Seminary 3 and 4
Work, energy, momentum and conservation laws

SEMINARY 3

The unsolved problems are given as homework.

1/ Work, Kinetic, Potential, Total Energy. Conservation laws

DISCUSSION: Briefly remember useful notions from the course: definitions of Kinetic, Potential energy, Work-Energy theorem, conservation laws.

1) A force acting on a body varies with the position following the law: **F(x) = 2x+1** N. Calculate the work of this force when moving the body between the positions x_1=1m and x_2=3m.

2) If it takes total work *W* to give an object a speed *v* and kinetic energy *K*, starting from rest, what will be the object's speed (in terms of *v*) and kinetic energy (in terms of K) if we do twice as much work on it, again starting from rest?

3) A force in the +x-direction with magnitude ***F(x)=18N-(0.530 N/m)x*** is applied to a 6.00-kg box that is sitting on the horizontal, frictionless surface of a frozen lake. F(x) is the only horizontal force on the box. If the box is initially at rest at x=0, what is its speed after it has traveled 14.0 m?

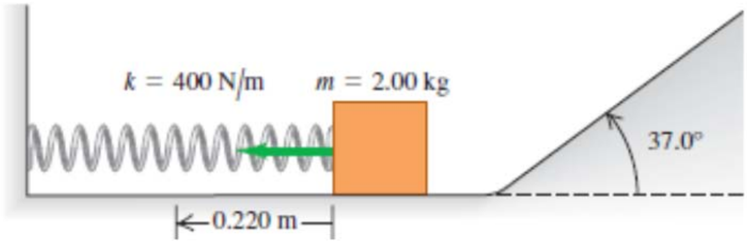
4) An object is attracted toward the origin with a force given by $F_x=-k/x^2$ (Gravitational and electrical forces have this distance dependence.) Calculate the work done by the force when the object moves in the x-direction from x_1 to x_2. If $x_2>x_1$ is the work done by F_x positive or negative?

5) A baseball is thrown straight up with initial speed v_0. (a) First, if air resistance is ignored, using energy conservation, calculate the maximum height and explain why, when the ball returns to its initial height, its speed would be equal to v_0. (b) If air resistance cannot be ignored, when the ball returns to its initial height its speed is less than v_0. Explain why, using energy concepts.

6) An object is released from rest at the top of a ramp. If the ramp is frictionless, does the object's speed at the bottom of the ramp depend on the shape of the ramp or just on its height? Explain. What if the ramp is not frictionless?

7) An ideal spring of negligible mass is 12.00 cm long when nothing is attached to it. When you hang a 3.15-kg weight from it, you measure its length to be 13.40 cm. If you wanted to store 10.0 J of potential energy in this spring, what would be its total length?

8) A 2.00-kg block is pushed against a spring with negligible mass and force constant k=400N/m compressing it 0.220 m. When the block is released, it moves along a frictionless, horizontal surface and then up a frictionless incline with slope 37^{o} (Fig.). (a) What is the speed of the block as it slides along the horizontal surface after having left the spring? (b) How far does the block travel up the incline before starting to slide back down?



9) A slingshot consists of a light leather cup, containing a stone that is pulled back against 2 rubber bands. It takes a force of 30 N to stretch the bands 1.0 cm (a) What is the potential energy stored in the bands when a 50.0 g stone is placed in the cup and pulled back 0.20 m from the equilibrium position? (b) With what speed does it leave the slingshot?

