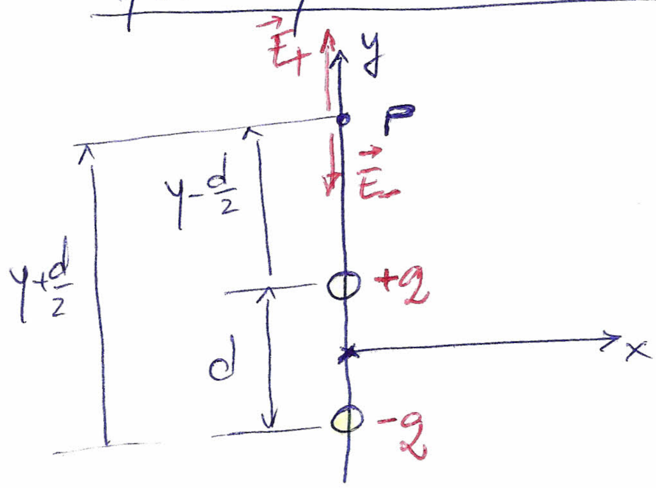


dipole positions where the electric force due to $\vec{E}$ is zero.

Field of an electric dipole

If we think to a dipole as a source of electric field, using the principle of superposition:

• for a point on d axis:



$$E_y = E_+ - E_- =$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{\left(y-\frac{d}{2}\right)^2} - \frac{1}{\left(y+\frac{d}{2}\right)^2}\right]$$

approximation: far from dipole $y \gg d$

$\frac{d}{2y} \ll 1$; using Mc'Lauren developement in series

$$\frac{q}{4\pi\varepsilon_0}\left[\frac{1}{\left(y-\frac{d}{2}\right)^2} - \frac{1}{\left(y+\frac{d}{2}\right)^2}\right] = \frac{q}{4\pi\varepsilon_0 y^2}\left[\left(1-\frac{d}{2y}\right)^{-2} - \left(1+\frac{d}{2y}\right)^{-2}\right]$$

$$\left(1-\frac{d}{2y}\right)^{-2} \simeq 1+\frac{d}{y}$$

$$\left(1+\frac{d}{2y}\right)^{-2} \simeq 1-\frac{d}{y}$$

$$\Rightarrow \quad E_y \simeq \frac{q}{4\pi\varepsilon_0 y^2}\left[1+\frac{d}{y}-\left(1-\frac{d}{y}\right)\right] = \frac{qd}{2\pi\varepsilon_0 y^3} = \frac{p}{2\pi\varepsilon_0 y^3}$$

$$\boxed{E_y = \frac{p}{2\pi\varepsilon_0 y^3}}$$

CAPACITANCE AND DIELECTRICS

Similarly to a spring which can store mechanic energy as elastic potential energy, a <u>capacitor</u> is a device that stores <u>electric potential energy</u> and electric charge. The energy stored in a charged capacitor is related to the electric field in the space between the conductors composing the capacitor. We will see that the electric potential energy can be regarded as being <u>stored in the field itself</u>. This represents the heart of the theory of electromagnetic waves and of our modern understanding of the nature of light.

[1] Capacitors and capacitance

Any two conductors separated by an insulator (or vacuum) form a capacitor.

In circuit diagrams it is represented by the symbol:

Charging the capacitor $\Longleftrightarrow$ each conductor have charges equals in magnitude but opposite sign $+Q$ and $-Q$

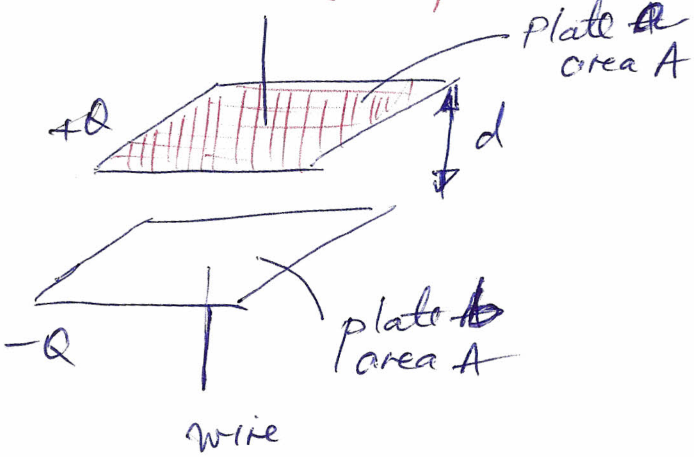
Then, we get a difference potential V_{ab} between the two conductors, proportional with the charge Q

