Structure of the course and Bibliography

Advanced Physics (2nd term)

- 7 seminaries (each 2 week)
- 2 main parts
  - Electromagnetism (7c)
  - Quantum mechanics (6-7c)

Electromagnetism:

- Electric charge, electric field
- Current, resistance electromotive force
- Magnetic field & magnetic forces
- Magnetic materials. Superconductivity
- Electromagnetic waves

Quantum mechanics (Intro - basis)

- Limitation of classical physics & hystorical hypotheses
- The wave-particle duality
- The ondulatory QM. Wave function. Schrödinger eq. Applications
- Introduction in spintronics.
- Quantum mechanics as a basis for atomic physics and solid state electronics

Bibliography:
- Zemanski & Sears
- teacher's notes (web).

ELECTROMAGNETISM

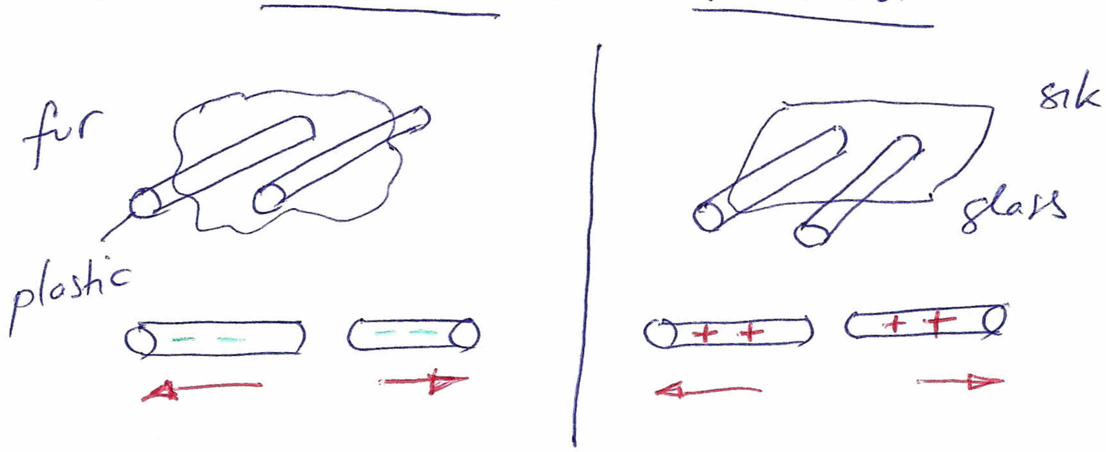
1/ ELECTRIC CHARGE AND ELECTRIC FIELD

Electromagnetic interactions involve particles that have a property called <u>electric charge</u>, an attribute that is fundamental as mass. Just as objects with mass are accelerated by gravitational forces, electrically charged objects are accelerated by electrical forces.

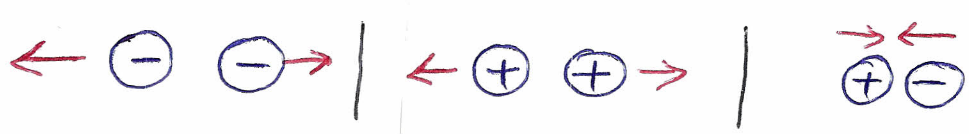
1 Electric charge

Ancient Greeks discovered in early 600 B.C. that after rubbing amber with wool, the amber attracts other objects. This is because the amber <u>acquired a net charge</u>, or has <u>become charged</u>. The word "electric" derives from the Greek "elektron" meaning amber.

The experiments have shown that there are <u>2 kinds</u> of charge: Benjamin Franklin (1706-1790) called them <u>NEGATIVE</u> and <u>POSITIVE</u>.



Two positive charges or negative charges repel each others. A positive charge and a negative charge attracts each other.



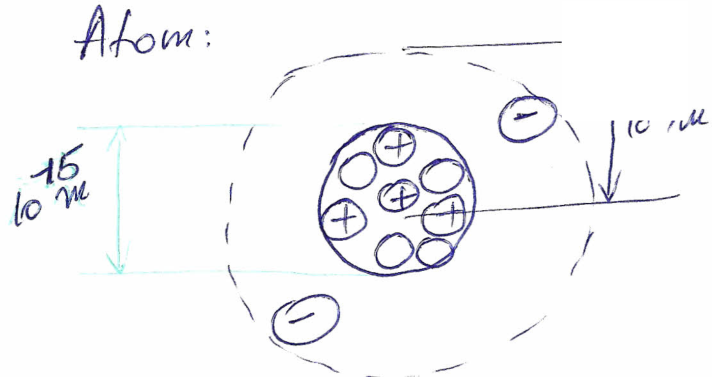
Application of electrostatic interactions

- the LASER printer. The printer's light sensitive imaging drum is given a positive charge. As the drum rotates, a laser beam shines on selected areas of the drum, leaving those areas negatively charged. Positivelly charged particles of the toner adhere "on the negative areas written by the laser. When a piece of paper is placed in contact with the drum, the toner particles sticks on the paper and form an image.

(homework: individually study the working principle of a LASER printer)

Electric charge and structure of matter

Atom:



Nucleus $\sim 10^{-15}$ m

Contains over 99,9% of atom's mass

→ protons: ⊕ p^+

→ neutrons ○ n^0

$M_P = 1{,}673 \cdot 10^{-27}$ kg ⊕ charge

$M_N = 1{,}675 \cdot 10^{-27}$ kg 0 charge

Electrons $m_e = 9{,}109 \cdot 10^{-31}$ kg ⊖ charge

→ $\overbrace{\text{Proton and neutron}}^{\text{nucleons}}$ are combination of quarks with fractionary $\pm\frac{1}{3}$ and $\pm\frac{2}{3}$ times the electron charge. (isolated quarks not yet observed).

→ Protons & neutrons are held within stable nuclei by an attractive force called STRONG NUCLEAR FORCE that overcomes the electric repulsion of protons.

The strong nuclear force has a short range, it is not extending far beyond the nucleus.

Atom: $\begin{cases} Z \\ A \end{cases}$ $\qquad$ Z electrons (−)
Z protons (+)

⇒ total neutral charge.

- If one electron is removed from an atom ⇒ positive ion M^+
- If the atom gains an electron ⇒ negative ion M^-

⇒ Charging electrostatically an object ⇔ creating ions (+) or (−)

Electric charge is conserved

Principle of charge conservation The algebraic sum of all the electrical charges in a closed system is constant. UNIVERSAL CONSERVATION LAW

If we rub together a plastic rod and a piece of fur, both initially uncharged, the fur acquires the same positive charge in magnitude as the plastic rod aquires negative ⇒ electrons are transfered from one body (fur) to the other (plastic rod)

$\Delta q_{tot} = 0$

Charge quantization principle The magnitude of charge of the electron or proton is the natural unit of charge

⇒ The charge of any macroscopic object is always either zero or a MULTIPLE (positive or negative) of the electron charge

Understanding the electric nature of matter gives us insight into many aspects of the physical world. The chemical bonds that hold atoms together to form molecules are due to electric interactions between atoms. They include strong ionic bonds that hold Na and Cl together to form NaCl (salt) and the relatively weak bonds between the strands of DNA recording the genetic code. The normal force $\vec{N}$ exerted on you by the chair in which you sits arises from electric forces between charged particles in the atoms of your seat and the atoms of your chair. The tension force in a stretching string and the adhesive force of glue are likewise due to the electric interactions ~~if~~ of atoms.

[2] Conductors, Insulators and Induced Charges

Some materials permit electric charge to move easily from one region of the materials to another, while others not.

⇒ charge conductors
insulators

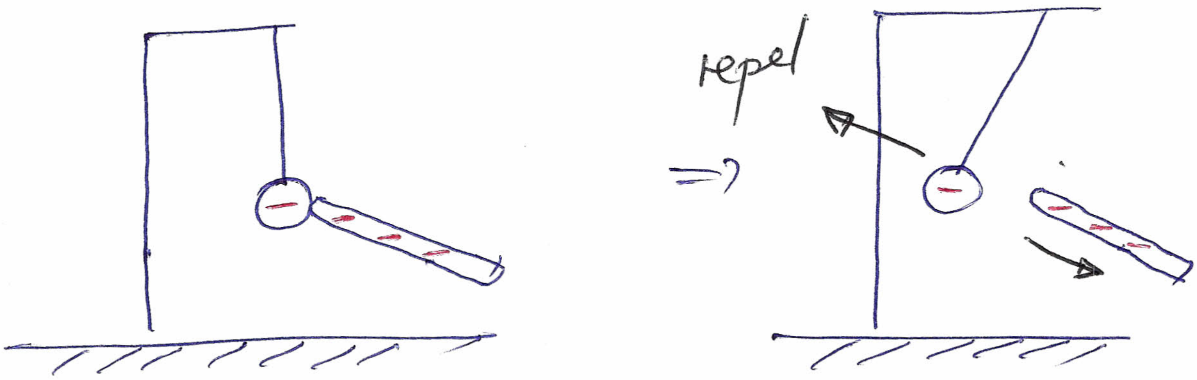
- conductors (Cu, metals...) ⟺ metals
- insulators (fur, silk, paper, ...) ⟺ nonmetals

semiconductors: - intermediate in their properties between good conductors and good insulators

→ we'll see later the physics behind (part of QM, Solid state electronics and applications)

Charging by induction

CONTACT We can charge a metal ball using a Copper wire and an electrically charged plastic rod. ⇒ Some excess electrons of the rod are transfered from it to the ball, leaving the rod with smaller negative charge. Then, opposite charges of rod and ball repels

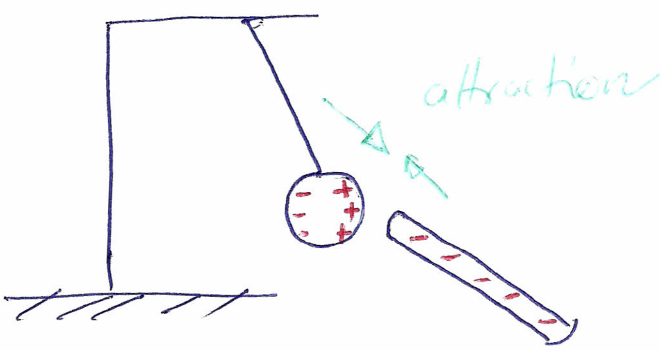


By contact, the objects charge identically
⇔ Charge redistribution between bodies