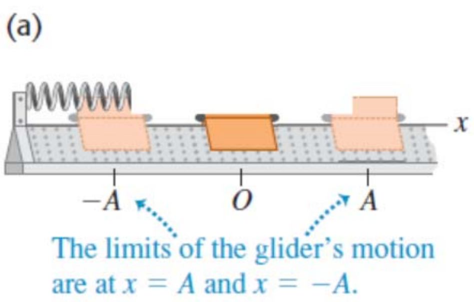
SEMINARY 4

2/ Force-potential energy and Energy diagrams

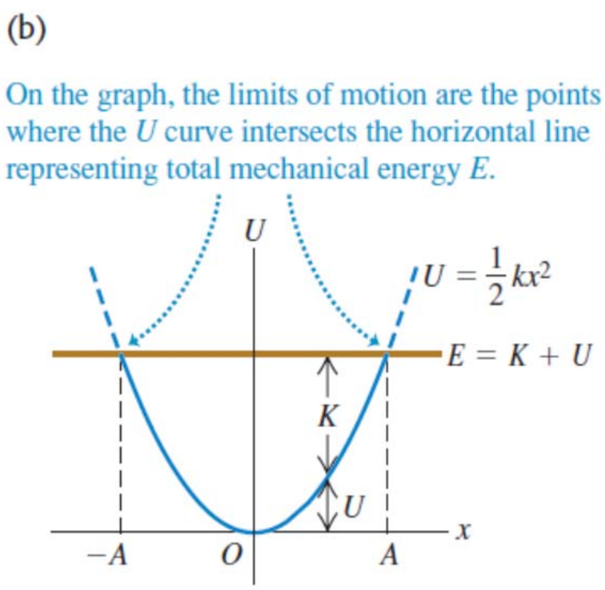
DISCUSSION: Briefly remember useful notions from the course: force-potential energy formula (1D and 3D): $F(x) = -\dfrac{dU(x)}{dx}$ and $\vec{F}(\vec{r}) = -grad(U(x,y,z)) = -\nabla U$

Energy diagrams

When a particle moves along a straight line under the action of a conservative force, we can get a lot of insight into its possible motions by looking at the graph of the potential-energy function *U(x)*. Figure below (a) shows a glider with mass m that moves along the x-axis on an air track. The spring exerts on the glider a force with x-component $F(x) = -kx$. Figure (b) is a graph of the corresponding potential-energy function $U(x)=(1/2)kx^2$. If the elastic force of the spring is the only horizontal force acting on the glider, the total mechanical energy $E=K+U$ is constant, independent of x. A graph of E as a function of x is thus a straight horizontal line. We use the term energy diagram for a graph like this, which shows both the potential-energy function *U(x)* and the energy of the particle subjected to the force that corresponds to *U(x)*.



(a) *A glider on an air track. The spring exerts a force Fx = -kx.*



(b) *The potential-energy function.*

The vertical distance between the U and E graphs at each point represents the difference E-U equal to the kinetic energy K at that point. We see that K is greatest at $x=0$. It is zero at the values of x where the two graphs cross, labeled ***A*** and ***–A*** in the diagram. Thus the speed v is greatest at and it is zero at $\boldsymbol{x=+/-A}$, the points of maximum possible displacement from $x=0$ for a given value of the total energy E. The potential energy U can never be greater than the total energy E; if it were, K would be negative, and that's impossible. The motion is a back-and-forth oscillation between the points $x=A$ and $x= - A$.

At each point, the force F_x on the glider is equal to the negative of the slope of the $U(x)$ curve: $F_x= - dU/dx$. When the particle is at $x=0$, the slope and the force are zero, so this is an equilibrium position. When x is positive, the slope of the $U(x)$ curve is positive and the force is negative, directed toward the origin. When x is negative, the slope is negative and F_x is positive, again directed toward the origin. Such a force is called a restoring force; when the glider is displaced to either side of the force tends to "restore" it back to $x=0$. An analogous situation is a marble rolling around in a round-bottomed bowl. We say that $x=0$ is a point of stable equilibrium. More generally, *any minimum in a potential-energy curve is a stable equilibrium position*.