The ratio $\boxed{C = \frac{Q}{V_{ab}} = \frac{Q}{V_a - V_b}}$ is called <u>CAPACITANCE</u>

$$[C]_{SI} = \frac{C}{V} = 1F$$ (Farad) (from M. Faraday)

Because the potential is potential energy per unit charge, the larger is the capacitance, the larger is the charge (energy) that the capacitor can store for a given potential difference V_{ab}.

Calculating capacitance: Capacitors in vacuum.



(1) Parralel plate capacitor

From Gauss law we deduced

$$E = \frac{\sigma}{\varepsilon_0}$$

σ = surface charge density (Q/A)

$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 A}$$

The field is uniform and then, the difference of voltage is

$$V_{ab} = Ed = \frac{1}{\varepsilon_0}\frac{Qd}{A} \Rightarrow \boxed{C = \frac{Q}{V_{ab}} = \frac{\varepsilon_0 A}{d}}$$

$V_{ab} = \int_a^b \vec{E}\,d\vec{r}$

depends only on the geometry (A, d)

Obs

1 Farad is a very large capacitance, typically we use submultiples:

MICRO $1\mu F = 10^{-6} F$ $\quad$ $1pF = 10^{-12} F$ pico

NANO $1nF = 10^{-9} F$

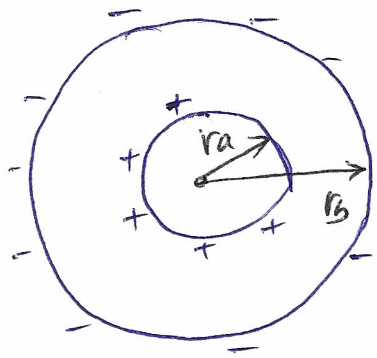
Obs: Which should be the size A of a plate of a capacitor with $A = 1F$ if plates are 1mm apart

$$C = \varepsilon_0 \frac{A}{d} \Rightarrow A = \frac{Cd}{\varepsilon_0} = \frac{1\cdot 10^{-3}}{8{,}85\cdot 10^{-12}} = \underline{\underline{1{,}1\cdot 10^{8} m^2}}\ !!$$

($\simeq$ square about 10 Km side)

② Spherical capacitor

(2 concentric spherical conducting shells)



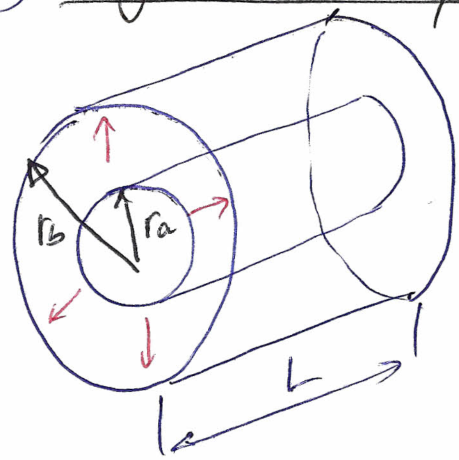
$$C = \frac{Q}{V_{ab}} \qquad V_{ab} = V_a - V_b = \int_a^b E(r)\,dr$$

(from Gauss) $V_{ab} = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r_a} - \frac{1}{r_b}\right)$

$$\Rightarrow \boxed{C = 4\pi\varepsilon_0 \frac{r_a r_b}{r_b - r_a}}$$

capacitance determined entirely by dimensions.

