Interference

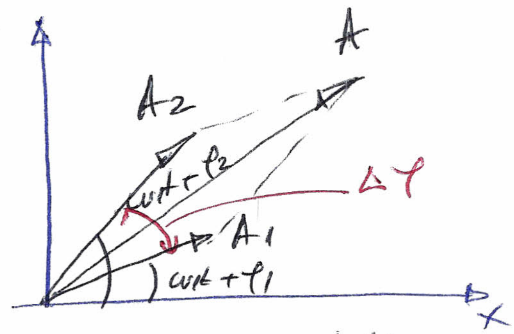
Goals

→ what happens when two waves combine or interfere in space

→ understand the interference pattern of two coherent waves.

Theory (brief): for more details see course

$$y_1 = A_1 \sin(\omega_1 t + \varphi_1)$$
$$y_2 = A_2 \sin(\omega_2 t + \varphi_2)$$
$$y = y_1 + y_2 = A \sin(\omega t + \varphi)$$



Phasor representation

$$\Delta\varphi = (\omega_2 - \omega_1)t + (\varphi_2 - \varphi_1)$$

$$A^2 = A_1^2 + A_2^2 + 2A_1A_2\cos\Delta\varphi$$

$\langle A^2 \rangle \propto$ wave intensity I

$$\langle A^2 \rangle = A_1^2 + A_2^2 + 2A_1A_2 \frac{1}{T}\int_0^T \cos[(\omega_2 - \omega_1)t + (\varphi_2 - \varphi_1)]\,dt$$

the integral $\neq 0$ if $\omega_2 = \omega_1$

and $\varphi_2 - \varphi_1$ = constant in time

$\Leftrightarrow$ coherent waves

In this case:

$$\langle A^2 \rangle = A_1^2 + A_2^2 + 2A_1A_2\cos(\varphi_2 - \varphi_1)$$

$\Delta\varphi = 2n\pi \quad n = 0,1,2,\dots \Rightarrow A^2 = \max = (A_1 + A_2)^2 = I_{max}$

$\Delta\varphi = (2n+1)\pi \quad n = 0,1,\dots \Rightarrow A^2 = \min = (A_1 - A_2)^2 = I_{min}$

In terms of λ $\quad \varphi = \omega t - Kx$

$$\Delta\varphi = K\Delta x = \frac{2\pi}{\lambda}\Delta x$$

$\Rightarrow$ Condition for maximum of interference:

$$K\Delta x = 2n\pi \qquad \frac{2\pi}{\lambda}\Delta x = 2n\pi \Rightarrow$$

$$\boxed{\Delta x = n\lambda} \qquad n = 1, 2, \ldots$$

constructive interference

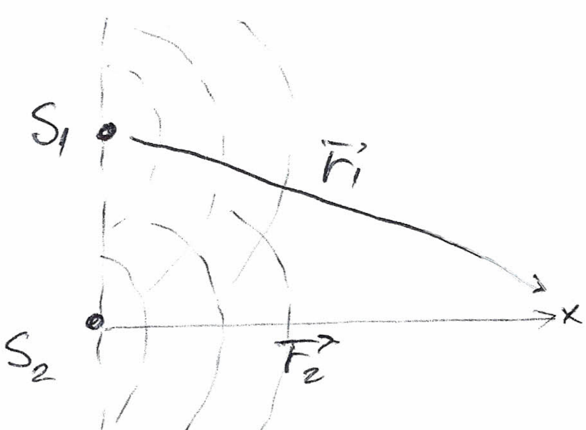
condition for minimum of interference:

$$K\Delta x = (2n+1)\pi \qquad \Delta x\frac{2\pi}{\lambda} = (2n+1)\pi$$

$$\boxed{\Delta x = \left(n+\tfrac{1}{2}\right)\lambda} \qquad n = 1, 2, \ldots$$

