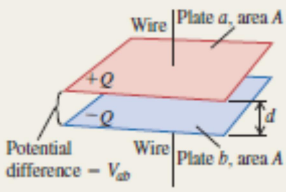
Capacitance and Dielectrics

Capacitors and capacitance: A capacitor is any pair of conductors separated by an insulating material. When the capacitor is charged, there are charges of equal magnitude Q and opposite sign on the two conductors, and the potential V_{ab} of the positively charged conductor with respect to the negatively charged conductor is proportional to Q. The capacitance C is defined as the ratio of Q to V_{ab}. The SI unit of capacitance is the farad (F): $1\ \text{F} = 1\ \text{C/V}$.

A parallel-plate capacitor consists of two parallel conducting plates, each with area A, separated by a distance d. If they are separated by vacuum, the capacitance depends only on A and d. For other geometries, the capacitance can be found by using the definition $C = Q/V_{ab}$. (See Examples 24.1–24.4.)

$$C = \frac{Q}{V_{ab}} \quad (24.1)$$

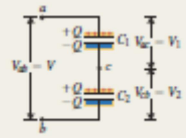
$$C = \frac{Q}{V_{ab}} = \epsilon_0 \frac{A}{d} \quad (24.2)$$

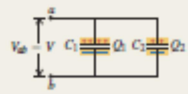


Capacitors in series and parallel: When capacitors with capacitances $C_1, C_2, C_3, \ldots$ are connected in series, the reciprocal of the equivalent capacitance C_{eq} equals the sum of the reciprocals of the individual capacitances. When capacitors are connected in parallel, the equivalent capacitance C_{eq} equals the sum of the individual capacitances. (See Examples 24.5 and 24.6.)

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots \quad (24.5)$$
(capacitors in series)

$$C_{eq} = C_1 + C_2 + C_3 + \cdots \quad (24.7)$$
(capacitors in parallel)

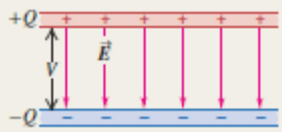




Energy in a capacitor: The energy U required to charge a capacitor C to a potential difference V and a charge Q is equal to the energy stored in the capacitor. This energy can be thought of as residing in the electric field between the conductors; the energy density u (energy per unit volume) is proportional to the square of the electric-field magnitude. (See Examples 24.7–24.9.)

$$U = \frac{Q^2}{2C} = \tfrac{1}{2}CV^2 = \tfrac{1}{2}QV \quad (24.9)$$

$$u = \tfrac{1}{2}\epsilon_0 E^2 \quad (24.11)$$



Dielectrics: When the space between the conductors is filled with a dielectric material, the capacitance increases by a factor K, called the dielectric constant of the material. The quantity $\epsilon = K\epsilon_0$ is called the permittivity of the dielectric. For a fixed amount of charge on the capacitor plates, induced charges on the surface of the dielectric decrease the electric field and potential difference between the plates by the same factor K. The surface charge results from polarization, a microscopic rearrangement of charge in the dielectric. (See Example 24.10.)

Under sufficiently strong fields, dielectrics become conductors, a situation called dielectric breakdown. The maximum field that a material can withstand without breakdown is called its dielectric strength.

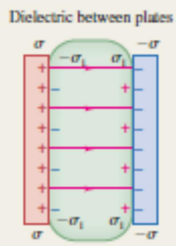
In a dielectric, the expression for the energy density is the same as in vacuum but with ϵ_0 replaced by $\epsilon = K\epsilon$. (See Example 24.11.)