INDUCTION

→ not direct contact, the negative charge of the rod is brough near (in proximity) of the ball. The free electrons in the metal ball are repelled by the excess electrons on the rod and they shift toward the other side of the ball, oppositied to the rod

⇒ INDUCED CHARGES

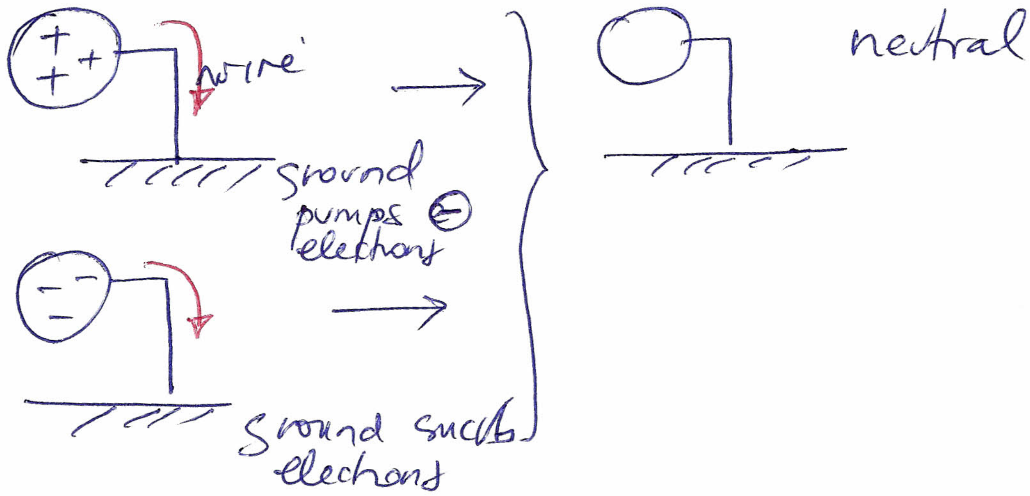


By induction, the body acquire local opposite charge ⇒ attraction

Charged system in contact to the Earth

The Earth is a conductor, and it is so large that it can act as a practically infinite source of extra electrons or sink of unwanted electrons.

⇒ descharging to the ground

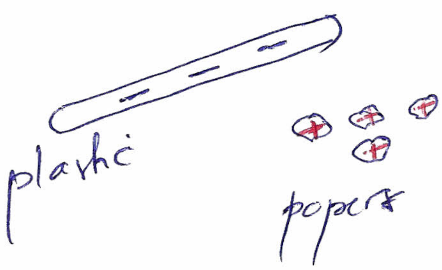


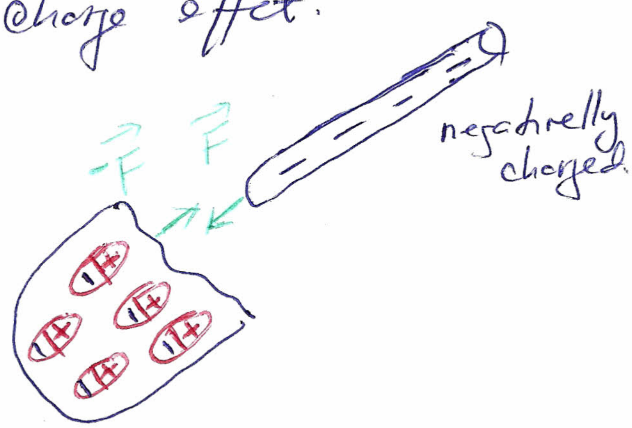
Electric force on uncharged objects

A charged body can exert forces even on objects that are not charged.

ex : comb electrified by hair attracts small pieces of paper.

?. Explaining mechanism: The interaction is an induced charge effect.



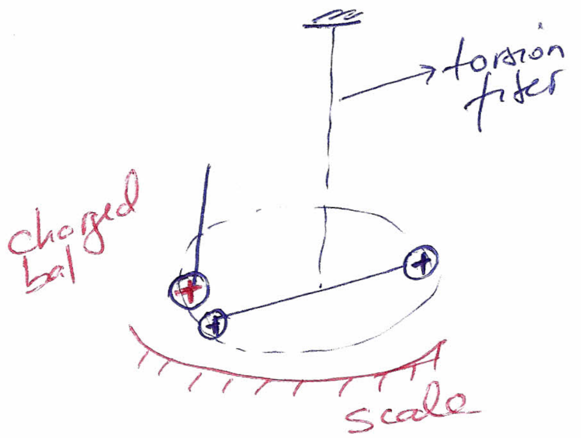


⇒ polarization

↳ The negativelly charged rod causes a slight shifting of charge in molecules of the neutral insulator

3 Culomb interactions

Charles Augustin de Coulomb (1736–1806) studied in detail the interaction forces of charged particles in 1784. Similar experiment, using a torsion balance, was used 13 years later by Cavendish to study the gravitational interaction.



For point charges (charged bodies with size smaller than distance r between them), Coulomb found that the:

→ electric force is $\sim \frac{1}{r^2}$

→ electric force depend on the charge quantity on each body: q_1, q_2

$$\boxed{F = k\,\frac{|q_1 q_2|}{r^2}}$$

force magnitude

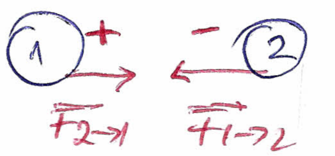
Coulomb's law

k = proportionality constant depending on the system of units used.

The direction of the force is always on the line joining the two charges.

Same sign q_1 and q_2 ⇒ Repulsion

opposite sign q_1 and q_2 ⇒ attraction



$$\boxed{|\vec{F}_{1\to2}| = |\vec{F}_{2\to1}|}$$

obeys Newton's 3rd law ⇔ equall magnitude, opposite direction

Obs: Electric and gravitational phenomena of interaction obey similar $1/r^2$ laws even if they are two distinct classes of interactions. Gravitational interactions are always attractive and mass always positive, while electrostatic interactions are attractive or repulsive, depending on the relative charges sign.

Fundamental electric constant

$$\boxed{k_{[SI]} \cong 8.988 \cdot 10^9 \, N \cdot m^2/C^2}$$

Obs: In problems we consider $\boxed{k = 9 \cdot 10^9 \cdot \frac{Nm^2}{C^2}}$

In terms of speed of light in vacuum:

$$c = 2.99792458 \cdot 10^8 \, m/s$$

$$\Rightarrow \boxed{k = \left(10^{-7} \frac{N \cdot A^2}{C^2}\right) c^2}$$

in SI $\quad k = \frac{1}{4\pi\varepsilon_0}$; $\quad \varepsilon_0 = 8{,}854 \cdot 10^{-12} \, C^2/N \cdot m^2$

permittivity of vacuum

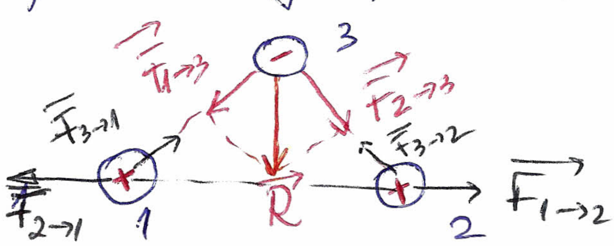
The unit of charge (electron/proton) charge is:

$$\boxed{e = 1{,}602 \cdot 10^{-19} C}$$

Superposition of forces

Coulomb law states the interaction between ~~two~~ point charges. For arbitrary number of charges, they interact in pairs of two, and for a certain charge one has to apply the principle of superposition of forces.

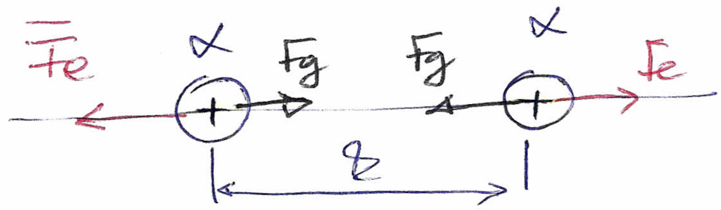
→ see pb. seminary



Electrostatic versus gravitational interaction

ex α particles (He_2^4-nuclei) ; $m = 6{,}64 \cdot 10^{-27}$ kg

$q = +2e = 3{,}2 \cdot 10^{-19}$ C



$$F_e = \frac{1}{4\pi\varepsilon_0} \frac{q^2}{r^2}$$

$$F_g = G \frac{m^2}{r^2}$$

$$\frac{F_e}{F_g} = \frac{1}{4\pi\varepsilon_0 G} \frac{q^2}{m^2} \longrightarrow 3{,}10 \cdot 10^{35} \quad !!!$$

=> at this scale the gravitational force is completelly negligible in comparison to the electric force. On the other hand, electrostatic interaction is negligible at macroscopic scale (person, planet, universe) where positive and negative charges are nearly equal in magnitude.

4 Electric field

When two electrically charged particles in empty space interact, how does each other know the other is there?

⇒ new concept required.

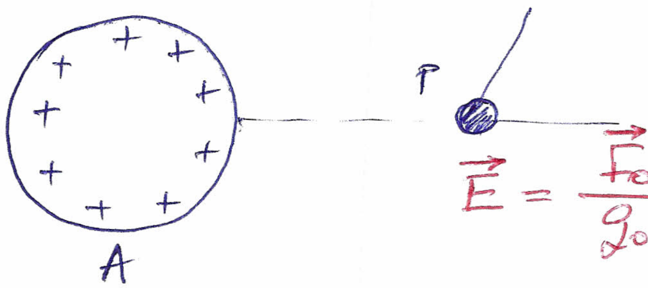
Electrically charged bodies modifies the properties of the space around it (similarly to a mass which changes the properties of the space leading to gravitational interactions with another mass in surroundings).

⇓

A charged body produces an ELECTRIC FIELD

This electric field exerts a force to any charged body brought inside.