destructive interference

General formulation



waves arrive in phase

$$\boxed{r_2 - r_1 = n\lambda}$$

constructive interference $\Rightarrow I_{max}$

wave arrive out of phase

$$\boxed{r_2 - r_1 = \left(n+\tfrac{1}{2}\right)\lambda}$$

destructive interference $\Rightarrow I_{min}$

$$n = 0, \pm 1, \pm 2, \pm 3, \ldots$$

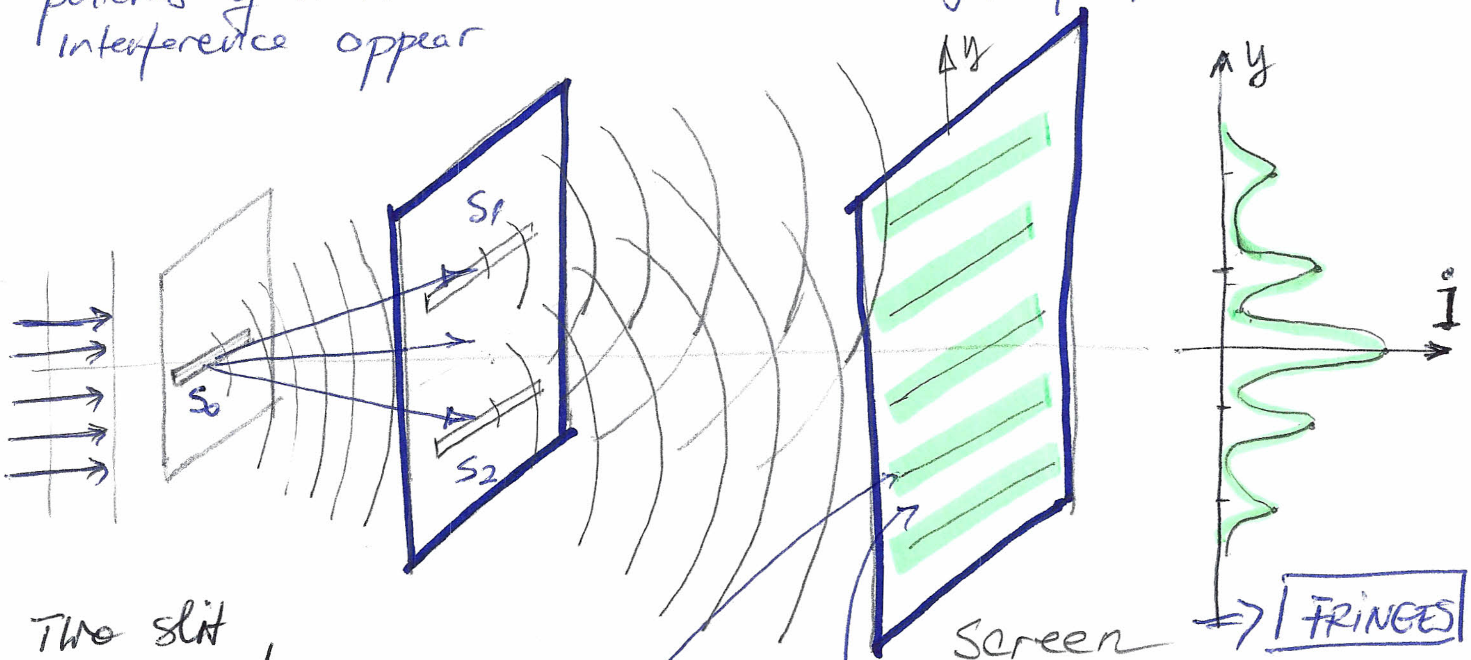
If the amplitude of two interfering oscillations are equal $A_1 = A_2 \Rightarrow$

$$I_{max} = (A_1 + A_2)^2 = 4A_0^2$$

$$I_{min} = (A_1 - A_2)^2 = 0$$

Young experiment

Performed in 1800 by Thomas Young. Using two light sources he demonstrated the wave nature of light, interference patterns of maximum and minimum intensity, specific to wave interference appear