Gauss's law in a dielectric has almost the same form as in vacuum, with two key differences: $\vec{E}$ is replaced by $K\vec{E}$ and Q_{encl} is replaced by $Q_{encl\text{-}free}$, which includes only the free charge (not bound charge) enclosed by the Gaussian surface. (See Example 24.12.)

$$C = KC_0 = K\epsilon_0 \frac{A}{d} = \epsilon \frac{A}{d}$$
(parallel-plate capacitor filled with dielectric) (24.19)

$$u = \tfrac{1}{2}K\epsilon_0 E^2 = \tfrac{1}{2}\epsilon E^2 \quad (24.20)$$

$$\oint K\vec{E} \cdot d\vec{A} = \frac{Q_{encl\text{-}free}}{\epsilon_0} \quad (24.23)$$



1/ Equation $C=\varepsilon_0 A/d$ shows that the capacitance of a parallel plate capacitor becomes larger as the plate separation decreases. However, there is a practical limit to how small can be made, which places limits on how large can be. Explain what sets the limit on (Hint: What happens to the magnitude of the electric field as $d \to 0$).

2/ Electrolytic capacitors use as their dielectric an extremely thin layer of nonconducting oxide between a metal plate and a conducting solution. Discuss the advantage of such a capacitor over one constructed using a solid dielectric between the metal plates.

3/ A conductor is an extreme case of a dielectric, since if an electric field is applied to a conductor, charges are free to move within the conductor to set up "induced charges." What is the dielectric constant of a perfect conductor? Is it $K=0$, $K \to \infty$ or something in between? Explain your reasoning.

4/ A parallel-plate vacuum capacitor with plate area *A* and separation *x* has charges $+Q$ and $-Q$ on its plates. The capacitor is disconnected from the source of charge, so the charge on each plate remains fixed. (a) What is the total energy stored in the capacitor? (b) The plates are pulled apart an additional distance *dx*. What is the change in the stored energy? (c) If *F* is the force with which the plates attract each other, then the change in the stored energy must equal the work $dW=Fdx$ done in pulling the plates apart. Find an expression for *F*. (d) Explain why F is not equal to *QE* where *E* is the electric field between the plates.

Solution

(a) $U = \dfrac{Q^2}{2C}$; $C = \dfrac{\epsilon_0 A}{x}$ so $U = \dfrac{xQ^2}{2\epsilon_0 A}$

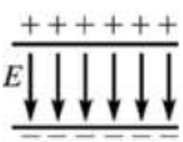
(b) $x \to x + dx$ gives $U = \dfrac{(x+dx)Q^2}{2\epsilon_0 A}$

$$dU = \frac{(x+dx)Q^2}{2\epsilon_0 A} - \frac{xQ^2}{2\epsilon_0 A} = \left(\frac{Q^2}{2\epsilon_0 A}\right)dx$$

(c) $dW = F\,dx = dU$, so $F = \dfrac{Q^2}{2\epsilon_0 A}$

(d)

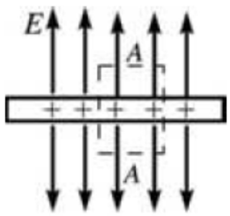
The capacitor plates and the field between the plates are shown in Figure



$$E = \frac{\sigma}{\epsilon_0} = \frac{Q}{\epsilon_0 A}$$

$$F = \tfrac{1}{2}QE, \text{ not } QE$$

The reason for the difference is that *E* is the field due to both plates. If we consider the positive plate only and calculate its electric field using Gauss's law



$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{encl}}}{\epsilon_0}$$

$$2EA = \frac{\sigma A}{\epsilon_0}$$

$$E = \frac{\sigma}{2\epsilon_0} = \frac{Q}{2\epsilon_0 A}$$

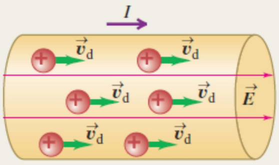
The force this field exerts on the other plate, that has charge $-Q$, is $F = \dfrac{Q^2}{2\epsilon_0 A}$.

