

## QUANTUM MECHANICS AND APPLICATIONS

### Multiple choice quiz

1/ The wave function for a particle must be normalizable because:

- a) the particle's charge must be conserved.
- b) the particle's momentum must be conserved.
- c) the particle cannot be in two places at the same time.
- d) the particle's angular momentum must be conserved.
- e) the particle must be somewhere.

2/ A particle is confined to a one-dimensional box on the x-axis between  $x = 0$  and  $x = L$ . The potential height of the walls of the box is infinite. The normalized wave function of the particle, which is in the ground state, is given by:

$$\Psi(x, L) = \sqrt{\frac{2}{L}} \sin \frac{\pi x}{L}; \quad 0 < x < L, \text{ the probability of finding the particle between } x = 0 \text{ and } x = L/3,$$

is closest to:

- (b) 0.22
- (c) 0.26
- (d) 0.28
- (e) 0.20
- (f) 0.24

3/ How does the probability of an electron tunneling through a potential barrier vary with the thickness of the barrier?

- (b) It decreases inversely with thickness.
- (c) It decreases sinusoidally with thickness.
- (d) It decreases linearly with thickness.
- (e) It decreases exponentially with thickness.
- (f) It is independent of the barrier thickness.

4/ Heisenberg's Uncertainty Principle states:

- (a) The more precise a particle's energy can be measured, the less precise its position can be measured.
- (b) A particle's position can be measured exactly.
- (c) A particle's energy can be measured exactly.
- (d) The more precise a particle's momentum can be measured, the less precise its position can be measured.
- (e) The more precise a particle's momentum can be measured, the less precise its energy can be measured.

### Problems

1/ Briefly define the photoelectric effect and give its quantum explanation writing the equation of Einstein and describing the physical signification of each term.

2/ The possible values of energies (energy spectrum) for an electron in an infinite potential well of width  $L=2\text{\AA}$  is given by the equation  $E_n = n^2 \frac{\pi^2 \hbar^2}{2mL^2}$ , with  $n=1,2,3,\dots$ . The corresponding electron wave

functions are  $\Psi_n(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right)$ .

- Calculate and represent graphically (at scale) the first three energy levels (in eV;  $1\text{eV}=1.6 \cdot 10^{-19}\text{J}$ ). Schematically, represent the corresponding wave functions  $\Psi_n(x)$ ,  $n=1,2,3$ .
- Calculate the energy, the frequency and the wavelength of the photon emitted during the transition from the level  $n=2$  to  $n=1$ . ( $m=9.1 \cdot 10^{-31}\text{kg}$ ,  $\hbar = 1.054 \cdot 10^{-34}\text{J}\cdot\text{s}$ )
- Briefly explain the physical origin of the discrete energy spectrum and give at least two other examples of quantum systems with discrete energy levels.
- Give the physical interpretation of the wave function and its square.

3/ (a) Using the energy band concept in crystalline solids, describe the metals, insulators and semiconductors.

(b) At room temperature, pure germanium has an almost completely filled valence band separated by a 0.67-eV gap from an almost completely empty conduction band. It is a poor electrical conductor, but its conductivity increases greatly when it is irradiated with electromagnetic waves of a certain maximum wavelength. What is that wavelength? To what spectral range it belongs?

4/ Accordingly to the Bohr model and to quantum mechanics, the possible values of energies (energy spectrum) for an electron in an atom are given by the equation:  $E_n[\text{eV}] = -\frac{13.6[\text{eV}]}{n^2}$

- Calculate and represent graphically (at scale) the first three energy levels.
- Calculate the energy, the frequency and the wavelength of the photon emitted during the transition from the level  $n=3$  to  $n=2$ .
- Give the physical interpretation of the wave function and its square in quantum mechanics.

5/ Using the phenomenological theory of conduction and the energy band concept in crystalline solids, briefly answer to the following questions:

(a) Indicate the significance of physical quantities in the expression:  $\vec{j} = nq\vec{v} = \sigma\vec{E}$ .

(b) The electrical conductivities of most metals decrease gradually with increasing temperature, but the intrinsic conductivity of semiconductors always *increases* rapidly with increasing temperature. What causes the difference?

6/ (a) Briefly explain the tunnel effect and give one application.

(b) Give the physical interpretation in quantum mechanics of the wave function  $\Psi(\vec{r})$  and its square  $|\Psi(\vec{r})|^2$ .

(c) Are the photons particles or waves? Justify the answer.

(d) Same question about electrons. Enounce the hypothesis of de Broglie.

7/ In what ways do photons resemble other particles such as electrons? In what ways do they differ? Do photons have mass? Do they have electric charge? Can they be accelerated? What mechanical properties do they have?

8/ According to the photon model, light carry its energy in packets called quanta or photons. Why then don't we see a series of flashes when we look at things?

9/ Most black-and-white photographic film (with the exception of some special-purpose films) is less sensitive to red light than blue light and has almost no sensitivity to infrared. How can these properties be understood on the basis of photons?

10/ Human skin is relatively insensitive to visible light, but ultraviolet radiation can cause severe burns. Does this have any- thing to do with photon energies? Explain.

11/ In a photoelectric-effect experiment, which of the following will increase the maximum kinetic energy of the photoelectrons? (a) Use light of greater intensity; (b) use light of higher frequency; (c) use light of longer wavelength; (d) use a metal surface with a larger work function. In each case justify your answer.

12/ (a) Draw typical band structures of: (1) *a metal*, (2) *an insulator* and (3) *a semiconductor* (b) Explain why the metal resistance (R) increases with increasing temperature (T) while the semiconductor resistance has an opposite temperature variation?

13/ At room temperature, pure germanium has an almost completely filled valence band separated by a 0.67-eV gap from an almost completely empty conduction band. It is a poor electrical conductor, but its conductivity increases greatly when it is irradiated with electromagnetic waves of a certain maximum wavelength. What is that wavelength? To what spectral range it belongs? (the Planck constant  $h=6.626 * 10^{-34} J.s$ ).