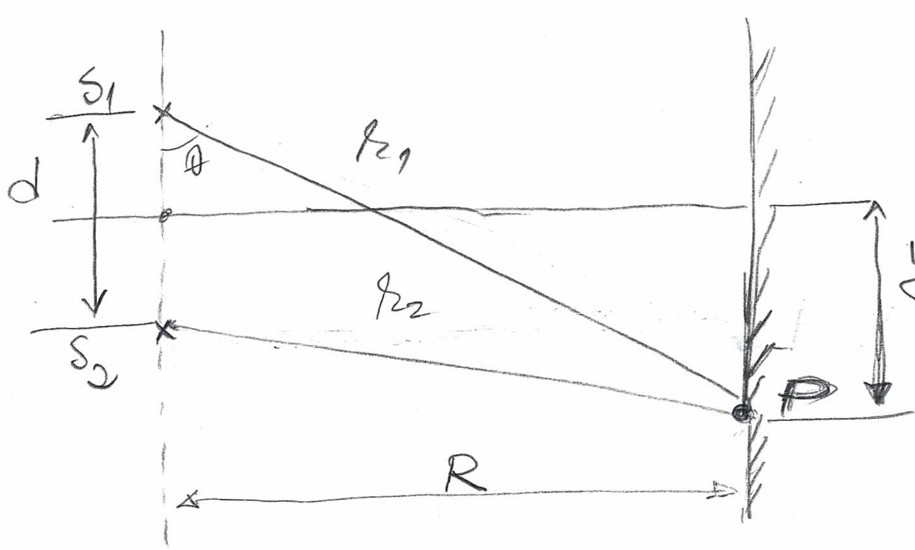


Two slit experiment

bright bands where waves arrive in phase and interfere constructivelly

dark bands where waves arrive out of phase and interfere destructivelly

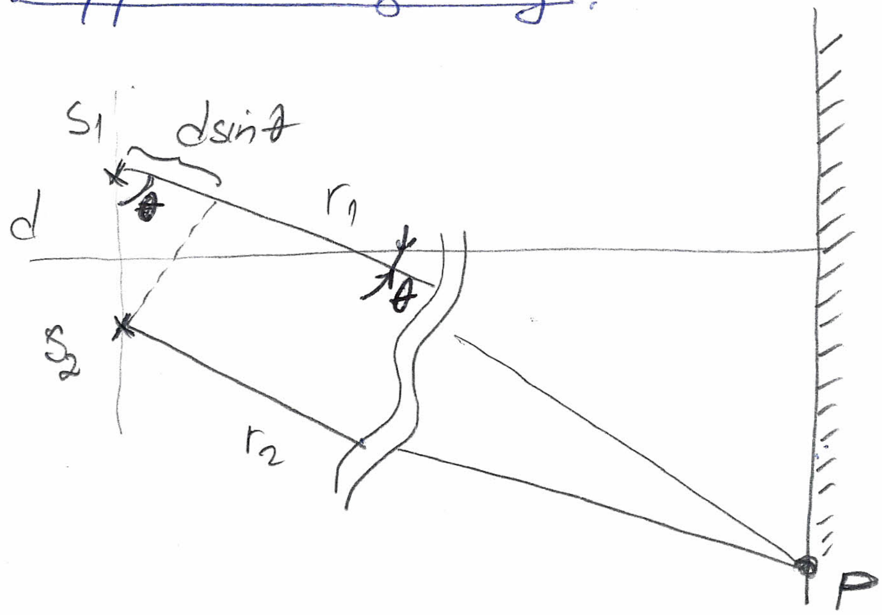
Geometry (seen from side)



$$\Delta r = r_2 - r_1$$

Obs: In real situations, the distance R to the screen is much larger than the distance d between the two slits S_1 and S_2 $\quad R \gg d \Rightarrow$

Approximate geometry:



$R \gg d$

We can threat the waves as parallel, in which case the path lenght difference in P is:

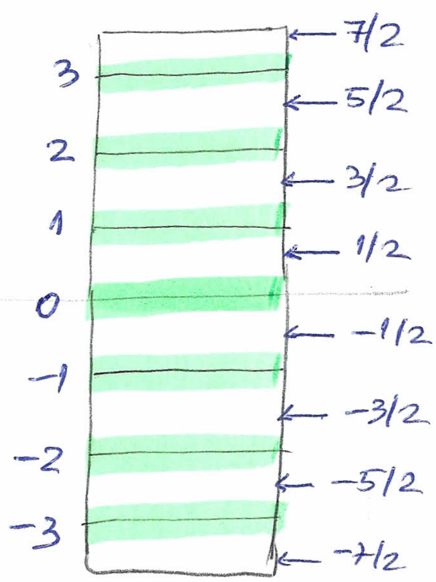
$$\boxed{r_2 - r_1 = d \sin\theta}$$

Constructive and destructive interference:

Constructive: $d \sin\theta = n\lambda$ $\quad n = 0, \pm 1, \pm 2, \ldots$ $\quad I_{max}$

destructive $d \sin\theta = \left(n + \frac{1}{2}\right)\lambda$ $\quad n = 0, \pm 1, \pm 2 \ldots$ $\quad I_{min} = 0$

Thus, the pattern on the screen is a succession of bright and dark bands or interference FRINGES. parallel to the slits S_1 and S_2. The center of pattern is a bright band corresponding to $n = 0$



The position of the center of a bright band (nth) to the screen:

$$y_n = R \tan\theta_m$$

but if $R \gg d$ also $R \gg y_n$ and in this case

$$\tan\theta \simeq \sin\theta = \frac{\Delta r}{d} = \frac{n\lambda}{d}$$

$$\Longrightarrow \boxed{y_n = R\, n \frac{\lambda}{d}}$$

valid for small angles only

We can measure R = the distance sources — screen
d = the distance between slits S_1 and S_2

by measuring the y_n = position of the n-th maximum one can calculate the wavelength λ

$$\boxed{\lambda = \frac{y_n\, d}{n\, R}}$$

Obs: ① The Young experiment was the first direct measurement of wavelengths of light.

② The distance between adjacent band = interfringe is inversly proportional to the distance d between slits. The closer together the slits are, the more patterns spread out. When slits are far-apart, the band pattern are closer together

$$\boxed{i = y_n - y_{n-1} = \frac{\lambda R}{d}}$$

(3) These results are valid for any type of waves. In the situation when $R \gg d$, the above formulas describe mathematically the positions of interference maxima.

Problem: Double-Slit Experiment

The geometry of the double-slit interference is shown in the Figure 14.2.3.

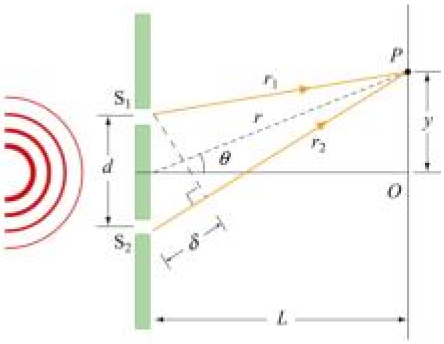


Figure 14.2.3 Double-slit experiment

Suppose in the double-slit arrangement, 0.150 mm, *d* = 120cm, λ = 833nm,L = and y= 2.00 cm

(a) What is the path difference δ for the rays from the two slits arriving at point *P*?

(b) Express this path difference in terms of λ.

(c) Does point *P* correspond to a maximum, a minimum, or an intermediate condition?

Solutions:

(a) The path difference is given by $\delta = d\sin\theta$. When $L \gg y$, θ is small and we can make the approximation $\sin\theta \approx \tan\theta = y/L$. Thus,

$$\delta \approx d\left(\frac{y}{L}\right) = \left(1.50\times10^{-4}\text{ m}\right)\frac{2.00\times10^{-2}\text{ m}}{1.20\text{ m}} = 2.50\times10^{-6}\text{ m}$$

(b) From the answer in part (a), we have

$$\frac{\delta}{\lambda} = \frac{2.50\times10^{-6}\text{ m}}{8.33\times10^{-7}\text{ m}} \approx 3.00$$

or $\delta = 3.00\lambda$.

(c) Since the path difference is an integer multiple of the wavelength, the intensity at point *P* is a maximum.