Theory of metallic conduction

Current and current density: Current is the amount of charge flowing through a specified area, per unit time. The SI unit of current is the ampere ($1\ \text{A} = 1\ \text{C/s}$). The current I through an area A depends on the concentration n and charge q of the charge carriers, as well as on the magnitude of their drift velocity $\vec{v}_d$. The current density is current per unit cross-sectional area. Current is usually described in terms of a flow of positive charge, even when the charges are actually negative or of both signs. (See Example 25.1.)

$$I = \frac{dQ}{dt} = n|q|v_d A \qquad (25.2)$$

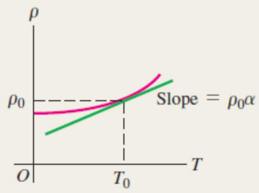
$$\vec{J} = nq\vec{v}_d \qquad (25.4)$$



Resistivity: The resistivity ρ of a material is the ratio of the magnitudes of electric field and current density. Good conductors have small resistivity; good insulators have large resistivity. Ohm's law, obeyed approximately by many materials, states that ρ is a constant independent of the value of E. Resistivity usually increases with temperature; for small temperature changes this variation is represented approximately by Eq. (25.6), where α is the temperature coefficient of resistivity.

$$\rho = \frac{E}{J} \qquad (25.5)$$

$$\rho(T) = \rho_0[1 + \alpha(T - T_0)] \qquad (25.6)$$

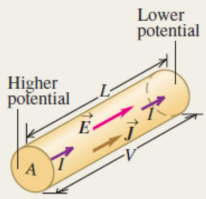


Metal: ρ increases with increasing T.

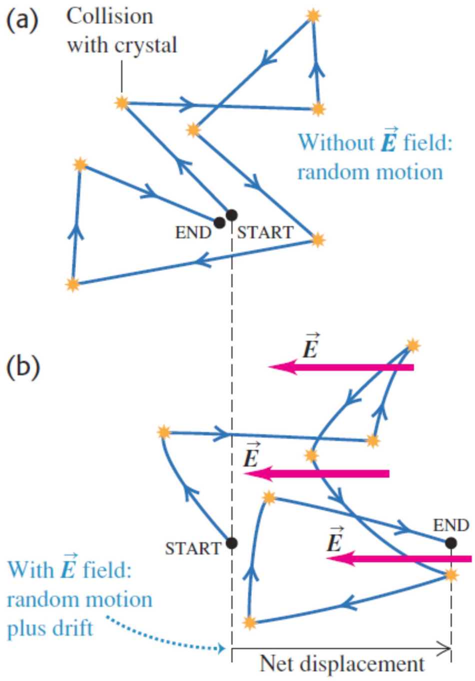
Resistors: The potential difference V across a sample of material that obeys Ohm's law is proportional to the current I through the sample. The ratio $V/I = R$ is the resistance of the sample. The SI unit of resistance is the ohm ($1\ \Omega = 1\ \text{V/A}$). The resistance of a cylindrical conductor is related to its resistivity ρ, length L, and cross-sectional area A. (See Examples 25.2 and 25.3.)

$$V = IR \qquad (25.11)$$

$$R = \frac{\rho L}{A} \qquad (25.10)$$

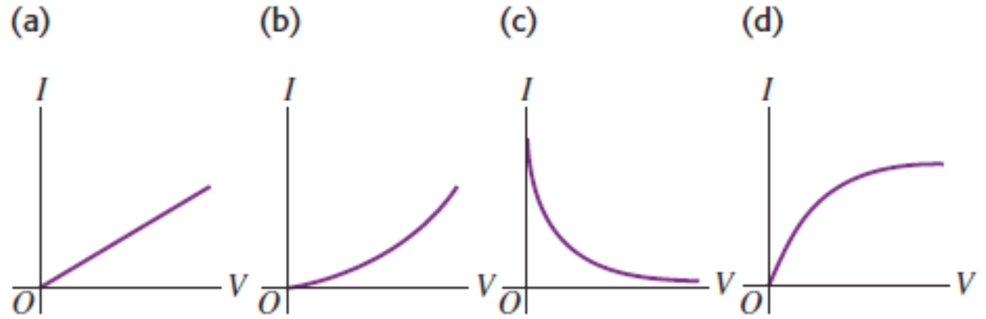


$$\vec{J} = nq\vec{v}_d = \frac{nq^2\tau}{m}\vec{E} \qquad \rho = \frac{m}{ne^2\tau}$$



Q1) Current causes the temperature of a real resistor to increase. Why? What effect does this heating have on the resistance? Explain.

