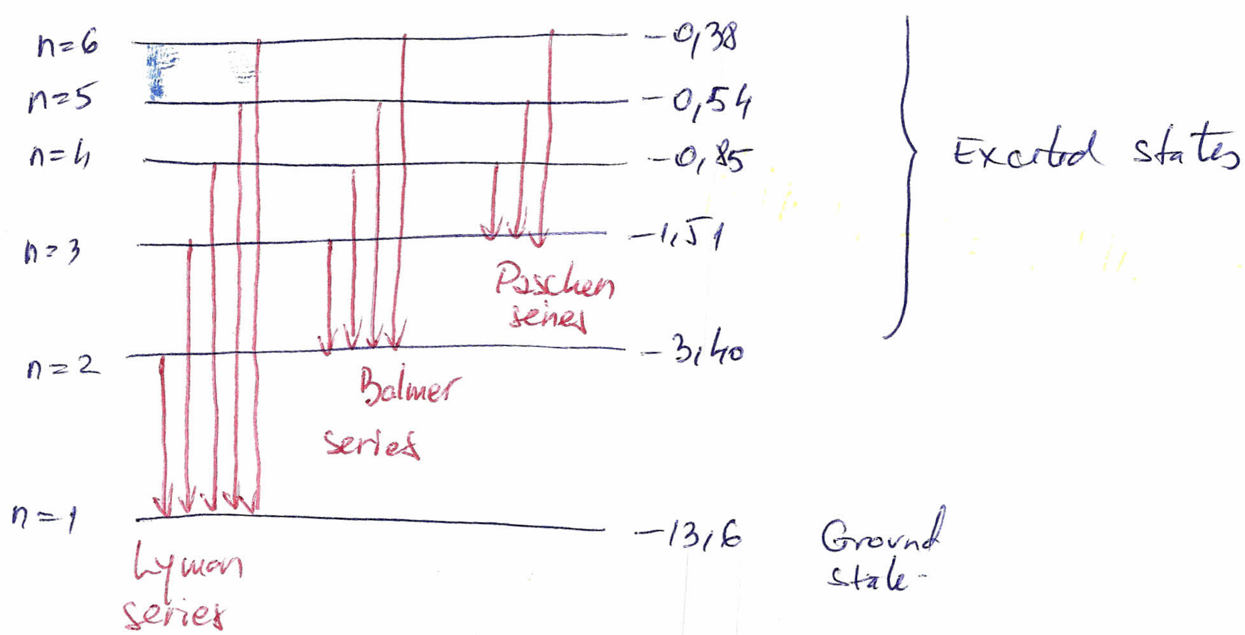
(1) Spectra & energy levels [PLANCK] [BOHR]

When light is spread out into its separate colours this is called spectrum. The light from a tungsten filament lamp produces a continuous spectrum. If the light is from a gas discharge tube, we observe a line spectrum when the light is passed through a slit, i.e. only certain lines of distinct colours are present, separated by gaps. These spectra are due to the emission of energy and are so-called emission spectra

(a) Explain the continuous spectrum of the light emitted by the heated W filament.

(b) Explain the emission spectrum of hydrogen, using the Bohr model and the energy diagram from the figure below, calculating the corresponding ν and λ for Lyman, Balmer, Paschen



(a) Continuous spectra of electromagnetic waves emitted by a heated body.

Any body heated at a temperature T emits radiation. The body contains atoms, and consequently electrons = negative charges. When the atoms vibrate at a certain T => accelerated movement of electrons => emission of electromagnetic waves, accordingly to the Maxwell theory. of each quantum harmonical oscillator

Following the hypothesis of Planck (1900) that an harmonical oscillator can only emit or absorb energy in quanta (steps) proportional with the frequency $\Delta E = h\nu$ (energy of quantum oscillator is quantified) one gets after a statistical calculation that the average energy of an oscillator is:

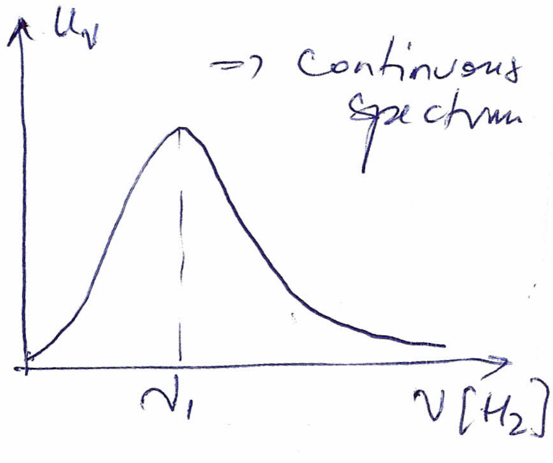
$$\langle E \rangle = \frac{h\nu}{e^{h\nu/kT} - 1}$$

by multiplying this with the number of modes (vibration modes) possible / unit volume within $[\nu, \nu + d\nu]$

$$\Rightarrow g(\nu) = \frac{8\pi\nu^2}{c^3}$$

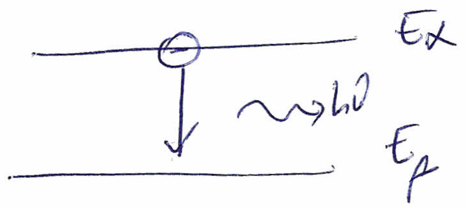
we get: the spectral density:

$$u_\nu = \frac{8\pi\nu^2}{c^3} \frac{h\nu}{e^{h\nu/kT} - 1}$$



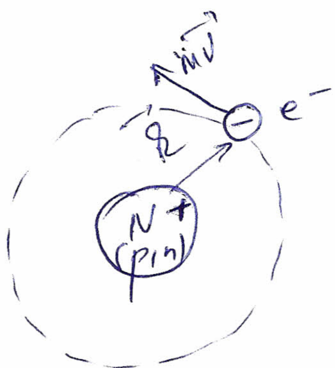
=> Continuous spectrum

(b) Line spectra can be explained using the Bohr model and Bohr's postulates on photon's emission. Each coloured line in an emission spectrum arises when an electron in an atom drops from one excited energy level to a lower energy level and a photon is emitted



$$\boxed{h\nu_{\alpha\beta} = E_\alpha - E_\beta}$$

The Bohr's model: (H^+ atom)



$\vec{L} = \vec{r} \times m\vec{v}$ – quantized

$L = mrv = n\hbar$

$$\frac{mv^2}{r} = \frac{e^2}{4\pi\varepsilon_0}\frac{1}{r^2} \Rightarrow$$

centripetal force

$$r_n = \frac{4\pi\varepsilon_0\hbar^2}{me^2} \cdot n^2$$

$n = 1 \Rightarrow r_1 = \frac{4\pi\varepsilon_0\hbar^2}{me^2}$

$m = 9{,}1\cdot10^{-31}\,kg$

$r_1 = 0{,}529\,Å$ (1^{st} Bohr radius)

The energy:

$$\left.\begin{aligned} E &= K + U = \frac{1}{2}mV_n^2 - \frac{e^2}{4\pi\varepsilon_0 r_n} \\ V_n &= \frac{n\hbar}{mr_n} \end{aligned}\right\} \Rightarrow$$

$$\Rightarrow \boxed{E_n = -\frac{me^4}{8\varepsilon_0^2 h^2}\frac{1}{n^2} = -\frac{13{,}6}{n^2}} \text{ (eV)}$$

$n=1$ $\quad E_1 = -13{,}6$ eV [ground state / fundamental level)

Putting $n = 1, 2, 3, 4, 5, 6, \ldots$ one can construct the energy diagram illustrated before.

Calculating frequencies

$$h\nu_{mn} = E_m - E_n = \left(-\frac{13{,}6}{m^2} + \frac{13{,}6}{n^2}\right) [eV]$$

$$\Rightarrow \nu_{mn} = \frac{13{,}6 \cdot 1{,}6 \cdot 10^{-19}}{6{,}625 \cdot 10^{-34}}\left[\frac{1}{n^2} - \frac{1}{m^2}\right]$$

$$\boxed{\nu_{mn} = 3{,}2840 \cdot 10^{15}\left(\frac{1}{n^2} - \frac{1}{m^2}\right)}$$

The corresponding wavelength $\boxed{\lambda_{mn} = \frac{c}{\nu_{mn}}}$

$c = 3 \cdot 10^8$ m/s

The Balmer series

Transition of n	3→2	4→2	5→2	6→2	7→2
Name	H_α	H_β	H_γ	H_δ	H_ε
Wavelength (nm)	656,3	486,1	434,1	410,2	397,0
Color	Red	Cyan	Violet	Violet	UV

∈ visible part of spectrum of light.

