ELECTRIC POTENTIAL

Calculating electric field and electric potential using Gauss' law

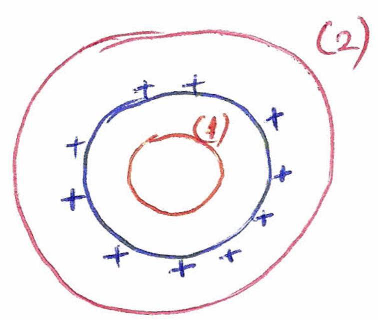
① Charged conducting sphere

Consider a conducting sphere of radius R having a total charge $+Q$ uniformly distributed over its surface. We denote by σ the surface charge density.

a) Using the Gauss's law, calculate $\vec{E}(\vec{r})$ for $r<R$ and $R>R$

b) Calculate the electrical potential for $r<R$ and $r>R$.

Solution:



a) Electric field

We consider a Gauss surface in each case $r<R$ (1) and $r>R$ (2) spherical of radius r

The Gauss's law writes: $$\oiint \vec{E}\cdot d\vec{A} = \frac{Q_{encl}}{\varepsilon_0}$$

1) $r<R$ $\quad Q_{encl}=0$

over the surface $4\pi R^2$ we have $E(r)=ct$

$$\Rightarrow E\oint dA = 0 \Rightarrow \boxed{E(r)=0} \quad r<R$$

inside the sphere

2) $r>R$ $\quad Q_{encl}=Q$

$$\Rightarrow E\cdot 4\pi r^2 = \frac{Q}{\varepsilon_0} \Rightarrow \boxed{E(r)=\frac{Q}{4\pi\varepsilon_0 r^2}} \quad r>R$$

Same as if a point charge is concentrated in $r=0$.

b) Electric potential

$$V(r) = \int_0^\infty \vec{E}(r)\, d\vec{r}$$

Inside $r < R$

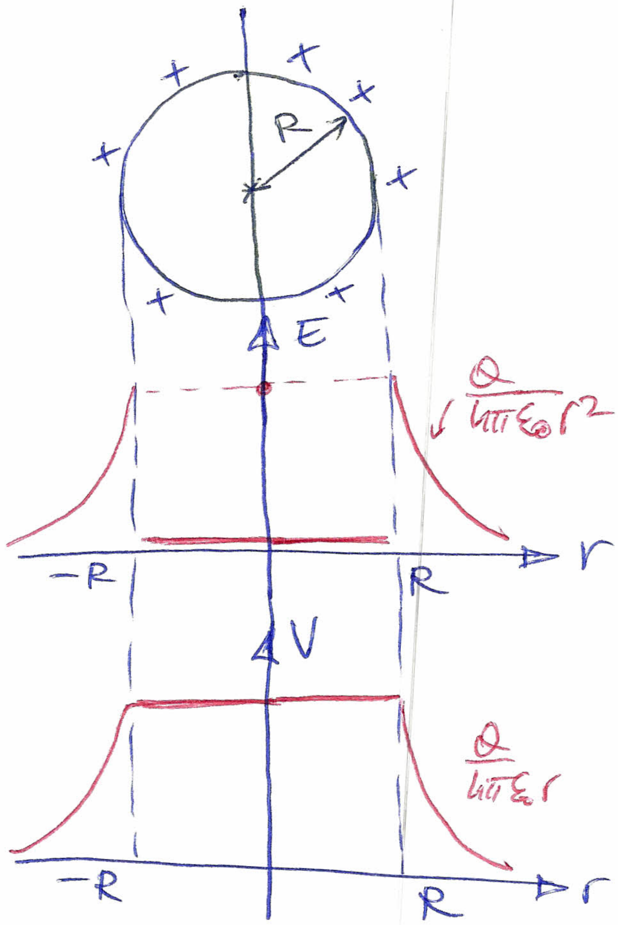
$$V(r) = \int_0^R \underset{0}{\underline{\vec{E}(r)}}\, d\vec{r} + \int_R^\infty \vec{E}(r)\, d\vec{r}$$

$$= \int_R^\infty \frac{Q}{4\pi\varepsilon_0} \frac{dr}{r^2} = \frac{Q}{4\pi\varepsilon_0 R} = \text{constant}$$

Outside $r > R$

$$V(r) = \int_r^\infty \vec{E}(r)\, d\vec{r} = \int_r^\infty \frac{Q\, dr}{4\pi\varepsilon_0 r^2}$$

$$\boxed{V(r) = \frac{Q}{4\pi\varepsilon_0} \frac{1}{R}}$$



zero field inside the metallic charged sphere

↕ Discuss the electrostatic screening effect!