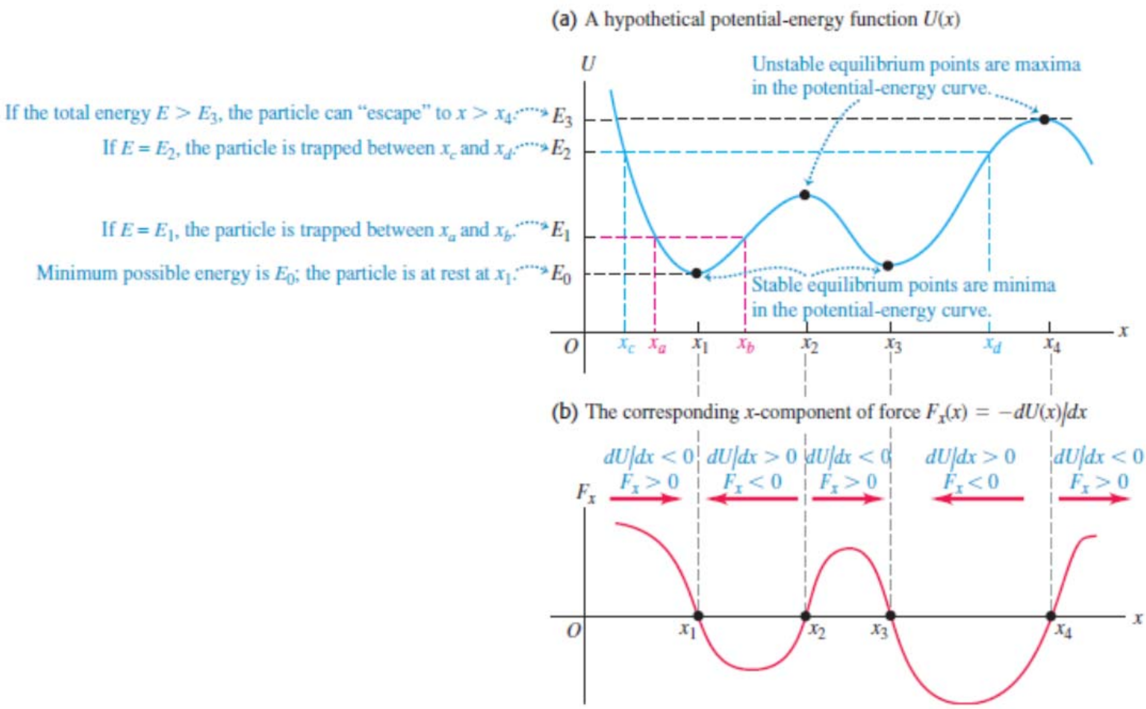


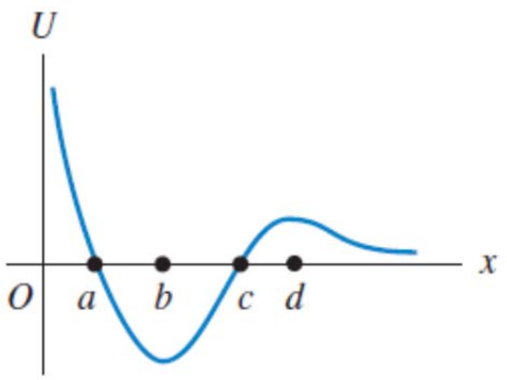
***Fig.** Hypothetical more general potential-energy function U(x). The maxima and minima of a potential-energy function U(x) correspond to points where Fx = 0.*

Problems

1) The potential energy of a pair of hydrogen atoms separated by a large distance x is given by $\boldsymbol{U(x)=C/x^6}$ where C is a positive constant. What is the force that one atom exerts on the other? Is this force attractive or repulsive?

2) A small block with mass 0.0400 kg is moving in the xy-plane. The net force on the block is described by the potential energy $\boldsymbol{U(x,y)= (5.8\ J/m^2)x2-(3.6\ J/m^3)y^3}$ function . What are the magnitude and direction of the acceleration of the block when it is at the point, x=0.3 and y=0.3m?

3) A particle moves along the x-axis. The potential-energy function is shown in Fig. below. (a) At which of the labeled x-coordinates is the force on the marble zero? (b) Which of the labeled x-coordinates is a position of stable equilibrium? (c) Which of the labeled x-coordinates is a position of unstable equilibrium? (d) Graphically, qualitatively represent the F(x) deduced from U(x).



4) A particle is in *neutral equilibrium* if the net force on it is zero and remains zero if the particle is displaced slightly in any direction. Sketch the potential-energy function near a point of neutral equilibrium for the case of one-dimensional motion. Give an example of an object in neutral equilibrium.