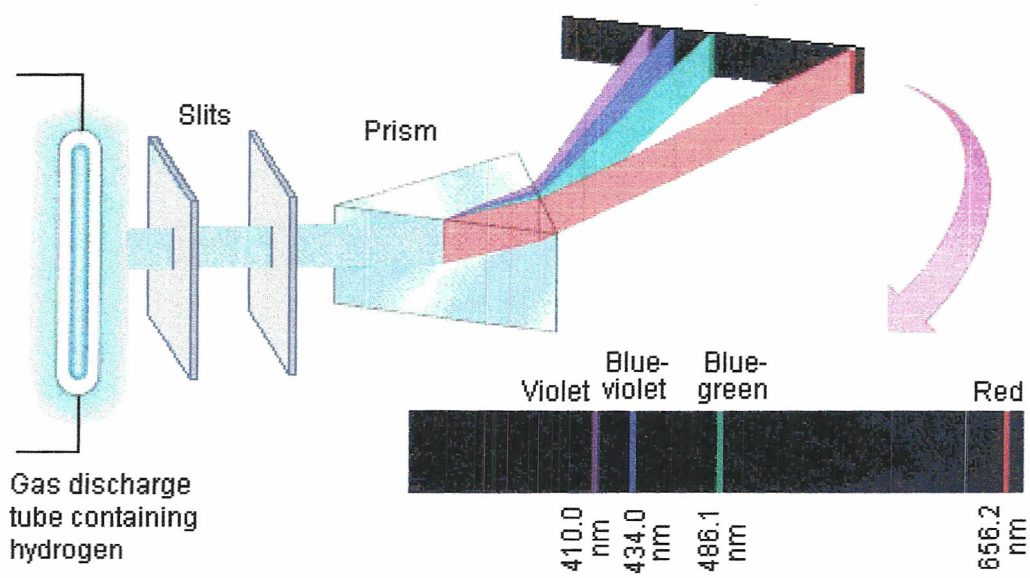
Observations

① Balmer spectrum & astronomy

The Balmer series are particularly useful in Astronomy because the Balmer lines appear in numerous stelar objects. Due to the abundence of the Hydrogen in the Universe, they will be commonly seen and relatively strong as compared to lines from other element.

The Balmer lines can be used to determine radial velocities of emitting stars, galaxies, by measuring the Doppler shift.

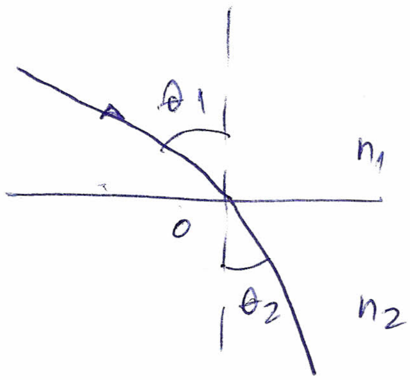
② Experimental setup for Hydrogen spectrum experiment



③ The electrons of an atom are only allowed to have certains values of energy. Each atom has its unique set of discrete (separate) energy levels and hence its own unique line spectrum =>