Test charge q_0



$$\vec{E} = \frac{\vec{F_0}}{q_0}$$

charge producing the field. $\vec{E}$

The electric field $\vec{E}$ at a point P as the electric force $\vec{F_0}$ experienced by a test charge q_0 at that point, divided by the charge q_0.

$$\vec{E} = \frac{\vec{F_0}}{q_0}$$

electric field = $\dfrac{\text{electric force}}{\text{unit charge}}$

$$[E]_{SI} = \frac{1\,N \to \text{Newton}}{1\,C \to \text{Coulomb}}$$

⇒ knowing $\vec{E}$ at a certain point, we can calculate the force experienced by any charge in that point:

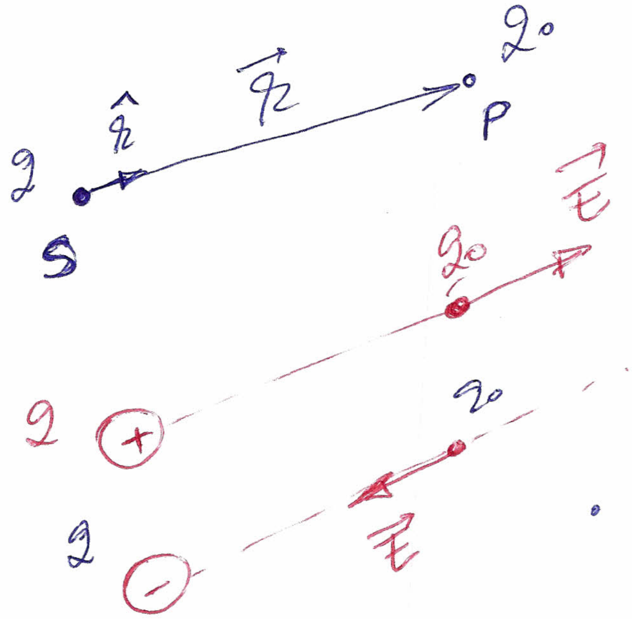
$$\boxed{\vec{F}_0 = q_0 \vec{E}}$$

equation equivalent with $\vec{F}_g = m_0 \vec{g}$

$\vec{g}$ = acceleration due to gravity or gravitational field

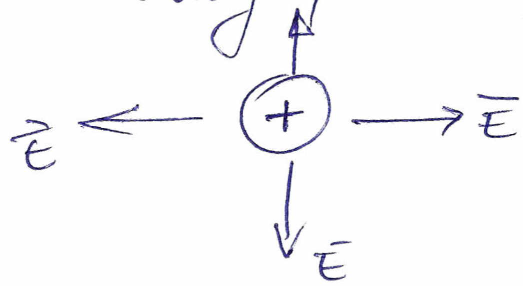
Electric field of a point charge

We call the location of the charge the SOURCE POINT (S) and the point P where the field is determined as the FIELD POINT.

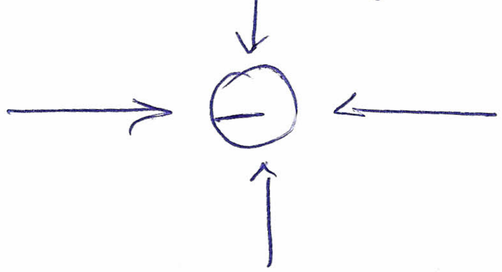


$$\underbrace{\vec{r} = \hat{r}\, r}$$
unit vector

- Electric field of a positive charge point away from the charge



- Electric field of a negative charge points toward the charge



From $F_0 = \frac{1}{4\pi\varepsilon_0} \frac{|q q_0|}{r^2}$ $\Rightarrow$ the magnitude E of the electric field in point P is:

$$E = \frac{F_0}{q_0} \Rightarrow \boxed{E = \frac{1}{4\pi\varepsilon_0} \frac{|q|}{r^2}}$$

Using the unit vector $\hat{r}$ we can write a vector equation: giving bot magnitude and direction of vector $\vec{E}$

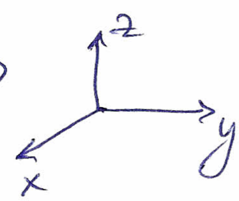
$$\boxed{\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} \hat{r}}$$

electric field of a point charge.

<u>Obs</u>: $\vec{E} = \vec{E}(r)$ varies from point to point $\Rightarrow$ it is not a single vector quantity but an <u>infinite set</u> of vector quantities, one associated to each point in space

This is an example of <u>VECTOR FIELD</u>

$\vec{E} = \vec{E}(\vec{r}) \longrightarrow$



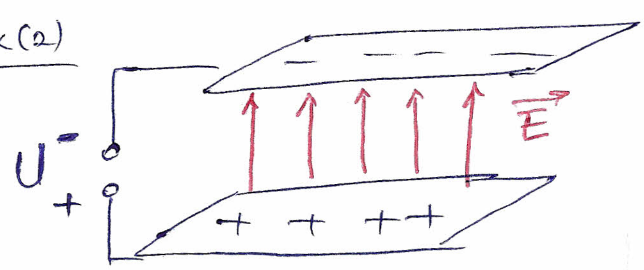
$\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$

$\Rightarrow \begin{cases} E_x(x,y,z) \\ E_y(x,y,z) \\ E_z(x,y,z) \end{cases}$

In some situations, the magnitude and the direction of the electric field have same values everywhere through a certain region $\Rightarrow$ the electric field is <u>uniform</u> in that region.

<u>ex(1)</u> the electric field inside of a conductor is <u>zero</u>.

<u>ex(2)</u>



uniform electric field between two parallel conducting plates connected at a battery

$\Leftrightarrow$ <u>plane capacitor</u>

Electric field calculations

Superposition of electric fields

$\Rightarrow$ field generated by a charge distribution $q_1, q_2, \dots$

In a point P, each charge produces a field $\vec{E}_1, \vec{E}_2 \dots$

So a test charge q_0 experiences a force:

$$\vec{F} = \vec{F}_1 + \vec{F}_2 + \dots = q_0 \vec{E}_1 + q_0 \vec{E}_2 + \dots$$

The combined effect of all the charges is described by the total electric field:

$$\vec{E} = \frac{\vec{F}}{q} = \vec{E}_1 + \vec{E}_2 + \dots$$

$$\Rightarrow \boxed{\vec{E} = \sum_i \vec{E}_i}$$

Principle of superposition of electric fields

(see seminary)

If the:

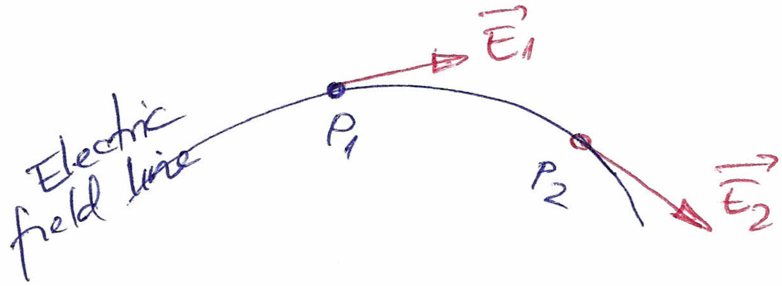
- charge is distributed along a line $\Rightarrow$ linear charge density
  - ~~over~~ a surface $\Rightarrow$ surface charge density
  - through a volume $\Rightarrow$ volume charge density

Examples for seminary

→ field of an electric dipole
→ field of a ring of charge
→ —"— charged line segment
→ —"— uniformly charged disk
→ field of two oppositelly charged infinite sheets.

Electric field lines

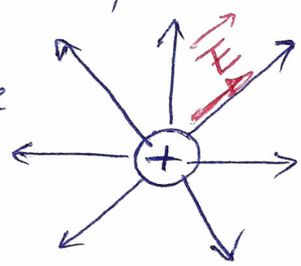
→ represent imaginary line or curve drawn through a region of space so that ~~its~~ tangent at any point is in the direction of the electric field in that point



Field lines never intersect, only one field line passes through each point.

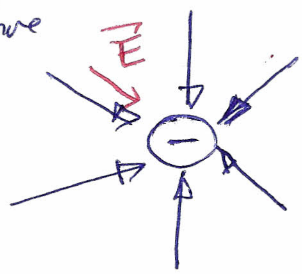
Examples.

① positive

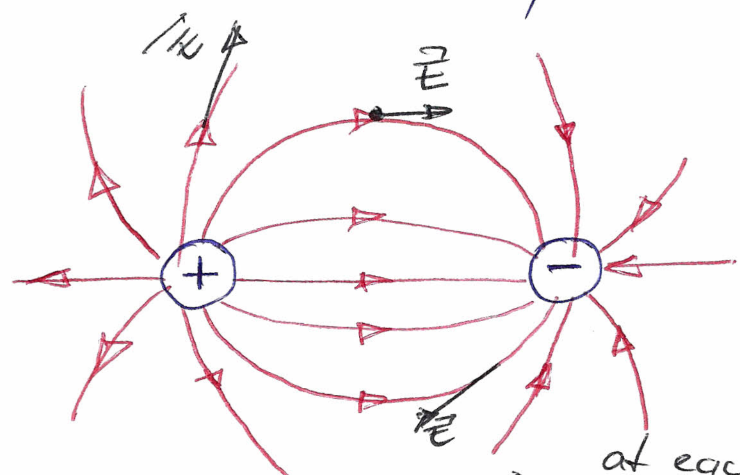


single charge

negative



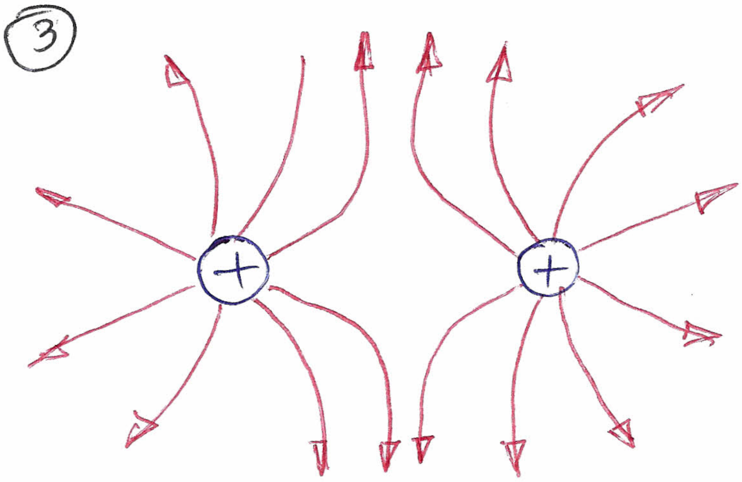
②



$\vec{E}$ is tangent at each point to the field line passing through the point

electric dipole : two equal and opposite sign charges

③



Obs: The spacing of $\vec{E}$ field lines gives idee à about magnitude of $\vec{E}$ at each point:

$\vec{E}$ strong ⇒ lines bunched together

$\vec{E}$ low (weak) ⇒ lines farther apart.