③ Cylindrical capacitor (long)



inner cylinder: radius r_a and linear charge density λ

outer cylinder: radius r_b and linear charge density $-\lambda$

The potential outside a charged cylinder has been calculated as:

$$V = \frac{\lambda}{2\pi\varepsilon_0} \ln\frac{r_0}{r}$$

where r_0 is an arbitrary finite radius where $V=0$

$Q = \lambda L$

$$\Rightarrow V_{ab} = \frac{\lambda}{2\pi\varepsilon_0} \ln\frac{r_b}{r_a}$$

$$C = \frac{Q}{V_{ab}} = \frac{\lambda L}{\frac{\lambda}{2\pi\varepsilon_0}\ln\frac{r_b}{r_a}} = \boxed{\frac{2\pi\varepsilon_0 L}{\ln\left(\frac{r_b}{r_a}\right)}}$$

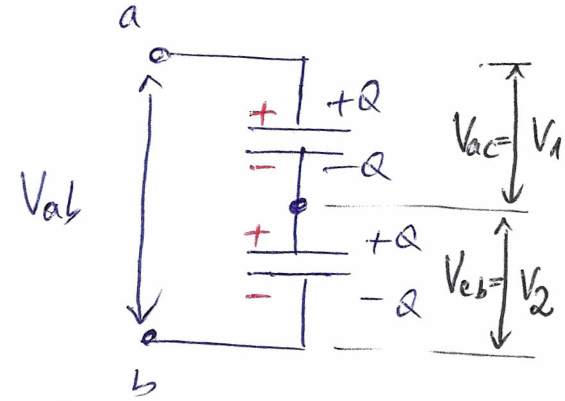
capacitance determined entirelly by dimentions

The capacitance per unit length is:

$$\boxed{\frac{C}{L} = \frac{2\pi\varepsilon_0}{\ln\frac{r_b}{r_a}}}$$

2 Capacitors in series and parallel

SERIES



the capacitors have
- same charge Q
- their potential differences add

$$V_{ac} = V_1 = \frac{Q}{C_1} \qquad V_{cb} = V_2 = \frac{Q}{C_2}$$

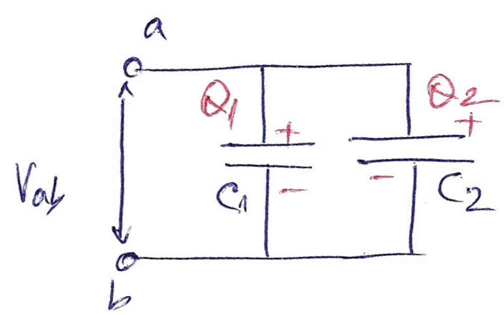
$$V_{ab} = V_1 + V_2 = Q\left(\frac{1}{C_1} + \frac{1}{C_2}\right)$$

$$\Updownarrow$$

$$\frac{\cancel{Q}}{C} = \cancel{Q}\left(\frac{1}{C_1} + \frac{1}{C_2}\right) \Rightarrow \boxed{\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}}$$

$$\Rightarrow \boxed{\frac{1}{C_{eq}} = \sum_i \frac{1}{C_i}}$$ for n capacitors in series.

PARALLEL



- same V_{ab}
- Q is distributed $Q = Q_1 + Q_2$

$$Q_1 = C_1 V_{ab} \quad ; \quad Q_2 = C_2 V_{ab} \qquad Q = C_{eq} V_{ab}$$

$$Q = Q_1 + Q_2 \Rightarrow$$

$$C_{eq} = C_1 + C_2$$

for n capacitors in parallel $\Rightarrow \boxed{C_{eq} = \sum_i C_i}$

3 Energy stored in capacitors

The electric potential energy stored in a charged capacitor is equal to the amount of work required to charge it – that is to separate opposite charges and place them on different conductors

When capacitor is charged the final charge is Q and the final potential difference is V;

$$V = \frac{Q}{C}$$

Let q and v be the charge and potential difference in an intermediate stage during charging, then $v = q/C$. At this stage, the work dW required to transfer additional element of charge dq is:

$$dW = v\,dq = \frac{q\,dq}{C} \Rightarrow$$

$$W = \int_0^W dW = \frac{1}{C}\int_0^Q q\,dq = \frac{Q^2}{2C}$$

If we defined the energy of a discharged capacitor to be zero

$$W = U_{charged} \Rightarrow \boxed{U = \frac{1}{2}CV^2 = \frac{1}{2}QV = \frac{Q^2}{2C}}$$

obs Analogy with mechanics:

$$U = \frac{Q^2}{2C} \longleftrightarrow U = \frac{1}{2}KX^2$$

$$\Downarrow$$

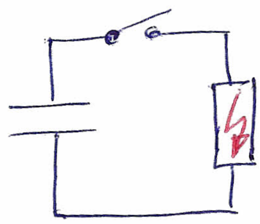
$$\frac{1}{C} \longleftrightarrow K$$

$$Q \longleftrightarrow X$$

The energy supplied to C to charge $\Leftrightarrow$ energy supplied to spring to stretch it.