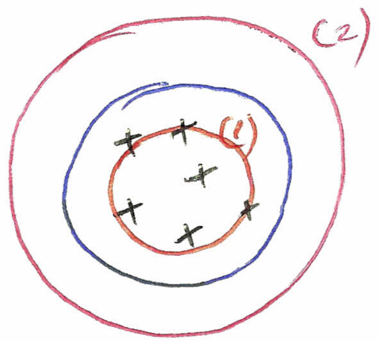
Constant potential inside the sphere

(H)

② Field and potential of uniformly charged dielectric sphere

Consider a dielectric sphere where the charge Q is uniformly distributed in its volume, the radius of the sphere is R. Calculate the electric field and the potential in arbitrary points, inside and outside the sphere.

Solution



To calculate the electric field we consider a Gaussian surface spherical of radius r in the two situations r<R and r>R.

Gauss law $\oiint \vec{E}\cdot d\vec{A} = \frac{Q_{encl}}{\varepsilon_0}$

for r<R

$$Q_{encl} = \frac{Q}{V}\cdot\frac{4\pi r^3}{3} = \frac{Q}{\frac{4\pi R^3}{3}}\cdot\frac{4\pi r^3}{3}$$

$$Q_{encl} = Q\frac{r^3}{R^3}$$

$$E\cdot 4\pi r^2 = \frac{Q}{\varepsilon_0}\cdot\frac{4\pi r^3}{R^3} \Rightarrow \boxed{E(r) = \frac{Q}{4\pi\varepsilon_0}\frac{r}{R^3}} \quad r<R$$

for r>R

$$Q_{encl} = Q \rightarrow E\cdot 4\pi r^2 = \frac{Q}{\varepsilon_0} \Rightarrow$$