ELECTRIC POTENTIAL

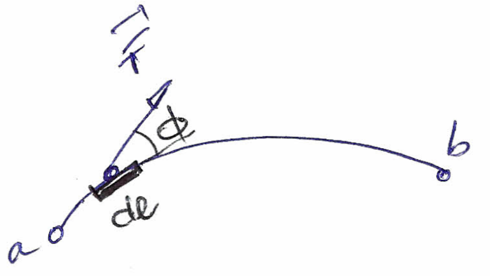
This chapter is about energy associated to electrical interactions. We are going to combine the concepts of WORK and ENERGY from mechanics with that we have learned about electric charge, electric forces and electric fields.

When a charged particle moves in an electric field, the field exerts a force that can do work on the particle. This work can be always be expressed in terms of electric potential energy. Just as the potential gravitational energy depends on the height of a mass above a surface, electric potential depends on the position of a charged particle in an electric field. We will describe electric potential energy using a new concept called electric potential or simply potential. In circuits, a difference of potential from one point to another is called VOLTAGE.

[1] Electric potential energy

From mechanics, the work done by a force $\vec{F}$ to move a particle from point a to b is:

$$W_{a \to b} = \int_a^b \vec{F} \cdot \vec{dl} = \int_a^b F \cos\phi \, dl$$



$\vec{dl}$ = infinitesimal displacement along the particle's path

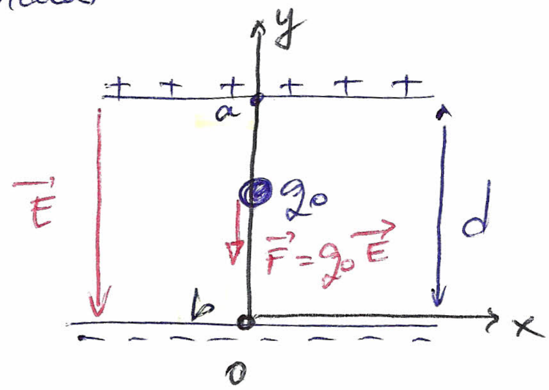
ϕ = angle between $\vec{F}$ and $\vec{dl}$ at each point along the path

If $\vec{F}$ is conservative this work can be expressed in terms of a potential energy $U \Rightarrow U_a, U_b$.

$$\boxed{W_{a \to b} = -\Delta U = U_a - U_b}$$

Electric potential energy in a uniform field

Let's consider an uniform field $\vec{E}$, between two infinite plates



$$W_{a\to b} = Fd = q_0 Ed$$

positive work since the force has same direction as displacement.

The y component of electric force: $F_y = -q_0 E$ is constant and exactly analogous to the gravitational force of a mass m $\Leftrightarrow$ $F_y = -mg$.

By analogy, we conclude that the electric force exerted by the (electric) field is conservative just as the gravitational force situation $\Rightarrow$ $W_{a\to b}$ is independent on path the particle takes from a to b.

We can, therefore, represent this work as a potential energy:

$$\text{by analogy } U = mgy \longrightarrow U = q_0 Ey$$

When the test charge moves from y_a to y_b, the work done on the charge will be:

$$W_{a\to b} = -\Delta U = -(U_b - U_a) = q_0 E(y_a - y_b)$$

When $y_a > y_b$ $W_{a\to b} > 0$ the charge moves ~~[illegible]~~ downwards along $\vec{E}$

For positive ~~negative~~ charge moving with field $W_{a\to b} > 0 \Rightarrow$ $\Delta U < 0 \Leftrightarrow U_b < U_a \Rightarrow$ the potential energy decreases

RULE

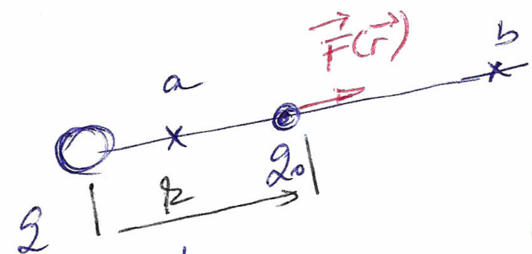
Whether charge is positive or negative, U decreases if q_0 moves in the same direction as the ~~field~~ force $\vec{F} = q_0\vec{E}$ and U increases if q_0 moves on opposite direction with respect to $\vec{F} = q_0\vec{E}$

Electric potential energy of two point charges

The ideea of electric potential energy is not restricted to the special case of an uniform electric field. It is valid for any electric field created by a charge distribution

a) The work done by the electric field of a single stationary charge q on a test charge q_0.

Consider a displacement along a radial line



$$\vec{F}(r) = \frac{1}{4\pi\varepsilon_0}\frac{q q_0}{r^2}$$

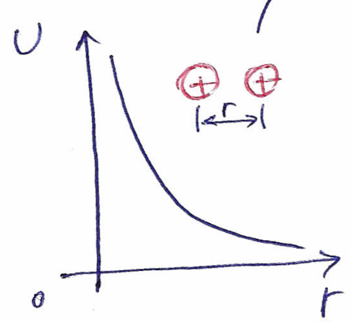
$$W_{a\to b} = \int_a^b \vec{F}(r)\cdot d\vec{r} = \int_{r_a}^{r_b} \frac{1}{4\pi\varepsilon_0}\frac{q q_0}{r^2}\,dr =$$

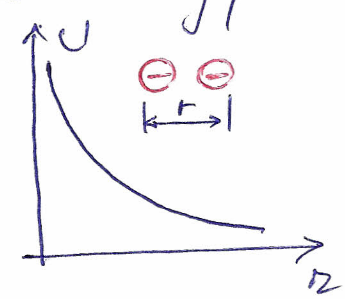
$$= \frac{q q_0}{4\pi\varepsilon_0}\left(\frac{1}{r_a} - \frac{1}{r_b}\right)$$

$$= U_a - U_b$$

valid for any other complex path (not simply radial) (because conservative force)

$\Rightarrow$ the potential energy is: $\boxed{U = \frac{1}{4\pi\varepsilon_0}\frac{q q_0}{r}}$



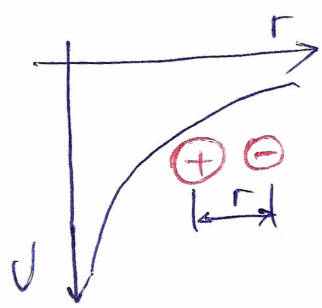


Same sign of charge

$U \to \infty$ when $r \to 0$

$U \to 0$ when $r \to \infty$

$\Rightarrow$ repulsion (more stable energetically position when $r \nearrow$)



opposite sign charges $\Rightarrow$ attraction (potential energy decreases when $r \searrow$)

$U \to -\infty$ when $r \to 0$

$U \to 0$ when $r \to \infty$

Obs (1) Potential energy is allways defined with some refference point where $U=0$. ($r=\infty$ for charges of same sign)

From

$$W_{a\to b} = \frac{q q_0}{4\pi\varepsilon_0}\left(\frac{1}{r_a} - \frac{1}{r_b}\right) = U_a - U_b$$

$$\text{if } r_b = \infty \implies W_{a\to\infty} = U_a$$

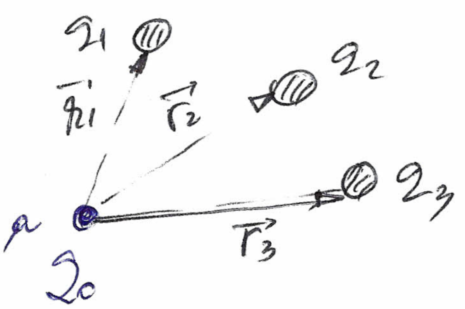
The potential energy U in a certain point is the work that would be done on a test charge q_0 by the field of q to move the test charge from that point to infinity.

(2) • The potential energy U defined above is a shared property of the two charges q and q_0 (likewise, if a mass m is at height h above the Earth's surface, the gravitational potential energy is a shared property between the mass m and Earth)

• The above concepts if the charge q_0 is outside a spherical charge distribution with total charge Q. (see Gauss' law).

Electric potential with several point charges

Suppose that the electric field $\vec{E}$ in which charge q_0 moves is caused by several point charges $q_1, q_2, \dots$ at distances $r_1, r_2, \dots$ from q_0 (e.g. q_0 could be a positive ion moving in the presence of other ions)



The electric field felt by q_0 is the vector sum of fields $\vec{E}_i$ produced by each charge q_i

The total work done on q_0 during any displacement is the sum of contributions from individual charges

For each couple q_i, q_0

$$W_{a\to b}^{(i)} = \frac{q_0 q_i}{4\pi\varepsilon_0}\left(\frac{1}{r_a}-\frac{1}{r_b}\right) = U_a - U_b$$

then we conclude that the potential energy associated with the test charge q_0 at point a is the algebric sum (not a vector sum)

$$\boxed{U = \frac{q_0}{4\pi\varepsilon_0}\left(\frac{q_1}{r_1}+\frac{q_2}{r_2}+\dots\right) = \frac{q_0}{4\pi\varepsilon_0}\sum_i \frac{q_i}{r_i}}$$

↳ potential energy associated to the presence of a test charge q_0 in the field $\vec{E}$ produced by $q_1, q_2, \dots$

We can also write the potential energy involved in assembling these charges. We can start with charges $q_1, q_2, q_3, \dots$ all separated initially by infinite distances and bring them together so at the distance between q_i and q_j is r_{ij} ⟹

the total potential energy is the sum of pair potential energy of interaction of each two charges

$$\boxed{U = \frac{1}{4\pi\varepsilon_0}\sum_{i<j}\frac{q_i q_j}{r_{ij}}}$$

Interpreting electric potential energy

$W_{a\to b} = U_a - U_b$ ⟹ The potential energy difference from a to b equals to the work done by the electric force to move the charge from a to b

2 Electric potential

Potential is potential energy per unit charge

We define the potential ~~charge~~ at any point in an electric field as the potential energy U per unit charge associated to the test charge q_0

$$\boxed{V=\frac{U}{q_0}} \quad \text{or} \quad \boxed{U = q_0 V}$$

↳ scalar quantity

$[V]_{SI} = 1V$ (Volt)

$$\boxed{1V = \frac{1J}{1C}}$$

From $\frac{W_{a\to b}}{q_0} = -\frac{\Delta U}{q_0} = -\left(\frac{U_b}{q_0}-\frac{U_a}{q_0}\right) = -(V_b-V_a) = V_a - V_b$

$\boxed{V_{ab}=V_a-V_b}$ ⇒ is called the potential of a with respect to b
(sometimes called potential difference between a and b)

⇒ V_{ab}, the potential of a with respect to b, equals the work done by the electric force when a unit charge moves from a to b.