Applications of capacitors: Energy storage

Capacitors have ability to store and release energy. In electronic flash units used by photographers, the energy stored in a capacitor is released depressing the camera's shutter button. This provide a conducting path from one capacitor plate to the other through the flash-tube
$\Rightarrow$ brief intense flash of light



Electric field energy

The energy per unit volume in the space between the two plates of a plane capacitor of area A separated by the distance d is:

$$u = \text{Energy density} = \frac{\text{Energy}}{\text{Volume}} = \frac{\frac{1}{2}CV^2}{Ad} \Rightarrow$$

but $C = \frac{\varepsilon_0 A}{d}$

$$\boxed{u = \frac{1}{2}\varepsilon_0 E^2}$$

this deduced eq. for the simple plane capacitor is VALID for ANY ELECTRIC FIELD CONFIGURATION IN VACUUM

(even for electromagnetic features)
— see later —

4 DIELECTRICS

Most capacitors have a non-conducting material, or DIELECTRIC, between their conducting plates

This dielectric has several (3) functions:

1. solves the mechanical problem of maintaining two large metal sheets at a very small distance without contact

2. increases the maximum possible potential difference between the capacitor plates.

⟺ The BREAKDOWN VOLTAGE is increased:

Breakdown: Any insulating material submitted to sufficiently large electric field experiences a partial ionisation and becomes locally conductor ⟹ dielectric breakdown.

(!) see later

Most dielectric materials tolerate stronger electric fields than air

3. enhances the capacitance. We can define the dielectric constant K as the ratio between the capacitance with dielectric and capacitance with vacuum Co

$$\boxed{K = \frac{C}{C_0}}$$

→ constant of material

Values of dielectric constant at 20°C

Material	K
vacuum	1
air	1,00059
teflon	2,1
mica	3–6
glass	5–10
water	80,4
$SrTiO_3$	$\boxed{310}$

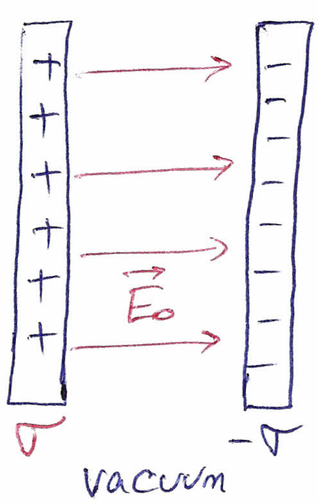
(!) Despite large K water is not a very practical dielectric for uses in capacitors. While pure water is a very poor conductor it is an excellent ionic solvent. Any disolved ions cause charge flow between capacitor plates ⟹ discharge !!!

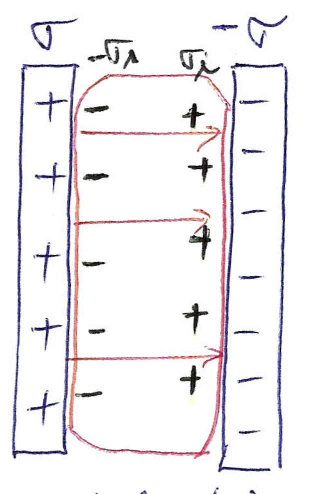
Induced charge and Polorization

When a dielectric material is inserted between the plates while the charge is kept constant, the potential difference between the plates is decreased by a factor K. Therefore, the electric field between plates must decrease by the same factor. If E_0 is the vacuum value and E the value with the dielectric =>

$$E = \frac{E_0}{K} \qquad (\text{when } Q = ct)$$

Smaller electric field magnitude would implicate smaller surface charge density which produces the field. The surface charge on the conducting plates do not change but ~~and~~ INDUCED charge of opposite sign appears on each surface of the dielectric. The dielectric was originally electrically neutral and still neutral, but the induce surface charges arise as a result of redistribution of positive and negative charges within the dielectric material, phenomenon called POLARIZATION.