$$\boxed{E(r) = \frac{Q}{4\pi\varepsilon_0 r^2}} \quad r>R$$

Potential

$$V(r) = \int_r^{\infty} \vec{E}(\vec{r})\, d\vec{r}$$

r<R

$$V(r) = \int_r^R E(r)\,dr + \int_R^{\infty} E(r)\,dr =$$

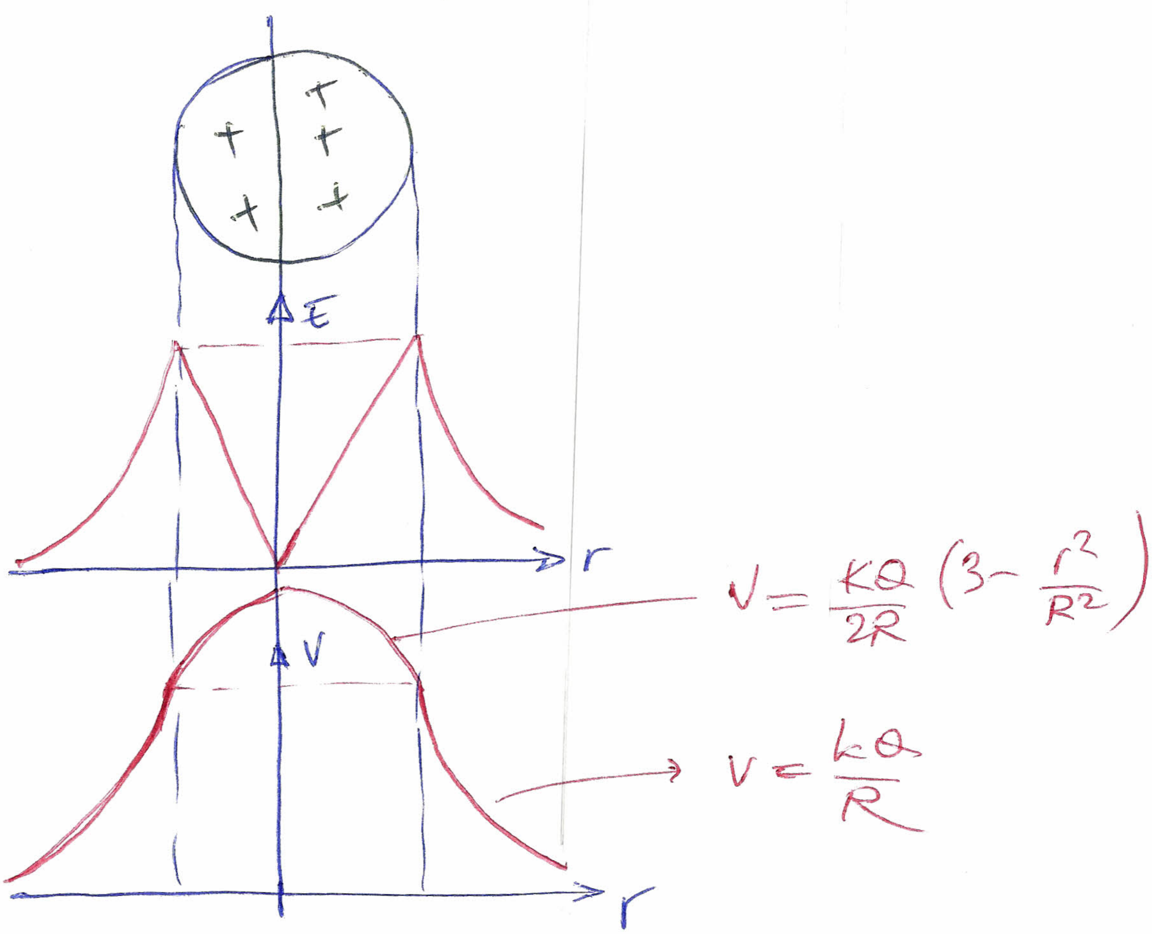
$$= \frac{Q}{4\pi\varepsilon_0}\left[\int_r^R \frac{r\,dr}{R^3} + \int_R^{\infty} \frac{dr}{r^2}\right] = \frac{Q}{4\pi\varepsilon_0}\left[\frac{R^2}{2R^3} - \frac{r^2}{2R^3} + \frac{1}{R}\right]$$

$$= \frac{Q}{4\pi\varepsilon_0}\left[\frac{3}{2R} - \frac{r^2}{2R^3}\right] = \frac{Q}{4\pi\varepsilon_0 R}\left[\frac{3}{2} - \frac{r^2}{2R^2}\right]$$

r>R

$$V(r) = \int_R^{\infty} E(r)\,dr = \frac{Q}{4\pi\varepsilon_0}\frac{1}{r}$$

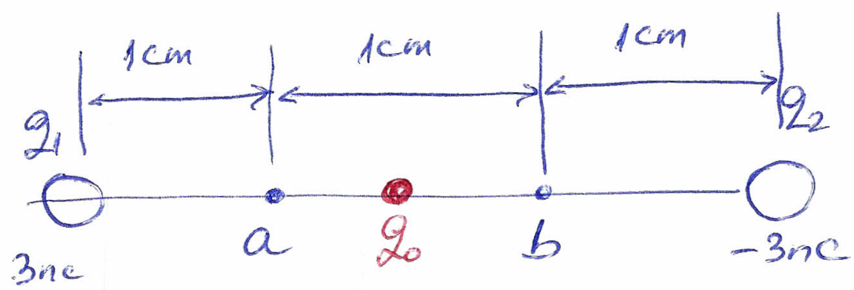
as in case of a point charge.



Moving a charge through a potential difference

③ A dust particle with mass $m = 5 \cdot 10^{-9}$ kg $= 5 \mu g$ and a charge $q_0 = 2nC$ starts from rest and moves in a straight line from point a to b. along the line between two charges $q_1 = 3nC$ and $q_2 = -3nC$. What is the speed v at point b?