Calculating electrical potential

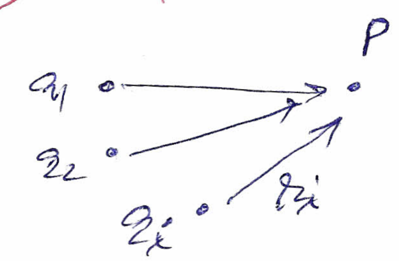
• for a single point q ; $V = \frac{U}{q_0}$ ⇒

$$\boxed{V=\frac{1}{4\pi\varepsilon_0}\frac{q}{r}}$$

• for a collection of charges: q_i

$$\boxed{V=\frac{1}{4\pi\varepsilon_0}\sum_i \frac{q_i}{r_i}}$$

→ scalar sum of potentials due to each charge q_i



• continuous charge distributions, along a line, over a surface, through a volume, we divide the charge into elements dq and the sum becomes an integral:

$$\boxed{V = \frac{1}{4\pi\varepsilon_0} \int \frac{dq}{r}}$$

Potential due to a continuous distribution of charge

Electric potential from electric field

$$\frac{W_{a\to b}}{q_0} = V_a - V_b = \frac{\int_a^b \vec{F}\cdot d\vec{l}}{q_0} = \int_a^b \vec{E}\cdot d\vec{l}$$

$$\Rightarrow \boxed{V_a - V_b = \int_a^b \vec{E}\cdot d\vec{l} = \int_a^b E\cos\phi \, dl}$$

Units

① The unit of V is volt. From the above equation an alternative unit of E, defined before as $\frac{N}{C}$ would be $\frac{Volt}{m}$ $\Rightarrow$ $1\frac{V}{m} = 1\frac{N}{C}$

In practice, the volt/meter is the usual unit for electric field.

② Electron-volts uses the magnitude of the charge e of an electron to define a new unit of energy useful especially in atomic and nuclear physics

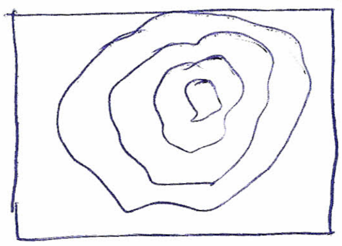
$U_a - U_b = q(V_a - V_b) = q V_{ab}$ if $q = e = 1{,}6\cdot 10^{-19}C$ and the potential difference is $V_{ab} = 1V$, the change in energy would be an electron volt

$$1eV = (1{,}6\cdot 10^{-19}C)(1V) = 1{,}6\cdot 10^{-19}J$$

Equipotential surfaces

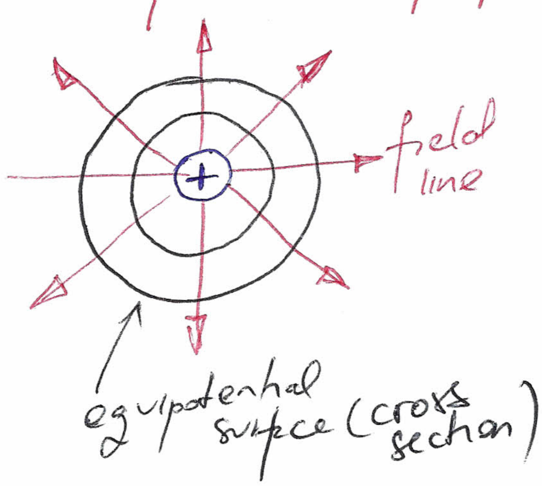
Field lines help us to visualise electric fields. Similarly, the potential at various points in an electric field can be represented graphically as <u>equipotential surfaces</u>.

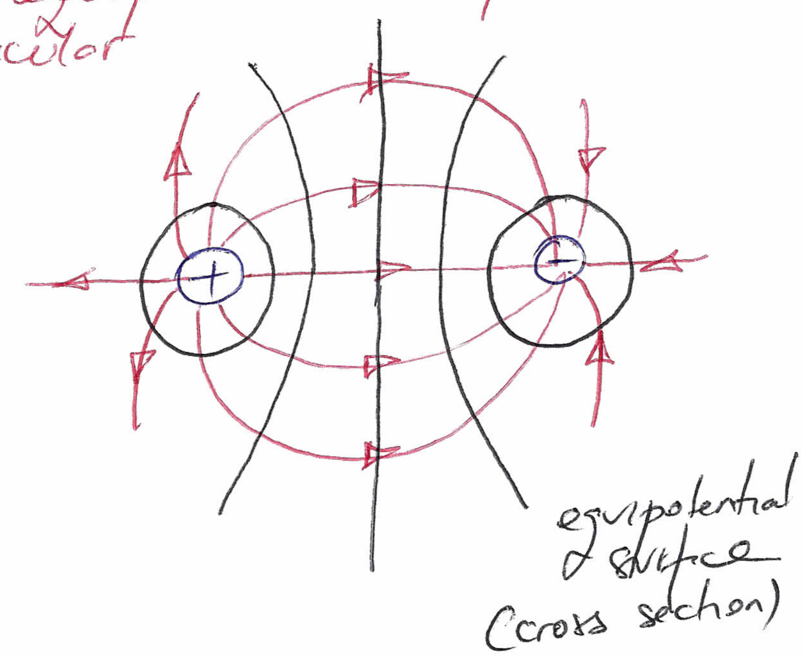
One can use some fundamental idea as <u>topographic maps</u> where contour lines are drawn through points which have same elevation (same potential gravitational energy for a mass m moved along that contour)



By analogy, an <u>equipotential surface</u> is a 3D (map) surface on which the electrical potential V is the same at every point.

! Field lines and equipotential surfaces are allways mutual perpendicular





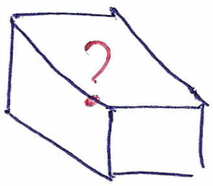
<u>Obs</u> E need not to be constant over an equipotential surface. V has to be constant.

GAUSS LAW

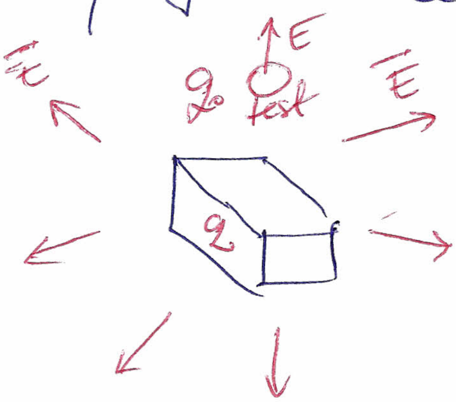
In physics, an important tool for simplifying problems is the symmetry properties of the system! Gauss law is a part of the key to using symmetry considerations to simplify electric field calculations.

[1] Charge and electric flux

Pb: How can you measure the charge inside a box without opening it? Using a test charge q_0 outside to probe the inside charge. By measuring the force $\vec{F}$ experienced by the test charge at different positions you can make a 3D

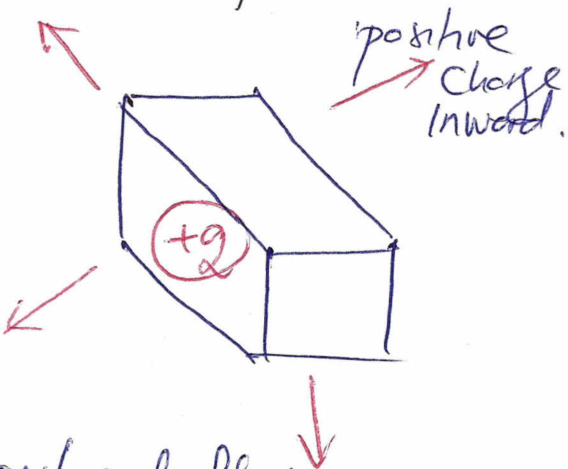


map of the electric field $\vec{E} = \frac{\vec{F}}{q_0}$ outside the box.

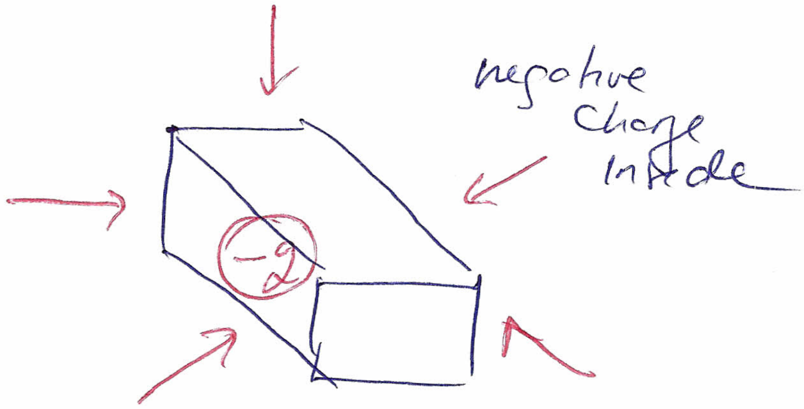


From the details of the map, you can find the exact value of the point charge inside the box!

Electric flux



outward flux

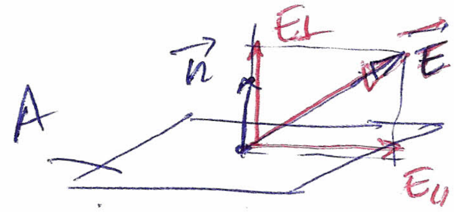


inward flux

Uniform field flux

Mathematically, one can write the electric flux of a vector $\vec{E}$ across a flat surface $\vec{A}$ as the scalar product

$$\Phi_E = \vec{E} \cdot \vec{A} = EA\cos\alpha = E_\perp A$$



$$\vec{A} = A\vec{n}$$

$$\Phi_{SI} = 1\,\frac{N \cdot m^2}{C}$$

For a closed surface we allways chose $\vec{n}$ to be outward.

$$\Rightarrow \text{outward electric flux} \Leftrightarrow \Phi_E > 0$$
$$\text{Inward electric flux} \Leftrightarrow \Phi_E < 0$$

Flux of a non-uniform electric field

$\vec{E}$ varies from point to point over the area $\vec{A}$. Then, we devide $\vec{A}$ into many small elements $d\vec{A}$, each of them having a unit vector $\vec{n}$ perpendicular to it and a vector area $d\vec{A} = \vec{n}\,dA$. We calculate the electric flux through an element and integrate the results to obtain the total flux

$$\boxed{\Phi_E = \oint\!\!\oint_A E\cos\varphi\, dA = \oint\!\!\oint_A E_\perp\, dA = \oint\!\!\oint_A \vec{E} \cdot d\vec{A}}$$

general definition of the electric flux.