5) The potential-energy function for a force $\vec{F}$ is $U=\alpha x^3$ where α is a positive constant. What is the direction of $\vec{F}$?

(II) Momentum Impulse and Collisions

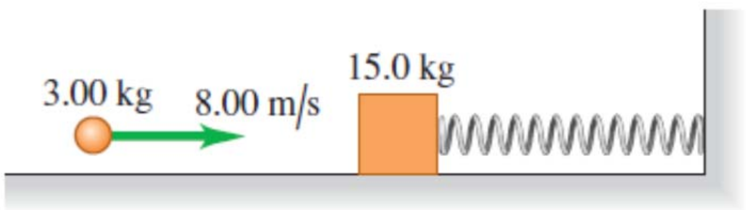
Briefly remember useful notions from the course: definitions and momentum conservation law.

1) In splitting logs with a hammer and wedge, is a heavy hammer more effective than a lighter hammer? Why?

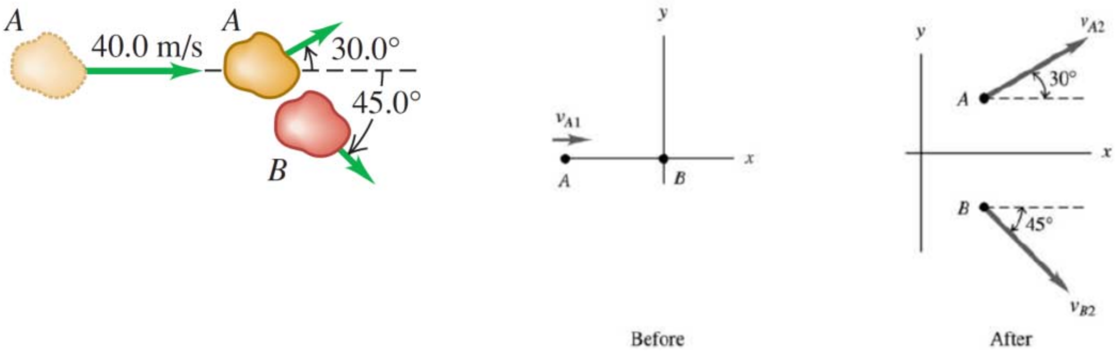
2) What is the magnitude of the momentum of a 10,000-kg truck whose speed is 60km/h (b) What speed would a 2000-kg SUV have to attain in order to have (i) the same momentum? (ii) the same kinetic energy?

3) A 68.5-kg astronaut is doing a repair in space on the orbiting space station. She throws a 2.25-kg tool away from her at relative to the space station. With what speed and in what direction will she begin to move?

4) Combining Conservation Laws. A 15.0-kg block is attached to a very light horizontal spring of force constant 500N/m and is resting on a frictionless horizontal table. (Fig.). Suddenly it is struck by a 3.00-kg stone traveling horizontally at 8m/s to the right, whereupon the stone rebounds at 2m/s horizontally to the left. Find the maximum distance that the block will compress the spring after the collision.



5) Asteroid Collision Two asteroids of equal mass in the asteroid belt between Mars and Jupiter collide with a glancing blow. Asteroid A, which was initially traveling at 40.0m/s is deflected 30.0° from its original direction, while asteroid B, which was initially at rest, travels at 45.0° to the original direction of *A* (Fig. 2). (a) Find the speed of each asteroid after the collision. (b) What fraction of the original kinetic energy of asteroid *A* dissipates during this collision?



Questions with short answer

1) What do we mean when we say a quantity is "conserved"?

2) What is the difference between a conservative and non-conservative force?

3) What is the difference between an elastic and an inelastic collision?

4) How do momentum and kinetic energy differ as velocity increases?

5) In an inelastic collision, when does the maximum loss of kinetic energy in the system occur?

6) Explain the work-energy theorem.

7) Say you apply a force F to an object for a time t and move that object a distance d. How much momentum does the object have now? How much kinetic energy does it have?

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9) How is the conservation of momentum related to Newton's third law?