Solution



Only conservative forces act on the particle, so total mechanical energy is conserved

$$K_a + U_a = K_b + U_b$$

K_a: kinetic energy = 0 (particle in rest in a)

U_a: potential energy = $q_0 V_a$

$K_b = \frac{m v_b^2}{2}$

$U_b = q_0 V_b$

$$\Rightarrow \quad 0 + q_0 V_a = \frac{1}{2} m v_b^2 + q_0 V_b \quad \Rightarrow$$

$$\boxed{v = \sqrt{\frac{2 q_0}{m}(V_a - V_b)}}$$

but $V(r) = \sum V_i(r)$ algebraic

$$V_i(r) = \frac{q_i}{4\pi\varepsilon_0 r} = \frac{k q_i}{r}$$

$$k = 9 \cdot 10^9 \frac{Nm^2}{C^2}$$

$$V_a = 9\cdot10^9\left(\frac{3\cdot10^{-9}}{0.01} - \frac{3\cdot10^{-9}}{0.02}\right) = 1350\ V$$

$$V_b = 9\cdot10^9\left(\frac{3\cdot10^{-9}}{0.02} - \frac{3\cdot10^{-9}}{0.01}\right) = -1350\ V$$

$$V_a - V_b = 1350 + 1350 = 2700\ V$$

$$\Rightarrow v = \sqrt{\frac{2(2\cdot10^{-9})\,2700}{5\cdot10^{-9}}} = 46\ m/s$$

(H)

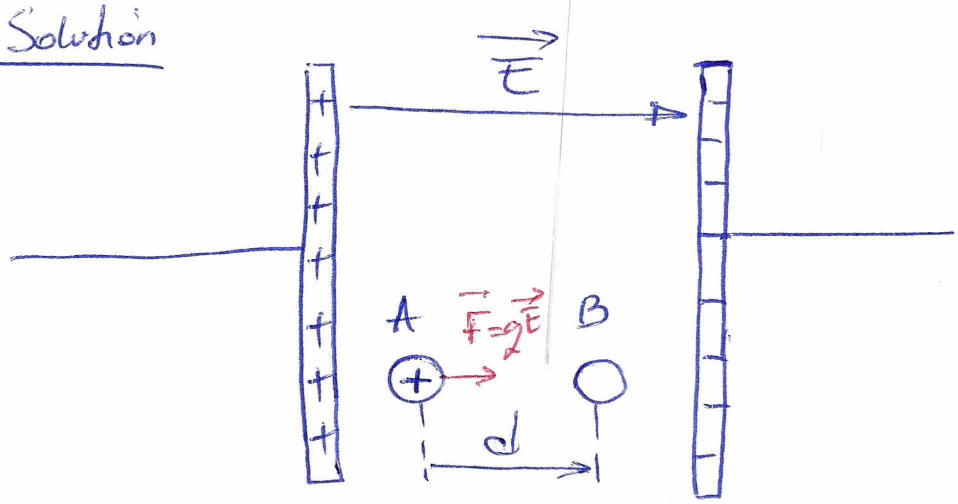
(4) A proton is left from rest in an uniform electric field $E = 8\cdot10^4\ \frac{V}{m}$. It will move on a distance $d = 0{,}5\ m$ in the direction of E

a) Calculate the potential difference between A and B (cf. fig)

b) Calculate the variation of the potential energy for the system field-proton.

c) Which is the speed of the proton in B after travelling $d = 0{,}5\ m$ in the field?

Solution



a)

$$W = -\Delta U$$ (work = − variation of potential energy)

$$Fd = -\Delta U$$

$$qEd = -\Delta U \Rightarrow Ed = -\frac{\Delta U}{q} = -\Delta V$$

$$\Rightarrow \Delta V = -Ed = -8\cdot 10^4 \cdot 0{,}5 = \underline{-4\cdot 10^4\,V} < 0$$

charge +e moves to lower V_B from larger V_A

b)