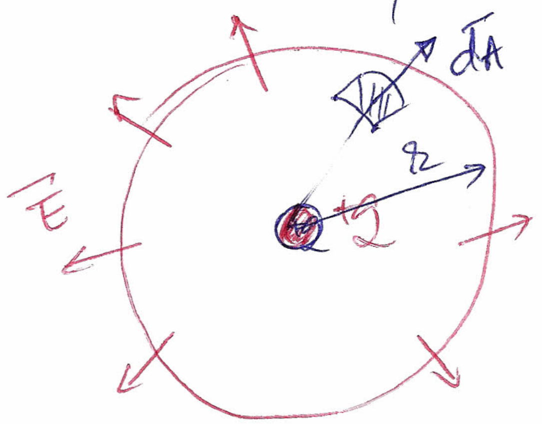
2 Gauss law

The Gauss law is an alternative to the Coulomb law, being completely equivalent to this. It was formulated by Carl Friedrich Gauss (1777–1855), famous mathematician.

The total electric flux through a closed surface is equal to the total (net) electric charge inside the surface, divided by ε_0.

$$\boxed{\Phi_E = \oiint \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\varepsilon_0}}$$

Obs: The Gaussian surfaces are imaginary closed surfaces.



Gaussian spherical surface around a positive charge.

$$\Phi_E = \oiint_A E(r)\, dA = E(r) \oiint_A dA$$

$$= E(r) \cdot 4\pi r^2 = \frac{q}{\varepsilon_0} > 0$$

$$\Rightarrow \boxed{E_+(r) = \frac{q}{4\pi\varepsilon_0 r^2}}$$

similar result obtained for inward flux (field) of a negative charge.

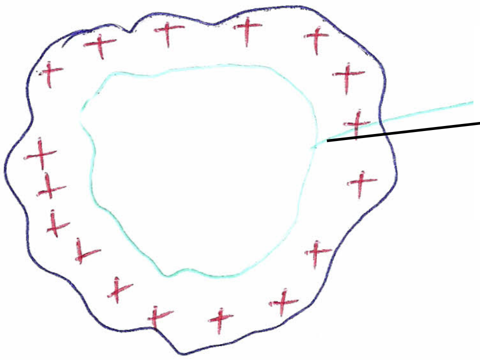
$$\Phi_E = -\frac{q}{\varepsilon_0} < 0$$

$$E_-(r) = -\frac{q}{4\pi\varepsilon_0 r^2}$$

Applications of Gauss law

Gauss' law is valid for any distribution of charges and for any closed surface. If we know the charge distribution and if we have enough symmetry to evaluate the integral in Gauss's law, we can find the field. Or, vice-versa, if we know the field, we can use Gauss's law to find the charge distribution, such as charges on conducting surfaces.

Remark: Under electrostatic conditions (charges not in motion), any excess charge on a solid conductor resides entirely on the conductor's surface, not in the interior of material.

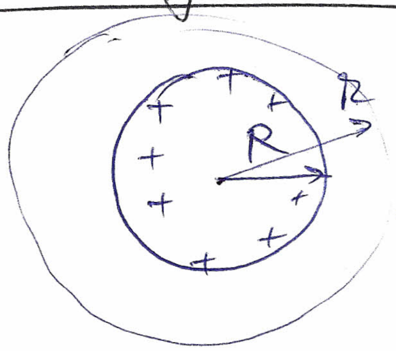


cross-section of Conductor.

↳ valid for a solid conductor without cavities inside.

Because $\vec{E} = 0$ everywhere on the Gaussian surface inside of a conductor, from the Gauss law $\Rightarrow$ $Q_{encl} = 0$.

Field of a charged conducting sphere of radius R



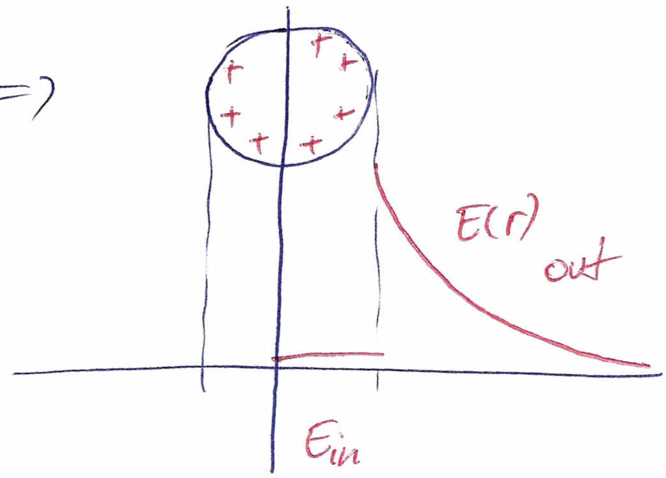
→ Gaussian surface at r

$$\oint_A \vec{E}\,d\vec{A} = \frac{Q_{encl}}{\varepsilon_0} \Rightarrow$$

$$E\,4\pi r^2 = \frac{Q}{\varepsilon_0} \quad \text{for } r > R$$

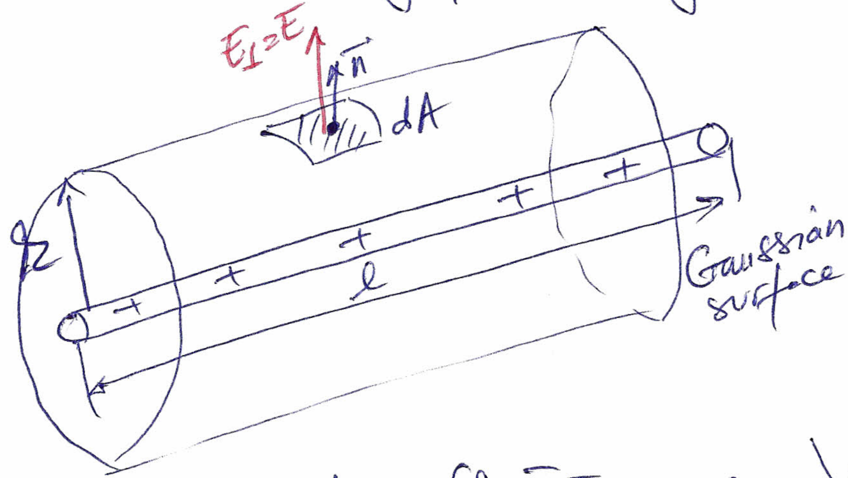
if $r < R$ $\quad Q_{encl} = 0 \Rightarrow E = 0$ (choosing a Gaussian surface inside the sphere).

$\Rightarrow$



Field of a uniform line charge

charge/unit length $= \lambda$



$$\vec{E}\cdot\vec{n} = E_\perp$$

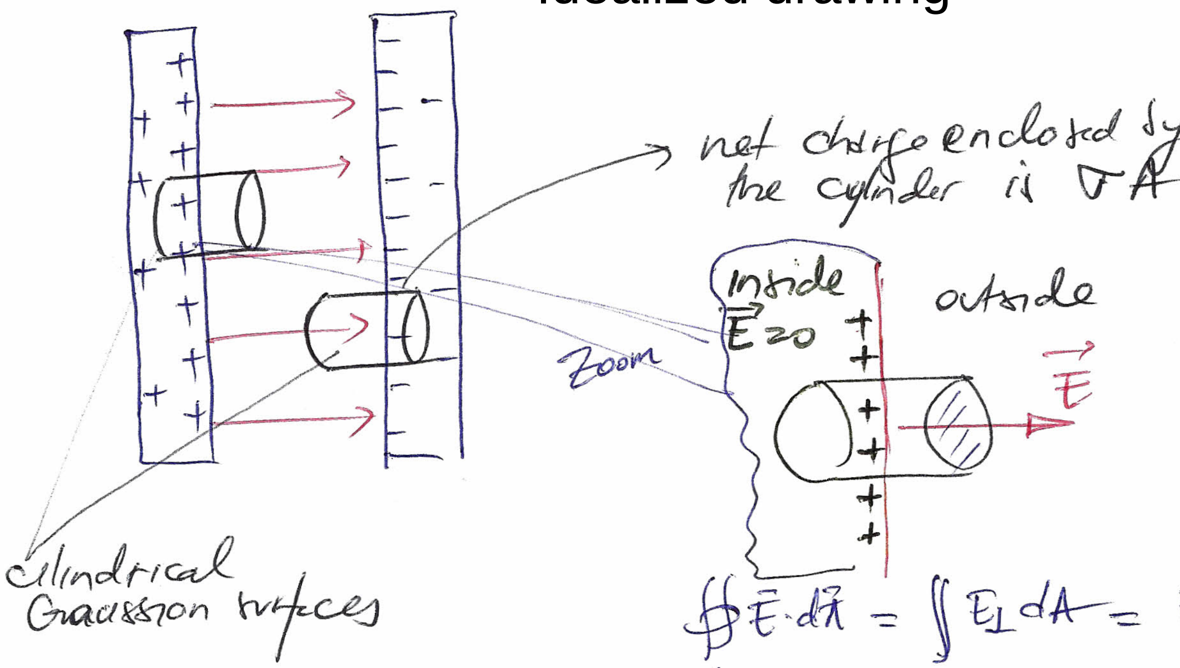
$$\Phi_E = \oint \vec{E}\,d\vec{A} = 2\pi r \cancel{l} E = \frac{Q_{encl}}{\varepsilon_0} = \frac{\lambda \cancel{l}}{\varepsilon_0}$$

$$\Rightarrow \boxed{E = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{r}}$$

Field between oppositelly charged parralel Conducting plates

The surface charge densities are $+\sigma$ and $-\sigma$

Idealized drawing



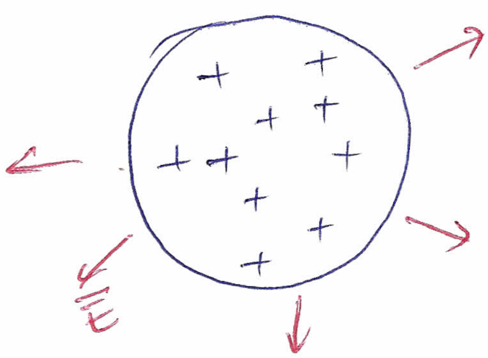
$$\oint_A \vec{E} \cdot d\vec{A} = \int E_\perp dA = EA$$

$$\Rightarrow EA = \frac{\sigma A}{\varepsilon_0} \Rightarrow \boxed{E = \frac{\sigma}{\varepsilon_0}}$$

in ANY POINT BETWEEN PLATES

field between oppositelly charged Conductive Infinite plates.