$$E_0 = \frac{\sigma}{\varepsilon_0} \qquad E = \frac{\sigma - \sigma_i}{\varepsilon_0}$$

and $E = \dfrac{E_0}{K}$ $\Bigg\}$ for a plane capacitor

=>

$$\boxed{\sigma_i = \sigma\left(1 - \frac{1}{K}\right)}$$

Induced surface charge density

obs: for large K; $\sigma_i \to \sigma$; $\frac{\sigma_i}{\sigma}$ nearly cancels

The product $K\varepsilon_0$ is called the permittivity of the dielectric, denoted by ε

$$\Rightarrow \boxed{\varepsilon = \varepsilon_0 K}$$

The capacitance is then, when the dielectric is considered:

- the electric field within dielectric

$$\boxed{E = \frac{\sigma}{\varepsilon}}$$

- capacitance:

$$\boxed{C = KC_0 = K\varepsilon_0\frac{A}{d} = \varepsilon\frac{A}{d}}$$

If we repeat calculations for energy and energy density, we get

$$u = \frac{U}{\text{Volume}} = \frac{1}{2}CU^2\frac{1}{Ad} \Rightarrow \text{if } C = \frac{\varepsilon A}{d}$$

$$\boxed{u = \frac{1}{2}\varepsilon E^2}$$

electric energy density in a dielectric

in vacuum $K = 1 \Rightarrow \varepsilon = \varepsilon_0$ and all equation reduce to those valid in vacuum.

Obs: From $C = \mathcal{E}\frac{A}{d} = \varepsilon_0 \frac{KA}{d}$ we can see that extremelly large capacities can be obtained with plates with large A separated by small d and dielectrics with large K. In an electrolytic double layer capacitor, tiny carbon granules adhere to each plate: the value of A is the combined area of granules which can be huge. These plates are separated by a very thin dielectric sheet. Such a capacitor can achieve a capacitance of 5000 farads yet fit in the palm of your hand.

Dielectric Breakdown

Occurs when the electric field is strong enough, above a critical value $E_{breakdn}$, so that electrons are pulled from their molecules and crash into another molecules, liberating even more electrons. This avalanche of moving charge forms a spark of an arc discharge. Lightning is a dramatic example of dielectric breakdown in air.

Because of dielectric breakdown, capacitors have a maximum voltage rating. For larger voltages, the dielectric breaks, an electric arc being creating burning or melting a hole through it. If this conducting path remains after the arc is extinguished, the capacitor device get broken.

The maximum electric field magnitude that a material can support without the occurance of breakdown is called DIELECTRIC STRENGTH. This quantity is strongly affected by temperature, irregularities or deffects, impurities, or other random factors.

Dielectric constant and dielectric strength of some materials

Material	Dielectric constant K	Dielectric strength E_m V/m
dry air	1	$3 \cdot 10^6$
polycarbonate	2,8	$3 \cdot 10^7$
polyester	3.3	$6 \cdot 10^7$
pyrex glass	4.7	$1 \cdot 10^7$

obs: • Large electrical fields can be attained if the separation distance is small (e.g. tunnel junctions $d \simeq 1\,nm = 10^{-9}\,m$).

A votage of $1\,V \Rightarrow E = 10^9\,V/m$ which may overcome the dielectric strength.

• Modern researches succeed to lead to dielectrics with $E_m \sim 10^9\,V/m$ (single crystal MgO formers)