$$U_B - U_A = -qEd$$

$$= q\Delta V = 1{,}6\cdot 10^{-19}(-4\cdot 10^4)$$

$$= -6{,}4\cdot 10^{-15}\,J$$

↑ "−" sign signifies that the potential energy lowered

$$U_B < U_A$$

c) Because the total energy in the conservative force field is constant ⇒

$$K_A + U_A = K_B + U_B$$

$K_A = 0$ (rest), $U_A = qV_A$

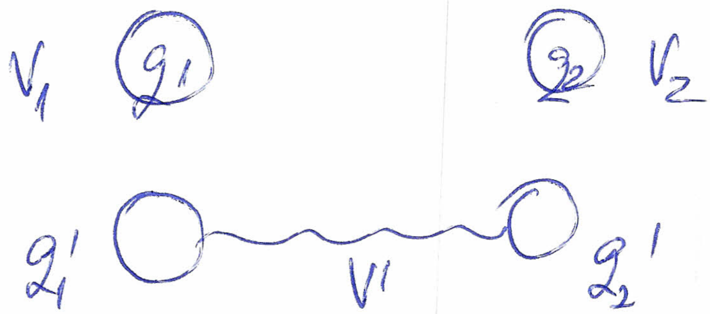
$$qV_A = \frac{mv^2}{2} + qV_B \Rightarrow$$

$$v_B = \sqrt{\frac{2q(V_A - V_B)}{m}} = \sqrt{\frac{2Edq}{m}}$$

$$v_B = \sqrt{\frac{2\cdot 1.6\cdot 10^{-19}}{1.66\cdot 10^{-27}}\,4\cdot 10^4} \simeq \underline{2{,}8\cdot 10^6\,m/s}$$

(5) Two metallic spheres of radius $R_1 = 1$ cm and $R_2 = 20$ cm have the potentials $V_1 = 9000$ V and $V_2 = 900$ V are situated in air at large distance. Which would be the potential V' of the system obtained by connecting the two spheres via a very thin conducting wire?

Solution:



When connecting the spheres:

(1) the potential is equalized $V_1' = V_2' = V'$

(2) the total charge is conserved

$$\boxed{q_1 + q_2 = q_1' + q_2'}$$

$$V_1 = \frac{q_1}{4\pi\varepsilon_0 R_1} \Rightarrow q_1 = 4\pi\varepsilon_0 R_1 V_1$$

$$V_2 = \frac{q_2}{4\pi\varepsilon_0 R_2} \Rightarrow q_2 = 4\pi\varepsilon_0 R_2 V_2$$

(1) $q_1' + q_2' = 4\pi\varepsilon_0 (R_1 V_1 + R_2 V_2)$ and

(2) $V' = \dfrac{q_1'}{4\pi\varepsilon_0 R_1} = \dfrac{q_2'}{4\pi\varepsilon_0 R_2} \Rightarrow \dfrac{q_1'}{R_1} = \dfrac{q_2'}{R_2}$

(1) and (2) system with 2 eq, 2 unknowns.

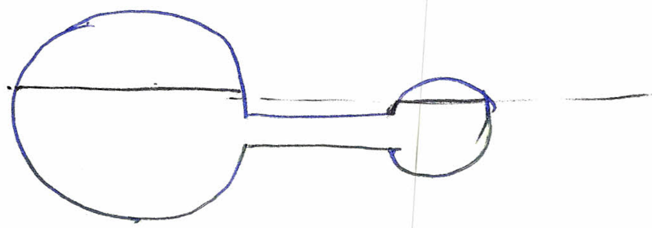
$$\begin{cases} q_1' + q_2' = 4\pi\varepsilon_0 (R_1 V_1 + R_2 V_2) \\ \dfrac{q_1'}{q_2'} = \dfrac{R_1}{R_2} \quad \Rightarrow \quad q_1' = q_2' \dfrac{R_1}{R_2} \end{cases}$$

$$\Rightarrow q_2' = \frac{4\pi\varepsilon_0 (R_1 V_1 + R_2 V_2)}{\frac{R_1}{R_2} + 1}$$

$$\Rightarrow V' = \frac{1}{4\pi\varepsilon_0 R_2} \frac{4\pi\varepsilon_0 (R_1 V_1 + R_2 V_2)}{\frac{R_1 + R_2}{R_2}} \Rightarrow$$

$$\boxed{V' = \frac{R_1 V_1 + R_2 V_2}{R_1 + R_2}}$$

Obs: The redistribution of charge to get same potential when connecting by wire is likewise the principles of communicating vessels



same level of fluid regardless of shape and volumes of containers.