Field of an uniformly charged sphere



Q – is distributed uniformly over the volume of a sphere with radius R

From the symmetry, $\vec{E}$ is radial at every point on the Gaussian surface, so

$E_\perp = E$;

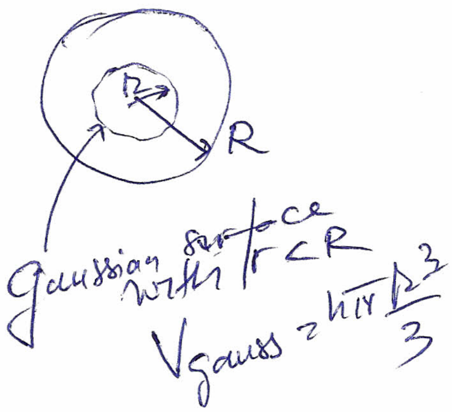
The flux across a Gaussian surface with a spherical shape of radius ~~R~~ r is:

$$\Phi_E = \oiint_A \vec{E}\, d\vec{A} = E(r)\cdot 4\pi r^2$$

Gauss law $\quad \Phi_E = \dfrac{Q_{encl}}{\varepsilon_0}$

→ <u>if $r < R$</u> (inside the sphere)

$$Q_{encl} = \frac{Q}{V}\cdot V_{gauss} = \frac{Q}{\cancel{\frac{4\pi R^3}{3}}}\cdot \cancel{\frac{4\pi r^3}{3}} = Q\,\frac{r^3}{R^3}$$



$$\Rightarrow E(r)\cdot 4\pi \cancel{r^2} = \frac{Q}{\varepsilon_0}\cdot\frac{r^{\cancel{3}}}{R^3}$$

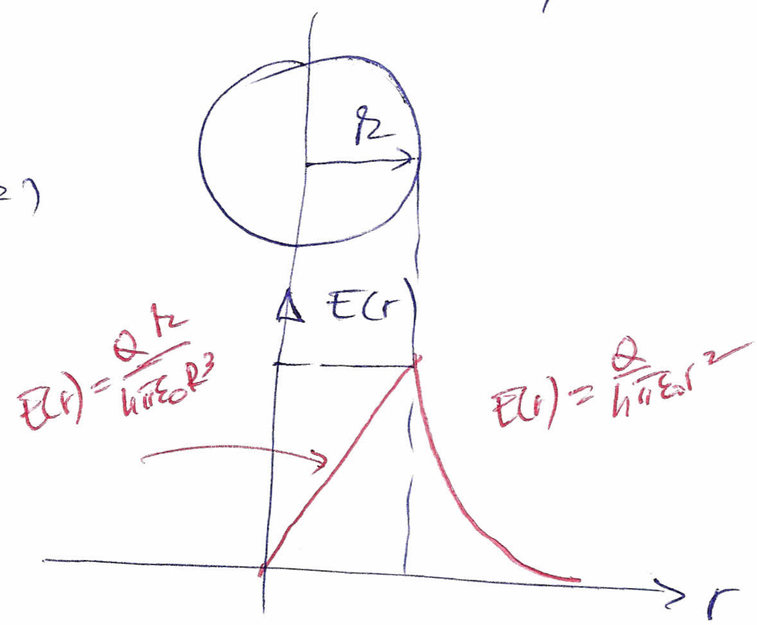
$$\boxed{E(r) = \frac{Q}{4\pi\varepsilon_0}\,\frac{r}{R^3} \sim r}$$

→ <u>if $r > R$</u> outside the sphere

$$Q_{encl} = Q \quad \Rightarrow$$

$$E(r)\cdot 4\pi r^2 = \frac{Q}{\varepsilon_0} \Rightarrow \boxed{E(r) = \frac{Q}{4\pi\varepsilon_0 r^2}}$$

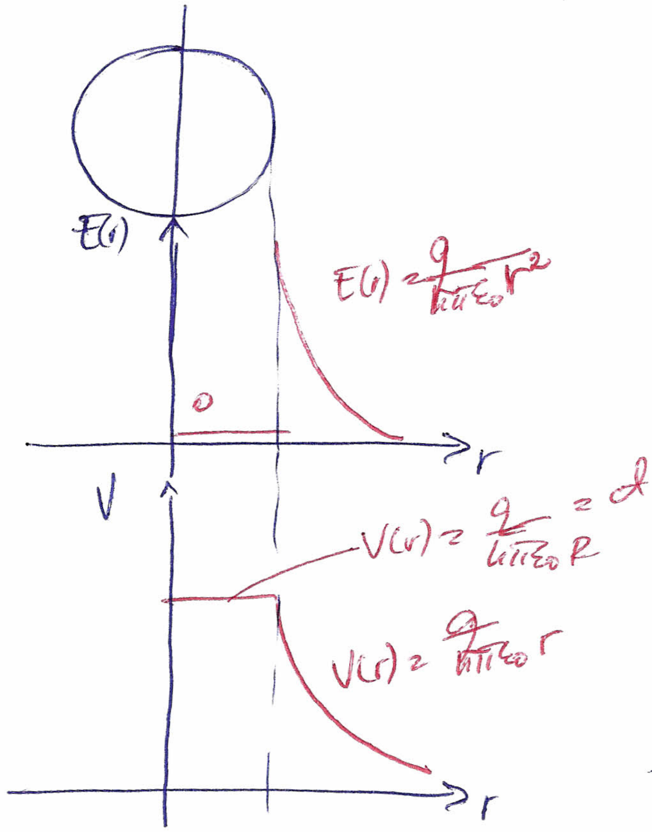
⇒



3 Calculating electrical potential

1 Charged conducting sphere (q)

From Gauss's law, we calculated E(r) for $r<R$, $r>R$



$$V(r) = \int_0^\infty \vec{E}(r)\, d\vec{r}$$

inside: $r<R$ $\quad E(r)=0$

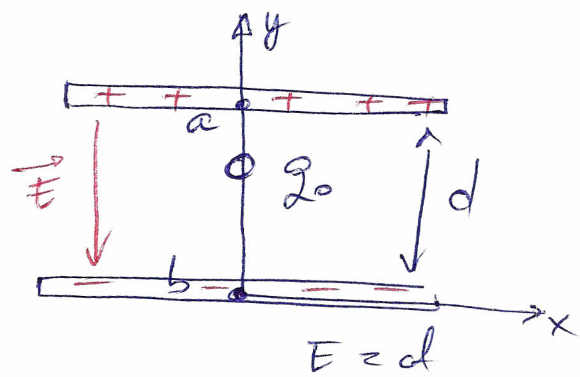
$$\Rightarrow V(r) = \int_0^\infty \vec{E}(r)d\vec{r} = \cancel{\int_0^R \vec{E}(r)\,d\vec{r}} +$$

$$\int_R^\infty \vec{E}(r)d\vec{r} = \frac{q}{4\pi\varepsilon_0}\frac{1}{R} = ct$$

outside: $r>R$

$$\boxed{V(r) = \frac{q}{4\pi\varepsilon_0}\frac{1}{r}}$$

2 Oppositely charge plates



$$V_a - V_b = \int_a^b \vec{E}(r)d\vec{r} = Ed$$

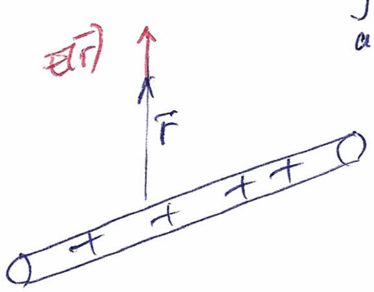
$$\Rightarrow \boxed{E = \frac{V_a - V_b}{d} = \frac{V_{ab}}{d}}$$

③ Infinite line charged conductor or conducting cillyndro

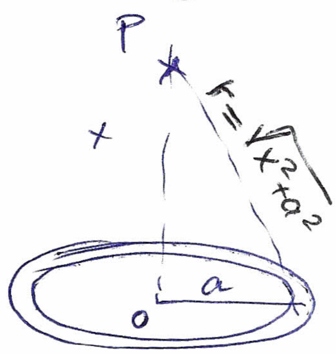
$\Rightarrow \lambda$ = linear charge density (charge/unit length)

From Gauss, we find that the electric field has only radial component: $E(r) = \dfrac{\lambda}{2\pi\varepsilon_0 r}$

$$V_a - V_b = \int_a^b \vec{E}(r)\,d\vec{r} = \int_{r_a}^{r_b} \frac{\lambda}{2\pi\varepsilon_0 r}\,dr = \frac{\lambda}{2\pi\varepsilon_0} \ln\frac{r_b}{r_a}$$



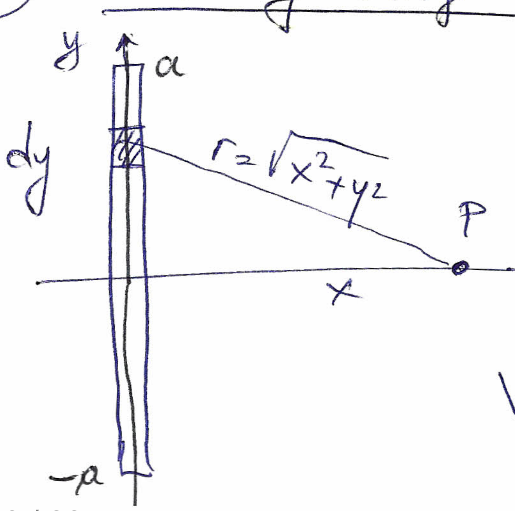
④ Ring of charge (Q) Electric charge distributed uniformly around a ring of radius a. Find the potential at a point P on the ring axis, at a distance x from the center of the ring



$$V = \frac{1}{4\pi\varepsilon_0} \int \frac{dq}{r} = \frac{1}{4\pi\varepsilon_0} \frac{1}{\sqrt{x^2+a^2}} \int dq$$

$$\boxed{V(x) = \frac{1}{4\pi\varepsilon_0} \frac{Q}{\sqrt{x^2+a^2}}}$$

⑤ Line of charge



The element of charge is

$$dQ = \frac{Q}{2a}\,dy$$

$$\Rightarrow V = \frac{1}{4\pi\varepsilon_0} \int \frac{dq}{r} = \frac{Q}{4\pi\varepsilon_0 \cdot 2a} \int_{-a}^{a} \frac{dy}{\sqrt{x^2+y^2}}$$