Q2) Which of the graphs in Fig. below best illustrates the current I in a real resistor as a function of the potential difference V across it? Explain. (Hint: See Discussion from previous question.) Discuss I(V) curve of an incandescent bulb if $R_{hot} = 10xR_{cold}$.



Q3) Lightning Strikes. During lightning strikes from a cloud to the ground, currents as high as 25,000 A can occur and last for about 40μs. How much charge is transferred from the cloud to the earth during such a strike?

P1) A silver wire 2.6 mm in diameter transfers a charge of 420 C in 80 min. Silver contains 5.8 x 10^{28} free electrons per cubic meter. (a) What is the current in the wire? (b) What is the magnitude of the drift velocity of the electrons in the wire?

IDENTIFY: $I = Q/t$. Use $I = n|q|v_d A$ to calculate the drift velocity v_d.

SET UP: $n = 5.8 \times 10^{28}\ \text{m}^{-3}$. $|q| = 1.60 \times 10^{-19}\ \text{C}$.

EXECUTE: **(a)** $I = \frac{Q}{t} = \frac{420\ \text{C}}{80(60\ \text{s})} = 8.75 \times 10^{-2}\ \text{A}.$

(b) $I = n|q|v_d A$. This gives $v_d = \frac{I}{n|q|A} = \frac{8.75\times10^{-2}\ \text{A}}{(5.8\times10^{28})(1.60\times10^{-19}\ \text{C})(\pi(1.3\times10^{-3}\ \text{m})^2)} = 1.78\times10^{-6}\ \text{m/s}.$

P2)
(1) Calculate the mean free time between collisions in copper at room temperature. For Cu n=8.5 x10^{28} free electrons per cubic meter and ρ=1.72 x10^{-8} Ωm, e=1.6 x10^{-19}C and m=9.1 x10^{-31} kg for the electrons.

(2) Pure silicon contains approximately contains 1.6 x 10^{16} free electrons per cubic meter. (a) If ρ=2300 Ωm, calculate the mean free time for silicon at room temperature. (b) Your answer in part (a) is much greater than the mean free time for copper given in (a). Why, then, does pure silicon have such a high resistivity compared to copper?

Solution
(1)

$$\tau = \frac{m}{ne^2\rho}$$

$$= \frac{9.11 \times 10^{-31}\ \text{kg}}{(8.5 \times 10^{28}\ \text{m}^{-3})(1.60 \times 10^{-19}\ \text{C})^2(1.72 \times 10^{-8}\ \Omega \cdot \text{m})}$$

$$= 2.4 \times 10^{-14}\ \text{s}$$

The mean free time is the average time between collisions for a given electron. Taking the reciprocal of this time, we find that each electron averages: $1/\tau = 4.2 \times 10^{13}$ collisions per second!

(2) From:

$$\rho = \frac{m}{ne^2\tau}$$

SET UP: For silicon, $\rho = 2300\ \Omega \cdot \text{m}$.

EXECUTE: **(a)** $\tau = \dfrac{m}{ne^2\rho} = \dfrac{9.11\times10^{-31}\ \text{kg}}{(1.0\times10^{16}\ \text{m}^{-3})(1.60\times10^{-19}\ \text{C})^2(2300\ \Omega\cdot\text{m})} = 1.55\times10^{-12}\ \text{s}.$

EVALUATE: **(b)** The number of free electrons in copper $(8.5\times10^{28}\ \text{m}^{-3})$ is much larger than in pure silicon $(1.0\times10^{16}\ \text{m}^{-3})$. A smaller density of current carriers means a higher resistivity.