⑥ (H) Two identical conducting spheres are spaced by 0,3m being charged with $q_1 = 12\,nC$ and $q_2 = -18\,nC$.

a) calculate the force between the two spheres

b) what is the new force, if the two spheres get connected by a thin metallic conductor

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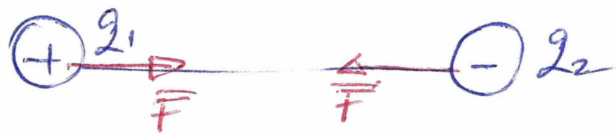
a) Before



attraction force

$$F = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2} = 9\cdot10^9\,\frac{|q_1||q_2|}{r^2} =$$

$$= 9\cdot10^9 \cdot \frac{12\cdot10^{-9}\;18\cdot10^{-9}}{0{,}3^2} = \underline{2{,}16\cdot10^{-5}\,N}$$

b) After:

Connexion ⇒ equalisation of potential $V_1' = V_2' = V'$

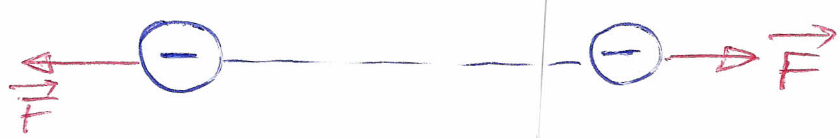
$$\frac{q_1'}{4\pi\varepsilon_0 r_1} = \frac{q_2'}{4\pi\varepsilon_0 r_2} \qquad r_1 = r_2 \Rightarrow \underline{q_1' = q_2'}$$

charge conservation:

$$q_1 + q_2 = q_1' + q_2' = 2q_1' \Rightarrow$$

$$q_1' = \frac{q_1 + q_2}{2} \qquad q_1' = \frac{(12-18)\,10^{-9}}{2} = -3\cdot10^{-9}\,C \;<0$$

⇒



repulsive force

$$F = 9\cdot10^9 \cdot \frac{3\cdot10^{-9}\cdot3\cdot10^{-9}}{0{,}3^2} = \underline{9\cdot10^{-7}\,N}$$

<u>Homework Questions/Problems</u>

1/ A student asked, “Since electrical potential is always proportional to potential energy, why bother with the concept of potential at all?” How would you respond?

2/ It is easy to produce a potential difference of several thousand volts between your body and the floor by scuffing your shoes across a nylon carpet. When you touch a metal doorknob, you get a mild shock. Yet contact with a power line of comparable voltage would probably be fatal. Why is there a difference?

3/ A high-voltage dc power line falls on a car, so the entire metal body of the car is at a potential of 10000V with respect to the ground. What happens to the occupants (a) when they are sitting in the car and (b) when they step out of the car? Explain your reasoning.

4/ The electric field inside a hollow, uniformly charged sphere is zero. Does this imply that the potential is zero inside the sphere?

5/ If a negative charge is initially at rest in an electric field, will it move toward a region of higher potential or lower potential? What about a positive charge? How does the potential energy of the charge change in each instance?

6/ A charge of –4.0 μC is initially 40 cm from a fixed charge of –6.0 μC and is then moved to a position 90 cm from the fixed charge. a) What is the change in the mutual potential energy of the charges? b) Does the change depend on the path through which one charge is moved (justify the answer)?

7/ How much work is required to bring two charges of +3.0 μC and +5.0 μC that are separated by an infinite distance to a distance of 0.50 m?

8/ (a) Compute the energy necessary to bring the charges together into the configuration shown in figure 1? (b) What is the electric potential at the center of the triangle in figure 1?

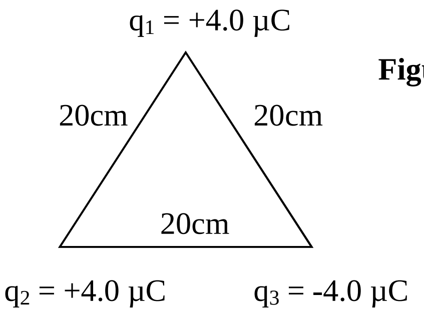


Figure 1

9/ Two electrons that are initially rest 10 cm apart are released. What will be their speed when they are 20 cm apart?

10/ In a television picture tube electron are accelerated from rest through potential difference of 10 kV in an electron gun. (a) What is the muzzle velocity of the electrons emerging from the gun? (b) If the gun is directed at a screen 35 cm away, how long does it take the electron to reach the screen?