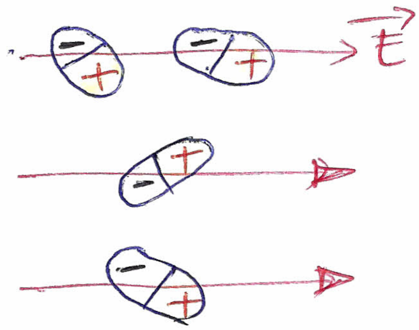
$$E_m > 0{,}75 \cdot 10^9\,V/m$$

5 Molecular model of induced charge

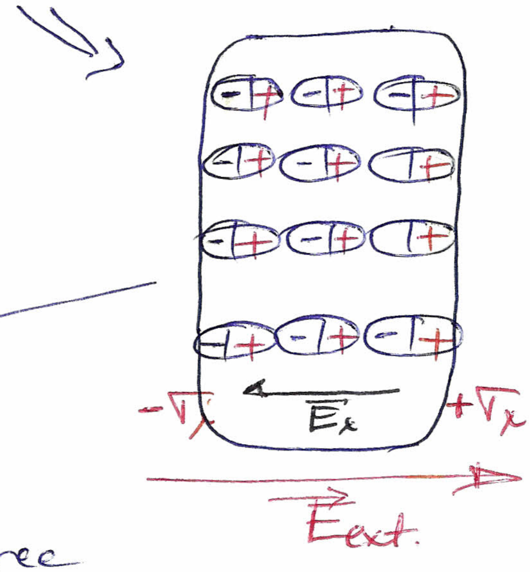
If the dielectric material would be a conductor, the mechanism of induced charges would be simple. The free electrons will redistribute within the electric field of the charges on plates. However, dielectrics are insulators, and no mobile charge is available.

We have to look to the charge rearrangement at a molecular level. In the presence of an electric field, non-polar molecules became polar, positive and negative charges separating slightly.

⇒ non-polar molecules become dipoles under applied electric field. ↳ <u>induced dipoles</u>



Then, the polar molecules tend to be aligned with the external electric field



This creates surface charge densities $+\sigma_x$, $-\sigma_x$

These charges are not free to move ⇒ <u>BOUND CHARGES</u>. In the bulk the total charge/unit volume cancels

This redistribution of charge is called <u>POLARIZATION</u>

The induced bound charges create an electric field $\vec{E}_x$ opposite to the external field $\vec{E}$ ⇒ decreasing the field amplitude within the dielectric:

$$\underline{\vec{E}_d = \vec{E}_{ext} - \vec{E}_x}$$

<u>Obs</u> 1) $\vec{E}_x$ is never enough long to cancel $\vec{E}_{ext}$ because charges in dielectric are not free to move indefinitelly

2) Polarization is also the reason a charged body (e.g. electrified plastic rod) exerts a force on uncharged body (bit of paper).

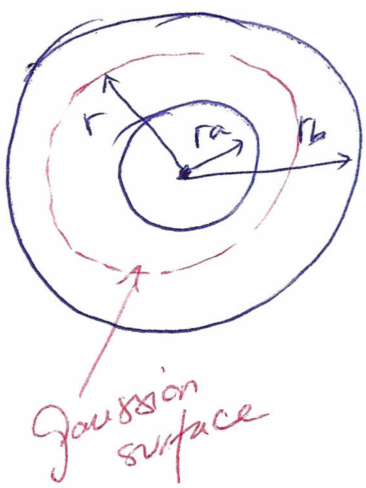
6 Gauss law in dielectrics

This has almost the same form as in vacuum with two key differences:

1. $\vec{E}$ is replaced by $K\vec{E}$
2. Q_{encl} is replaced by $Q_{encl\text{-}free}$ which includes only the free-charge (not bond charge) enclosed by the Gaussian surface.

$$\boxed{\oint K\vec{E}\cdot d\vec{A} = \frac{Q_{encl\text{-}free}}{\varepsilon_0}}$$

Example: Spherical capacitor with dielectric
K - dielectric constant



$$\oint K\vec{E}\cdot d\vec{A} = \oint KE\,dA = KE.4\pi r^2$$

$$= \frac{Q}{\varepsilon_0}$$

$$\Rightarrow E = \frac{Q}{4\pi K\varepsilon_0 r^2} = \frac{Q}{4\pi\varepsilon r^2}$$

with $\varepsilon = K\varepsilon_0$

compared with vacuum $\Rightarrow E = \frac{E_0}{K}$

$$\Rightarrow V_{ab} = \frac{V_0}{K} \quad \Rightarrow C = \frac{Q}{V_{ab}} = KC_0$$

$$V_{ab} = \int_{r_a}^{r_b} E\,dr = \int_{r_a}^{r_b} \frac{Q\,dr}{4\pi\varepsilon r^2} = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r_a} - \frac{1}{r_b}\right)$$

$$\Rightarrow \boxed{C = \frac{4\pi K\varepsilon_0 r_a r_b}{r_b - r_a} = KC_0}$$