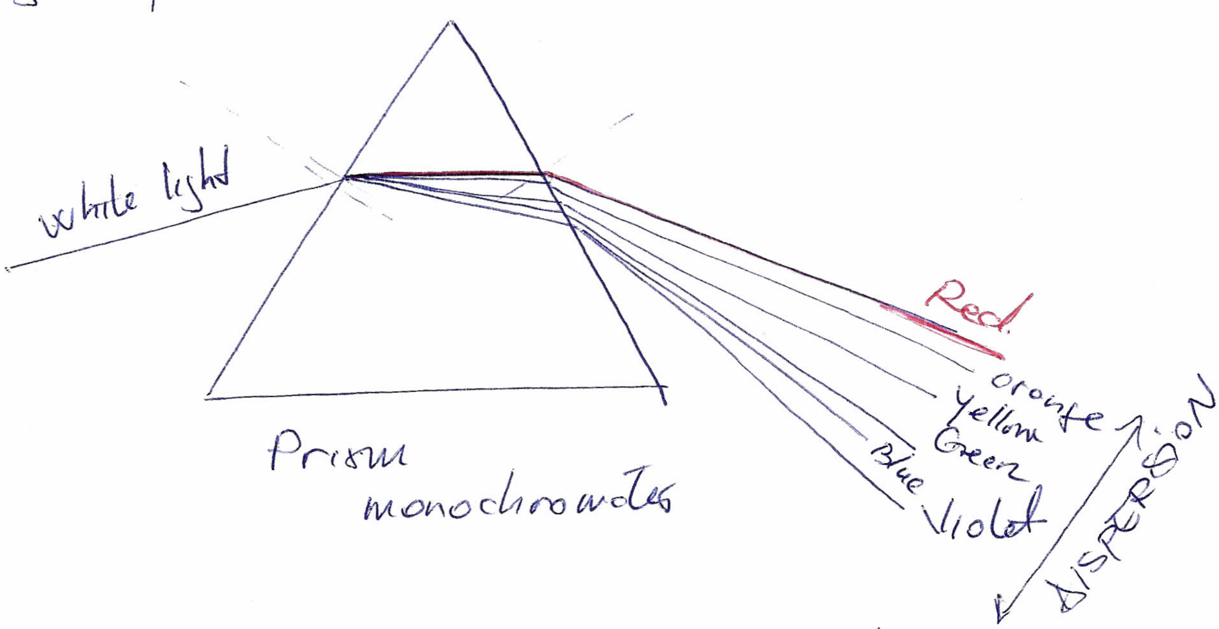
Spectroscopy <=> allows identifying atoms that compose a gas in a gas discharge. ↳ see. emission spectroscopy

(4) Snell's refraction law of monochromatic radiation



$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2} = \frac{n_2}{n_1}$$

=) radiation with larger λ is less deviated by a glass prism



=) spectral decomposition of light.

In case when the "source" contain only discrete lines =) the dispersion spectrum is formed by lines. (see observation 2)

(11) Laser pointer photons

[EINSTEIN]

A LASER pointer with a power output of 5 mW emits red light ($\lambda = 650$ nm). (a) What is the magnitude of the momentum of each photon? (b) How many photons does the LASER pointer emit each second?

(a) $\lambda = 650 \text{nm} = 650 \cdot 10^{-9}$ m

The momentum of the photon is given by:

$$p = \frac{h}{\lambda} = \frac{6{,}626 \cdot 10^{-34} \text{ J}\cdot\text{s}}{6{,}50 \cdot 10^{-7} \text{ m}} = 1{,}02 \cdot 10^{-27} \text{ Kg} \frac{\text{m}}{\text{s}}$$

(b) The energy of a single photon is:

$$E = pc = h\frac{c}{\lambda}$$

$$\Rightarrow E = 1{,}02 \cdot 10^{-27} \text{ Kg} \frac{\text{m}}{\text{s}} \cdot 3 \cdot 10^{8} \text{ m/s} = 3{,}06 \cdot 10^{-19} \text{ J}$$
$$= 1{,}91 \text{ eV}$$

$$\boxed{1 \text{ eV} = 1{,}6 \cdot 10^{-19} \text{ J}}$$

The laser pointer emits energy at a rate of $5 \cdot 10^{-3}$ J/s, so it emits photons at the rate of:

$$\frac{5 \cdot 10^{-3} \text{ J/s}}{3{,}06 \cdot 10^{-19} \text{ J/photon}} = \underline{1{,}63 \cdot 10^{16} \text{ photons}}$$

↳ huge number of photons emitted each second ⇒ discreteness of photons not observed ⇒ apparently continuous flow.