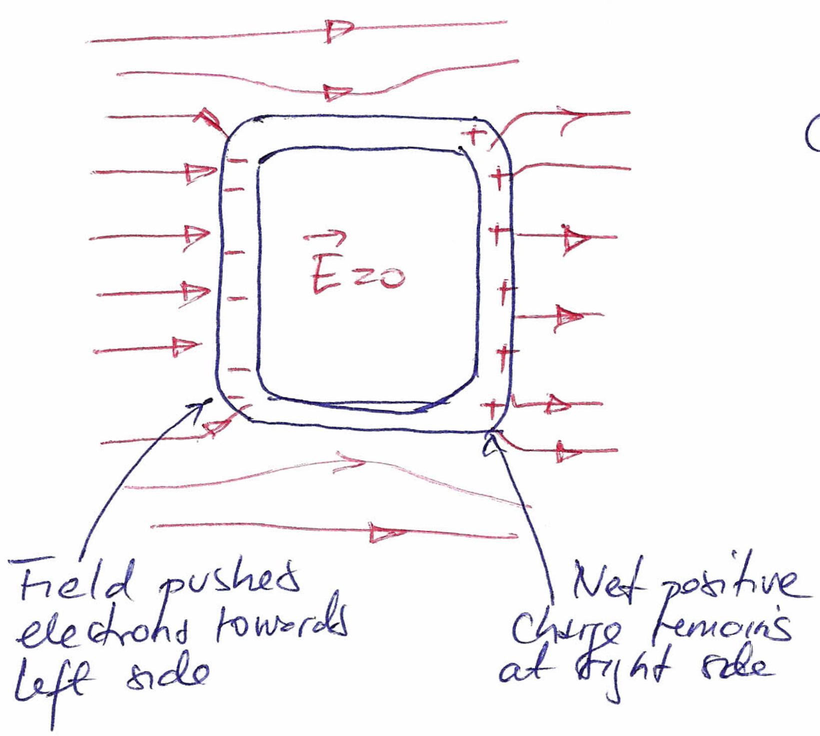
$$V = \frac{1}{4\pi\varepsilon_0} \frac{Q}{2a} \ln\left(\frac{\sqrt{a^2+x^2}+a}{\sqrt{a^2+x^2}-a}\right)$$

Charges on conductors

Electrostatic shielding

i.e. protecting of sensitive electronic instruments from stray electric fields.

We surround the instrument with a conducting box, or we line the walls, floor or ceiling or room with a conducting material such as a sheet of copper. The external electric field redistributes the free electrons in the conductor, leaving a net outer surface on some regions and a negative charge in the others. This charge distribution causes an additional electric field such as the total field at every point inside the box is zero, as states the Gauss-law. The charge distribution on the box also alters the shapes of the field lines near the box, as the figure shows. Such setup is called a <u>FARADAY CAGE</u>



Conducting box imersed in external electric field

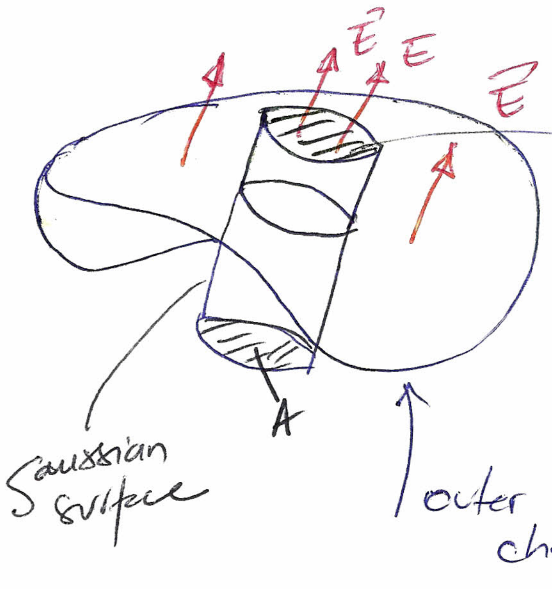
<u>EX</u> ① one of safest places to be in a lightning storm is inside an automobile, the charge of lightning remains on the metal skin of automobile, no electric field being produced inside

② <u>Shielding of electromagn fields</u>

no/bad signal inside metallic cages (radio, TV, phone cells)

$\Rightarrow$ outward antenas necessary.

Field at the surface of a conductor



σ – surface charge density

$\to E_\perp = E$

OUTSIDE

$E_\perp A = \dfrac{\sigma A}{\varepsilon_0}$; $\boxed{E_\perp = \dfrac{\sigma}{\varepsilon_0}}$

INSIDE $E = 0$

$\vec{E}$ is allways $\perp$ to the surface of the conductor,
In general, σ may vary with the point of the surface

Equipotential and conductors

When all charges are in rest ① the surface of a conductor is allways an equipotential surface.

② all the points in the interior of a conductor are at the same potential

③ When a cavity within a conductor contains no charge, the entire cavity is an equipotential region and there is no surface charge anywhere on the surface of the cavity.

5 Potential gradient

Electric field and potential are closely related.

$$\left.\begin{aligned} V_a - V_b &= \int_a^b \vec{E}\cdot\vec{dl} \\ V_a - V_b &= \int_b^a dV = -\int_a^b dV \end{aligned}\right\} \Rightarrow$$

$$\int_a^b \vec{E}\,\vec{dl} = -\int_a^b dV \quad \Rightarrow \quad -dV = \vec{E}\cdot\vec{dl}$$

but: $\vec{E} = E_x\vec{i} + E_y\vec{j} + E_z\vec{k}$

$$\vec{dl} = dx\,\vec{i} + dy\,\vec{j} + dz\,\vec{k}$$

$$\vec{E}\,\vec{dl} = E_x dx + E_y dy + E_z dz \quad \Rightarrow$$

$$-dV = E_x dx + E_y dy + E_z dz$$

if $V = V(x, y, z) \quad \Rightarrow$

$$E_x = -\frac{\partial V}{\partial x} \qquad E_y = -\frac{\partial V}{\partial y} \qquad E_z = -\frac{\partial V}{\partial z}$$

$$\Rightarrow \vec{E} = -\left(\frac{\partial V}{\partial x}\vec{i} + \frac{\partial V}{\partial y}\vec{j} + \frac{\partial V}{\partial z}\vec{k}\right)$$

$$\boxed{\vec{E} = -\nabla V(x,y,z)}$$

with ∇ (nabla) gradient operator

$$\nabla = \frac{\partial}{\partial x}\vec{i} + \frac{\partial}{\partial y}\vec{j} + \frac{\partial}{\partial z}\vec{k}$$

$\boxed{\text{Electric field} = -\text{gradient of potential}}$

at each point $\vec{E}$ is in the direction in which V decreases more rapidly and is allways perpendicular on the equipotential surface through the point.

Obs: If $\vec{E}$ is radial with respect to a point or an axis and r is the distance from the point or the axis,

$$\Rightarrow \boxed{E_r = -\frac{\partial V}{\partial r}}$$

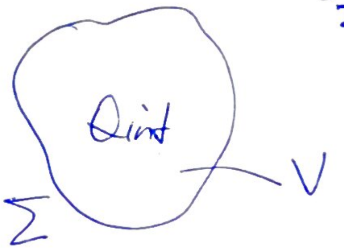
EXAMPLES:

Potential and field of a point charge

$$V(r) = \frac{q}{4\pi\varepsilon_0 r} \qquad E(r) = -\frac{\partial V}{\partial r} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$

Poisson & Laplace equations

Gauss theorem $\oint\!\!\oint_{\Sigma} \vec{E} \cdot d\vec{A} = \frac{Q_{int}}{\varepsilon_0}$



Σ = surface including the volume V

Gauss - Ostrogradski theorem:

$$\oint\!\!\oint_{\Sigma} \vec{E} \cdot d\vec{A} = \int_{V} \nabla \cdot \vec{E} \, dV$$

transforms a surface integral in volume integral

Divergence

$$\vec{\nabla} \cdot \vec{E} = \mathrm{div}\, \vec{E} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}$$

divergence of $\vec{E}$ vector

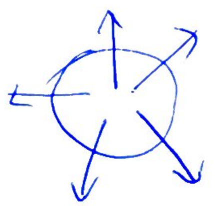
scalar product between $\vec{\nabla}$ and $\vec{E}$

$$\begin{cases} \vec{\nabla} = \frac{\partial}{\partial x}\vec{i} + \frac{\partial}{\partial y}\vec{j} + \frac{\partial}{\partial z}\vec{k} \\ \vec{E} = E_x\vec{i} + E_y\vec{j} + E_z\vec{k} \end{cases}$$

nabla operator

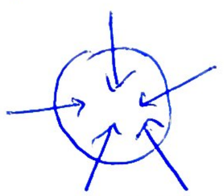
vector field $\vec{E}$

The divergence measures the expansion or the contraction of a vector field.



source

$\mathrm{div}\, \vec{E} > 0$



sink

$\mathrm{div}\, \vec{E} < 0$

describes the net flow going in vs. going out for a vector field. In absence of creation or destruction of matter the density within a region can change only by having it flow into or out of the region. $\longrightarrow$ principle of continuity fundamental in Physics

$$\Rightarrow \oiint \vec{E} \cdot d\vec{A} = \int_V \nabla \cdot \vec{E} \, dV = \frac{q_{int}}{\varepsilon_0} = \frac{1}{\varepsilon_0} \int \rho \, dV$$

$\rho = \frac{dQ}{dV}$ = charge density

$$\Rightarrow \boxed{\nabla \cdot \vec{E} = \rho / \varepsilon_0}$$

Gauss law for the electric field in the differential formulation

because $\vec{E} = -\nabla V$

$$\Rightarrow \vec{\nabla} \cdot (\nabla V) = -\rho/\varepsilon_0 \quad \Leftrightarrow \boxed{\nabla^2 V = -\rho/\varepsilon_0}$$

Poisson equation

$$\Rightarrow \boxed{\Delta V = -\rho/\varepsilon_0}$$

$$\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$ = Laplace operator (2nd order differential operator)

Knowing $\rho(x,y,z)$, by solving (numerically, analytically) the Poisson equation one can get the electric potential $V(x,y,z)$.

If $\rho(\vec{r}) = \rho(x,y,z) = 0 \Leftrightarrow$ no charge density source

$$\Rightarrow \boxed{\Delta V = 0}$$

Laplace equation $\Leftrightarrow$ 2nd order differential eq + boundary conditions $\Rightarrow V(x,y,z)$