(III) Photoelectric effect

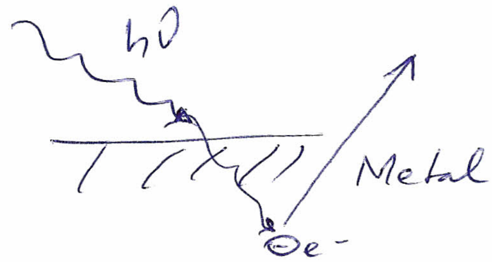
The photoelectric work-function W of alluminium is $W = 4{,}08\,eV$

a) Find the cutoff frequency and the corresponding cutoff length for photoemission from Al.

b) Find the maximum photoelectron kinetic energy and the stopping potential when UV light of wavelength 200 nm illuminates an Al plate.

$$h\nu = W + E_c = W + \frac{mv^2}{2}$$

a) the cutt-off frequency corresponds to $E_c = 0$



$\Rightarrow h\nu_{min} = W \qquad \nu_{min} = \frac{W}{h} = \frac{4{,}08 \cdot 1{,}6 \cdot 10^{-19}}{6{,}626 \cdot 10^{-34}} \quad \frac{J}{J \cdot s}$

$\nu_{min} = 9{,}8 \cdot 10^{14}\ Hz$

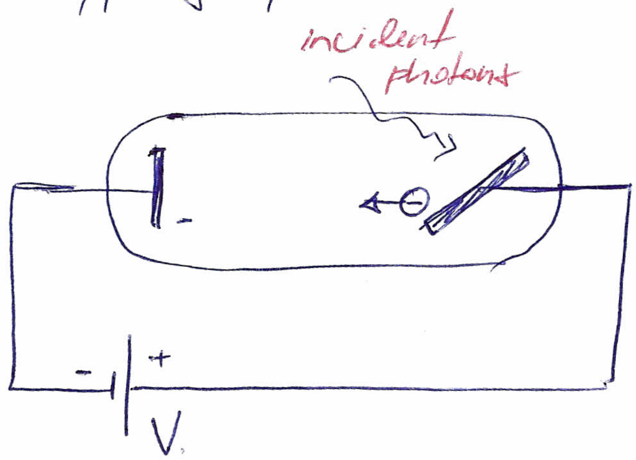
$\lambda_{max} = \frac{c}{\nu_{min}} \Rightarrow \lambda_{max} = \frac{3 \cdot 10^8}{9{,}8 \cdot 10^{14}} = \underline{304{,}1\ nm}$

b) $h\nu = W + E_{c_{max}} \Rightarrow E_{c_{max}} = h\nu - W = \frac{hc}{\lambda} - W$

$E_{c_{max}} = \frac{6{,}626 \cdot 10^{-34} \cdot 3 \cdot 10^8}{200 \cdot 10^{-9}} - 4{,}08 \cdot 1{,}6 \cdot 10^{-19} = 3{,}411 \cdot 10^{-19}\ J$

$= \underline{2{,}13\ eV}$

The stopping potential: corresponds to the complete braking of e^- in the electric field



$\Rightarrow E_{c_{max}} = e V_0$

$\Rightarrow \boxed{V_0 = 2,13 \, eV}$

(III) HOMEWORK (questions)

1/ In what ways do photons resemble other particles such as electrons? In what ways do they differ? Do photons have mass? Do they have electric charge? Can they be accelerated? What mechanical properties do they have?

2/ According to the photon model, light carries its energy in packets called quanta or photons. Why then don't we see a series of flashes when we look at things?

3/ Most black-and-white photographic film (with the exception of some special-purpose films) is less sensitive to red light than blue light and has almost no sensitivity to infrared. How can these properties be understood on the basis of photons?

4/ Human skin is relatively insensitive to visible light, but ultraviolet radiation can cause severe burns. Does this have any- thing to do with photon energies? Explain.

5/ In a photoelectric-effect experiment, which of the following will increase the maximum kinetic energy of the photoelectrons? (a) Use light of greater intensity; (b) use light of higher frequency; (c) use light of longer wavelength; (d) use a metal surface with a larger work function. In each case justify your answer.